[Towards] Coupled-channels analysis of N^* in finite box and Lattice QCD

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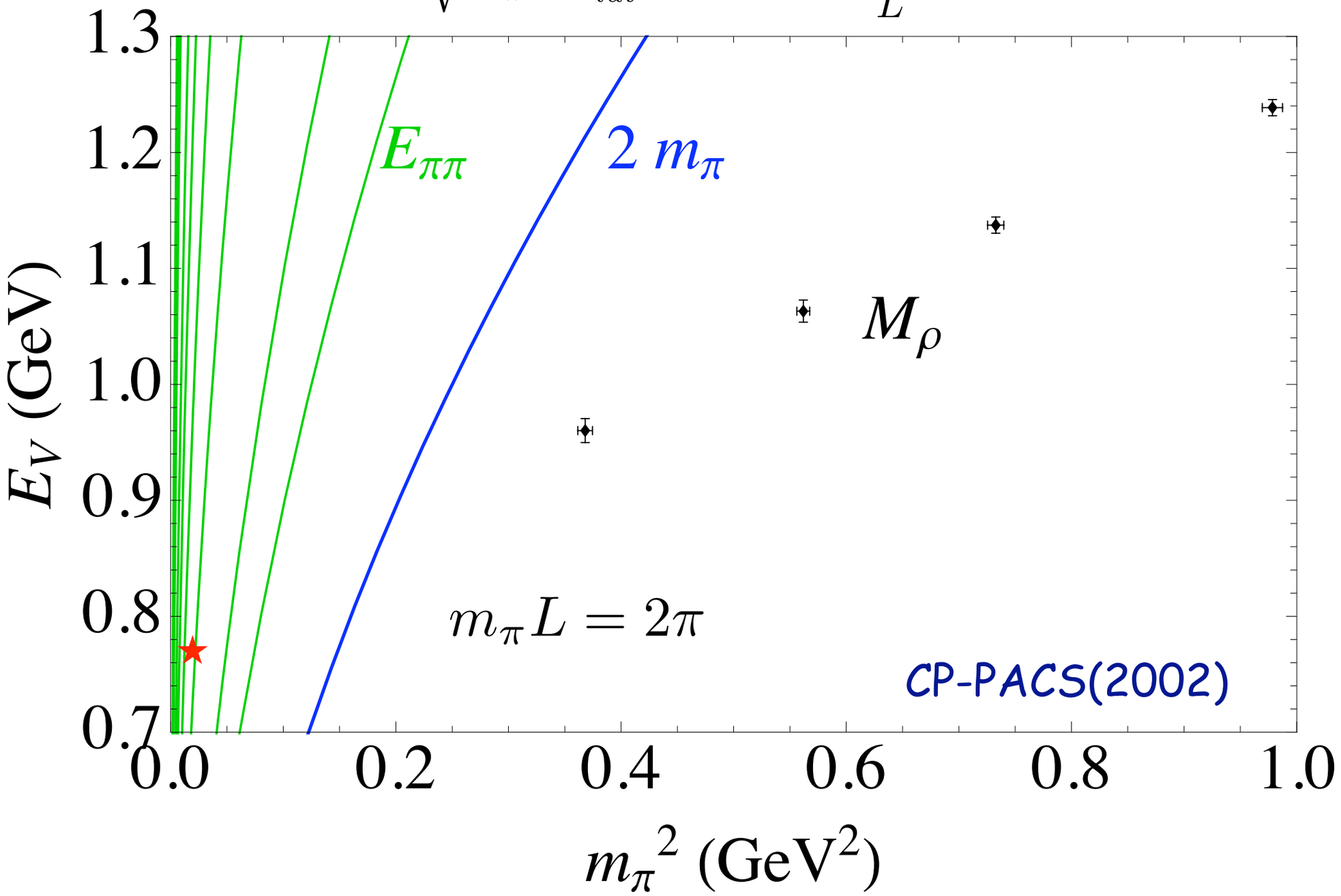
Resonance at large quark masses

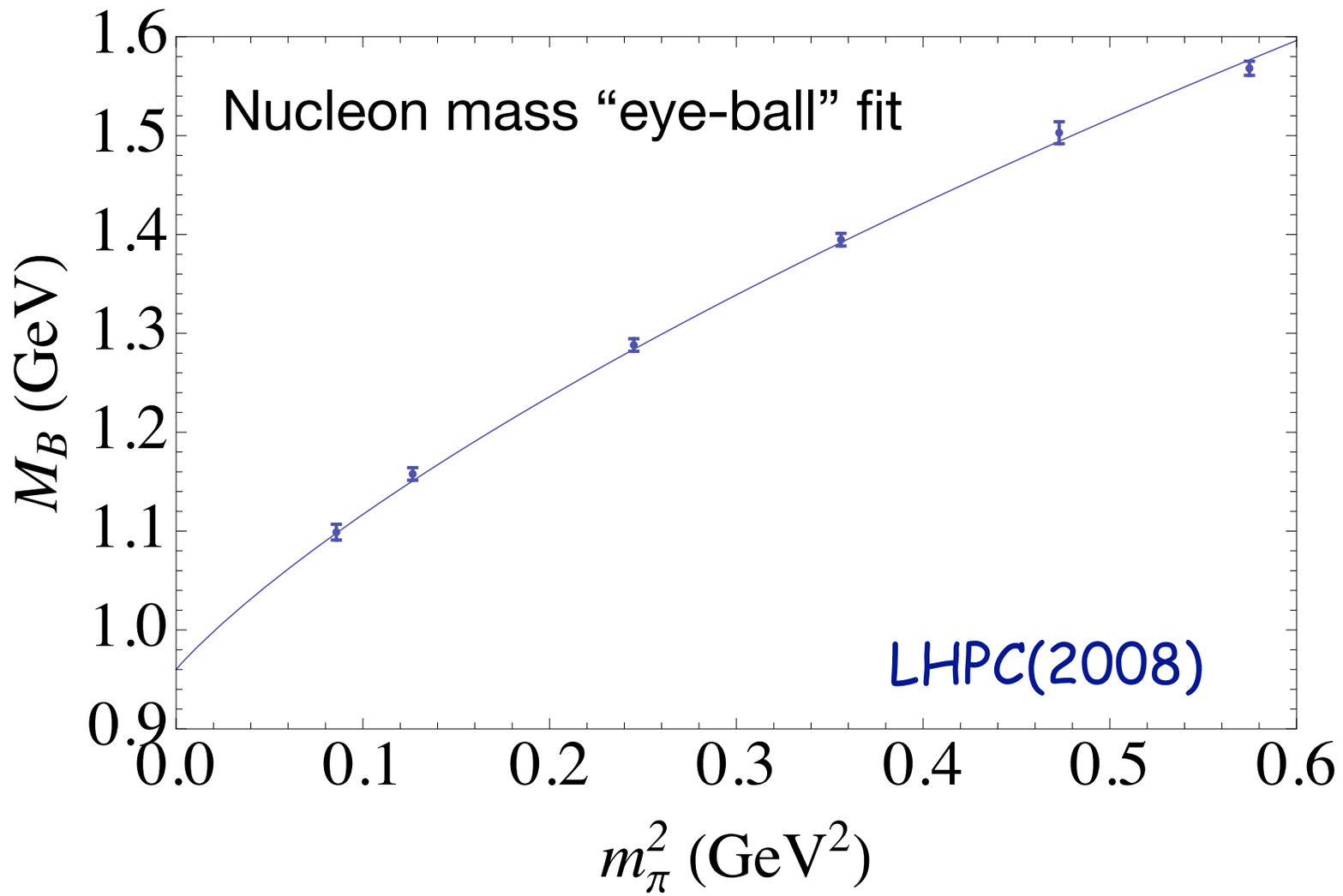
$$E_{\pi\pi} = 2\sqrt{m_\pi^2 + \vec{p}_{lat}^{\,2}} \qquad \vec{p}_{lat} = \frac{2\pi}{L}\vec{n}$$



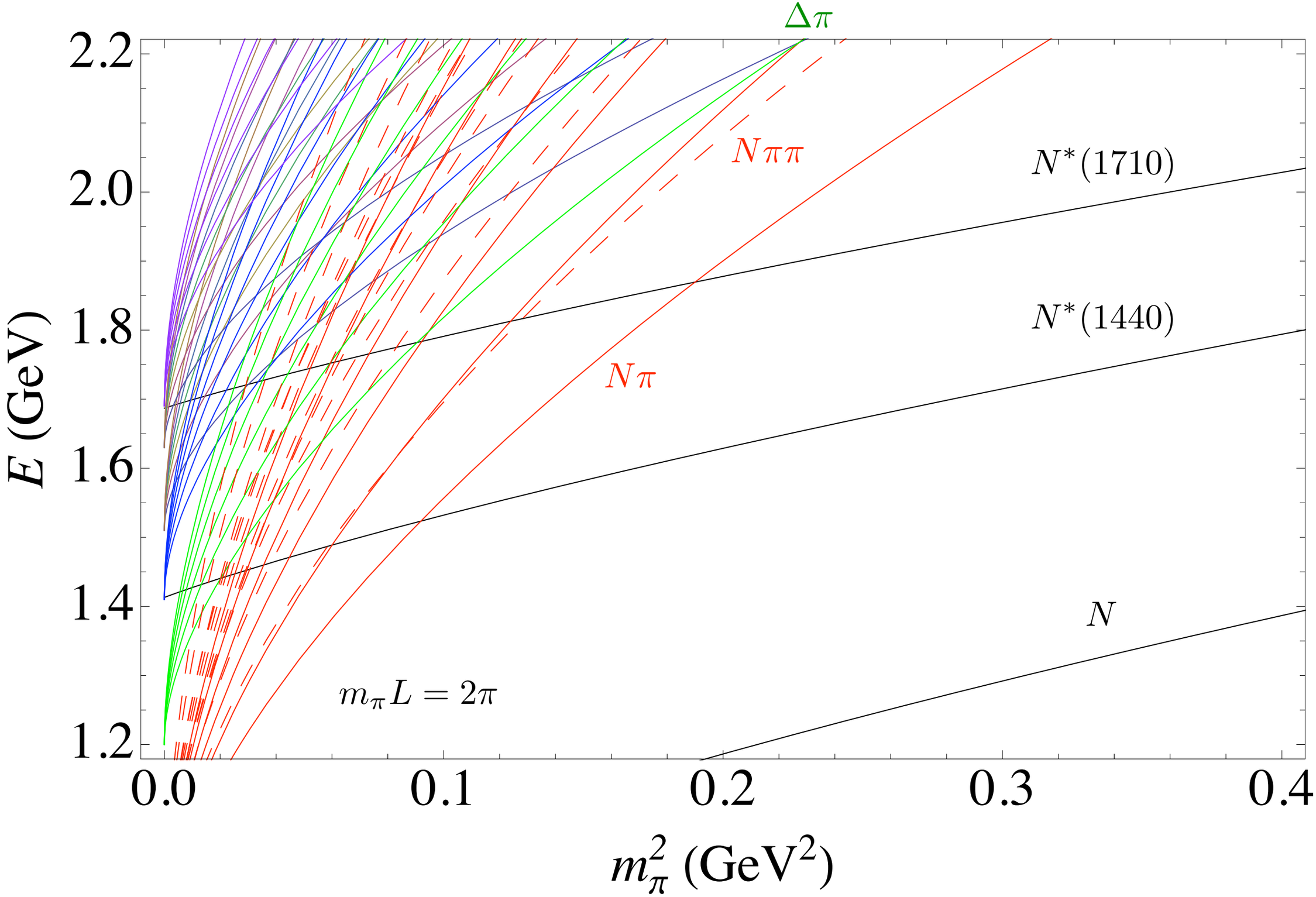
Simple parameterisation

- First “guess” at baryons; use simple (one parameter) form

$$M_B = M_B^0 \left(\frac{1}{1 + \frac{3}{2}\frac{m_\pi}{M_B^0}} + \frac{3}{2}\frac{m_\pi}{M_B^0} \right)$$

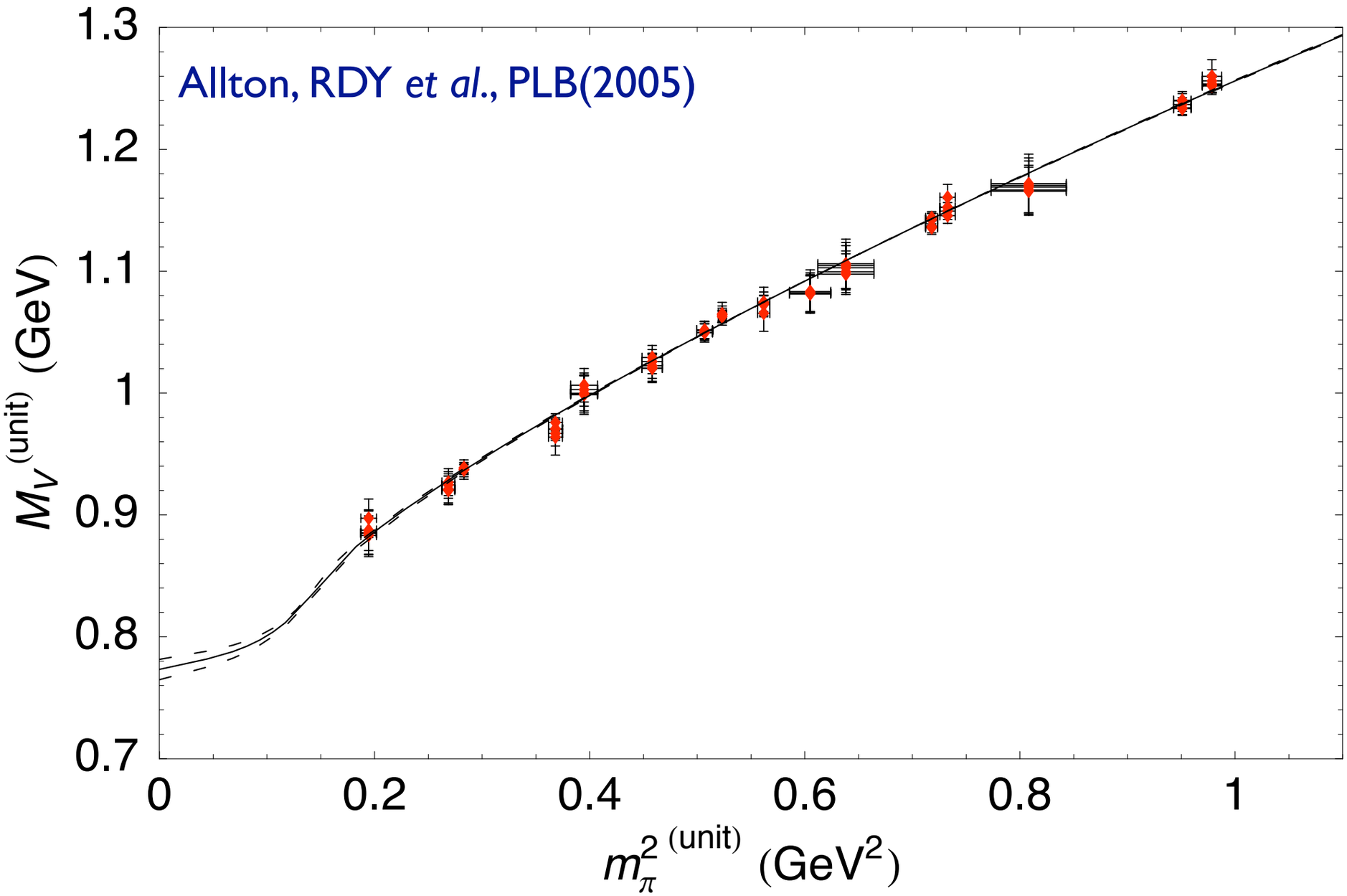


Non-interacting energies



Interactions: much more interesting

- Real part of rho meson mass



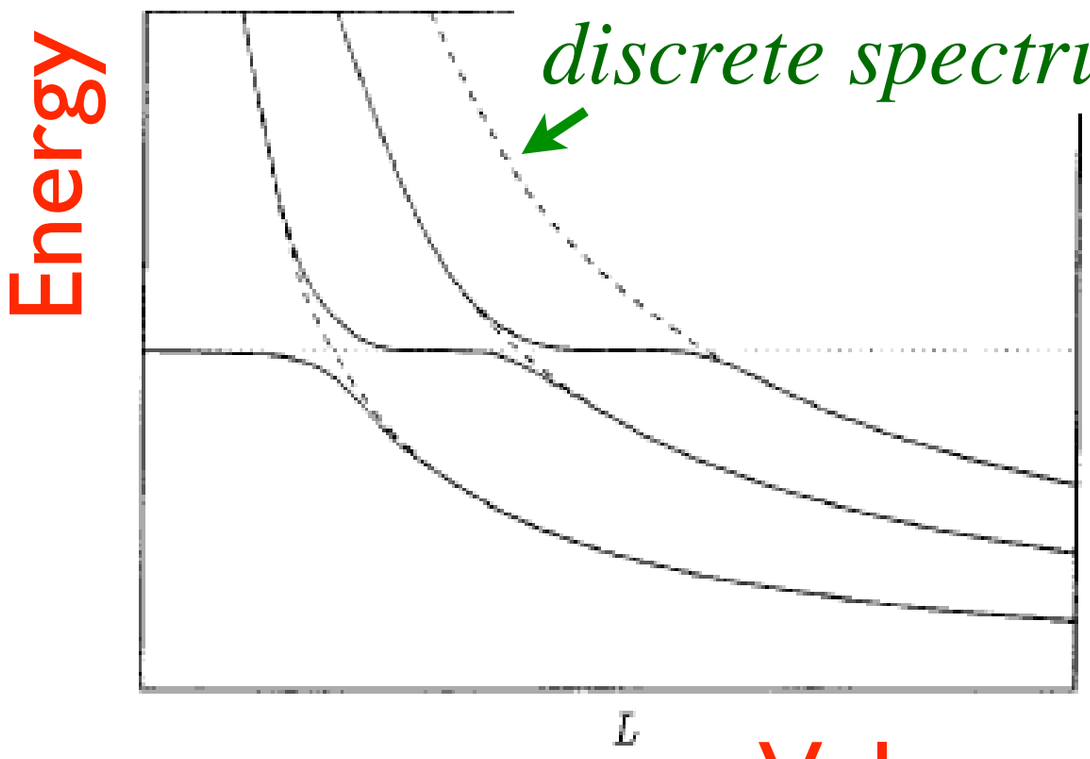
Lüscher

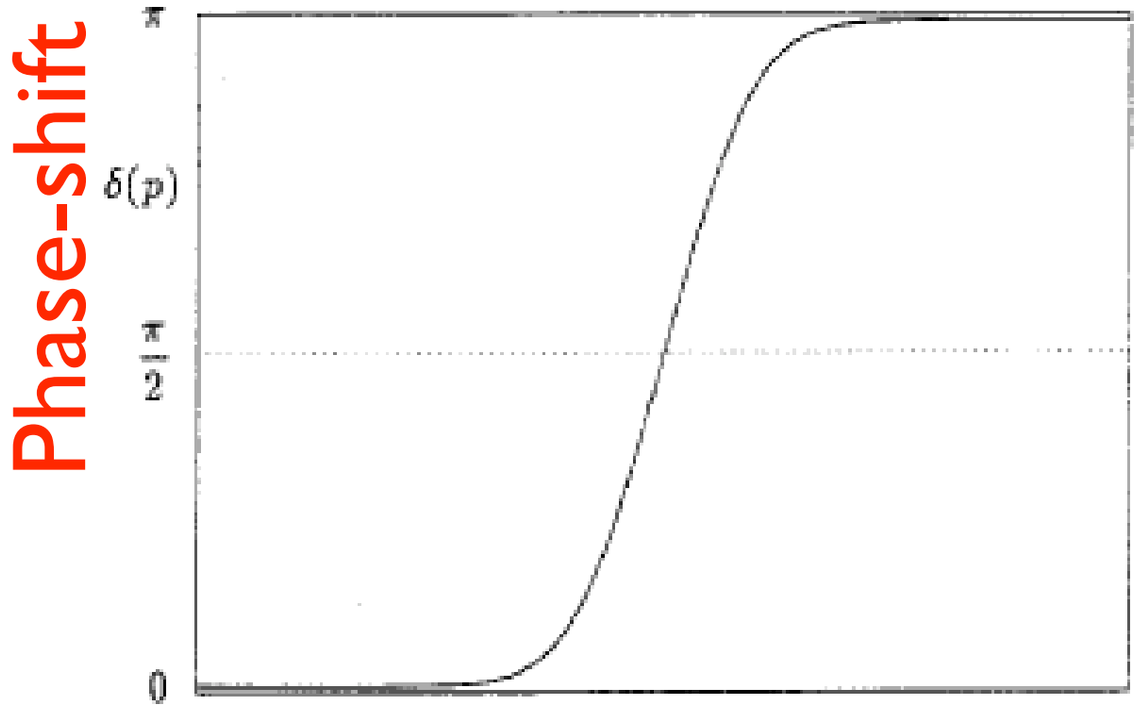
Lüscher NPB(1991)

Map out volume-dependence of energy levels

Wiese [Lat88]

"free" 2-particle discrete spectrum





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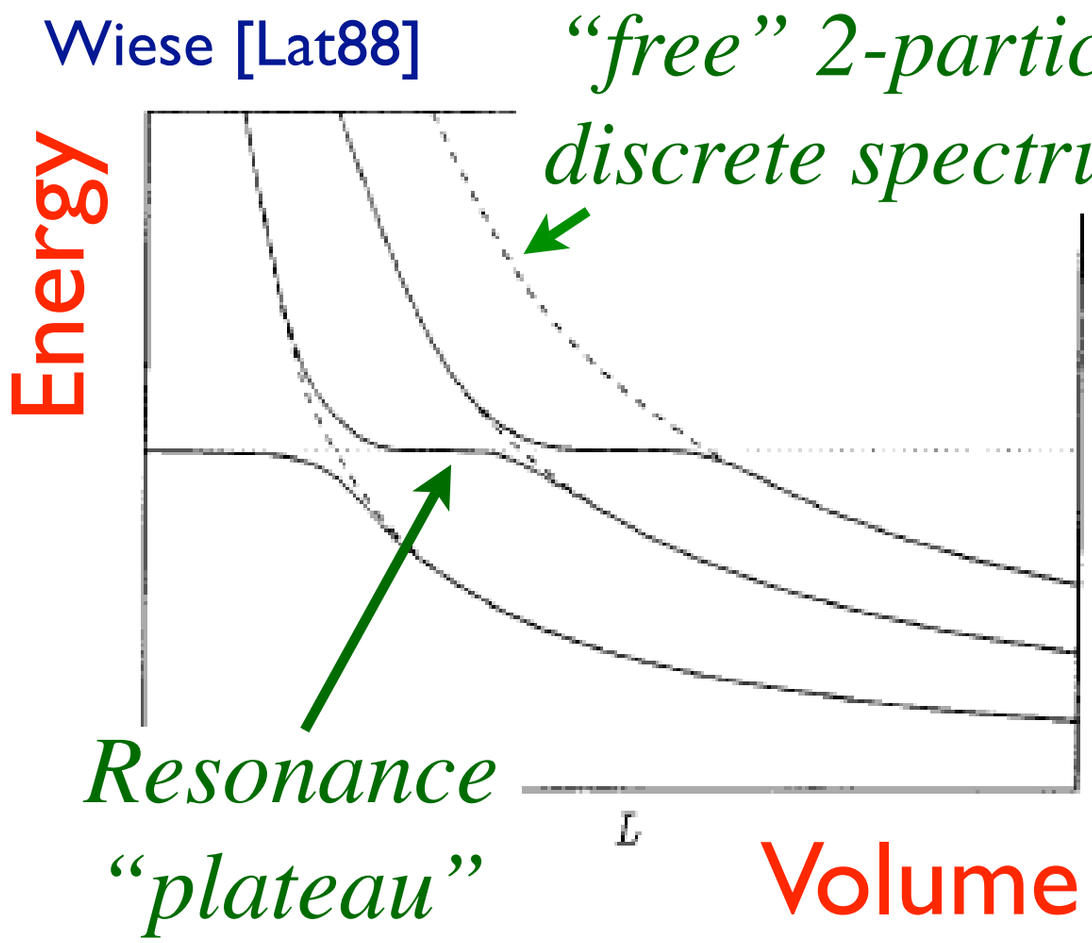
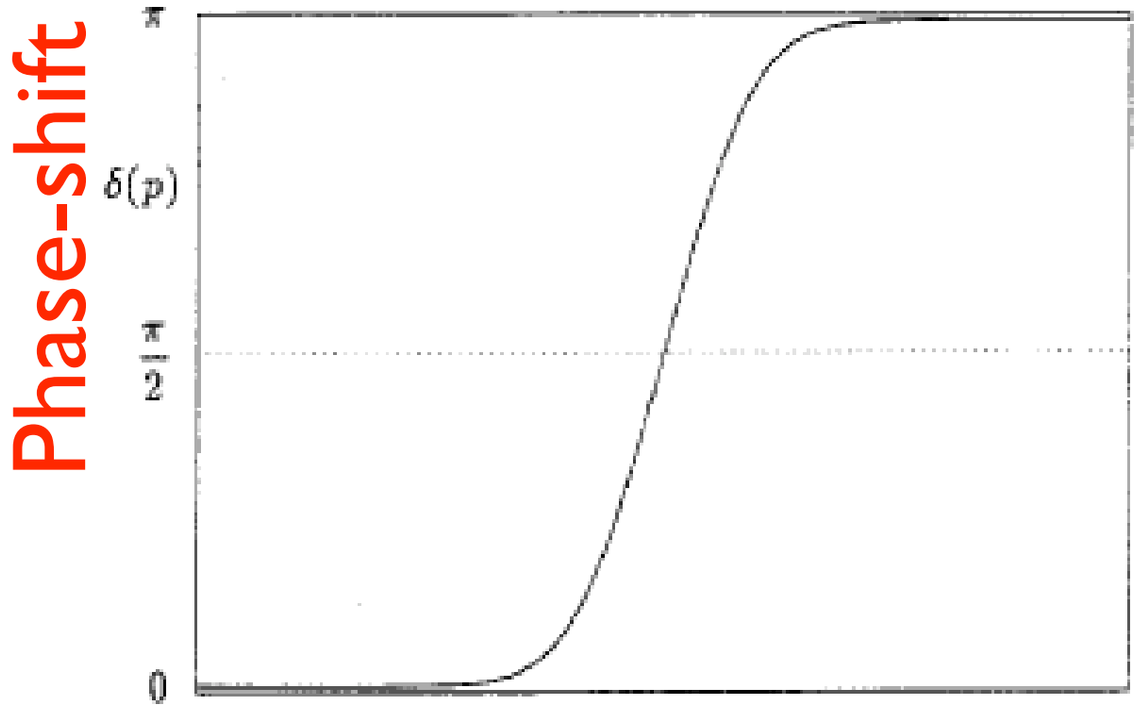
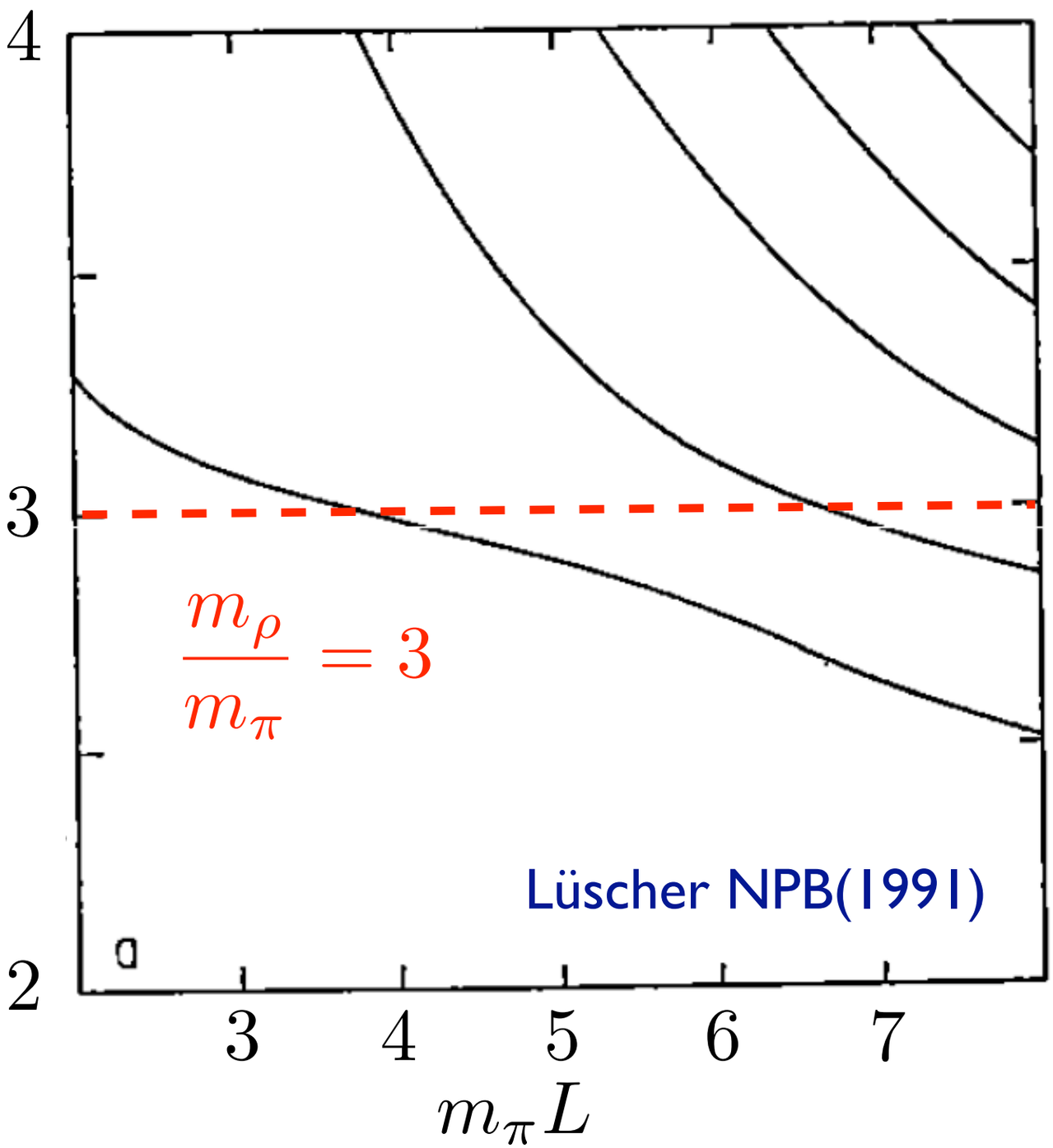


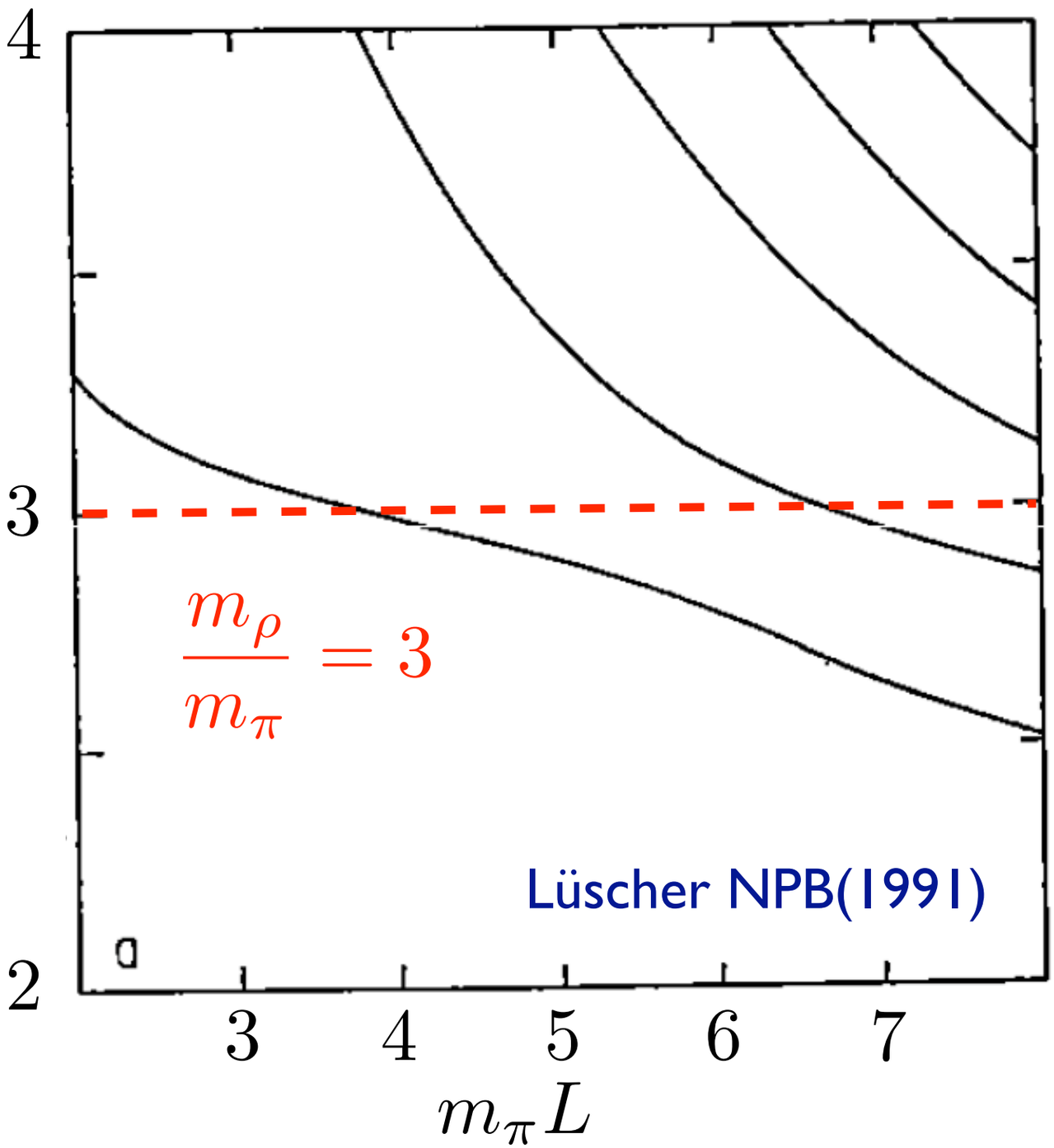


Resonance "plateau" for weakly interacting system ie. small width

Large width



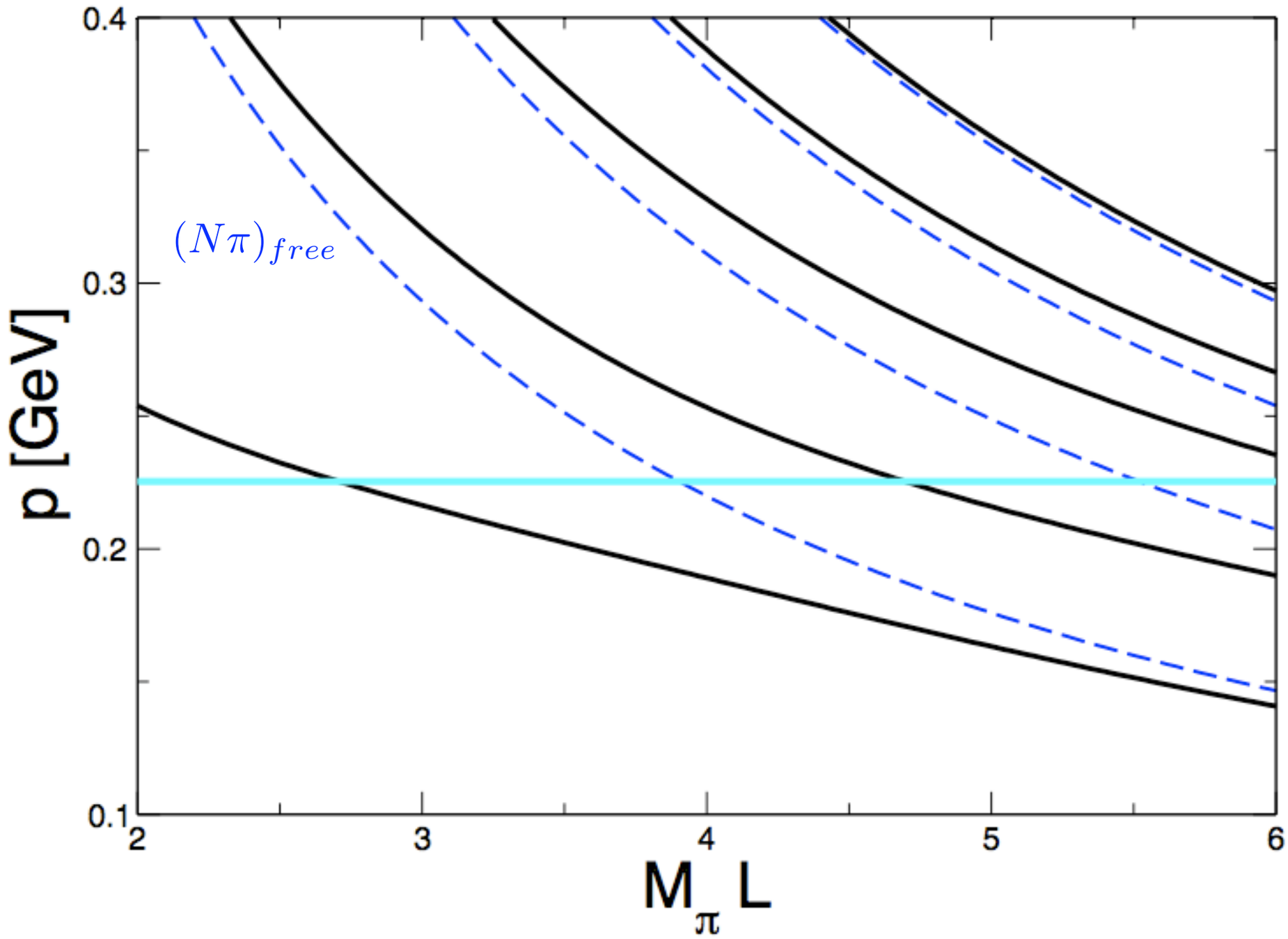
Large width



Need to extract full phase-shift analysis for strongly-coupled (large-width) systems

Delta

Bernard *et al.* JHEP(2008)



Finite-momentum trick

Lattice volume:

$$m_\pi L \gtrsim 2\pi$$

Lowest non-zero momentum:

$$p_{\rm min} = \frac{2\pi}{L} \sim m_\pi$$

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Rest system: rho couples to 2 back-to-back pions

2-pion energy

$$2\sqrt{m_\pi^2 + p_{\rm min}^2} \sim 2\sqrt{2} m_\pi$$

Rho "decay" requires

$$\frac{m_\pi}{m_\rho} \lesssim 0.35$$

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BOOST rho to $p_{\min}$ Rummukainen & Gottlieb, NPB(1995)

Rho energy $\sqrt{m_\rho^2 + p_{\min}^2}$

2-pion energy $m_\pi + \sqrt{m_\pi^2 + p_{\min}^2}$

"decay" $\frac{m_\pi}{m_\rho} \lesssim 0.45$

Finite momentum trick

CP-PACS(2007)

$m_\pi L = 4.3$ $\quad \frac{m_\pi}{m_\rho} = 0.41$

Rho "stable" at rest, but (just) "decays" in boosted frame

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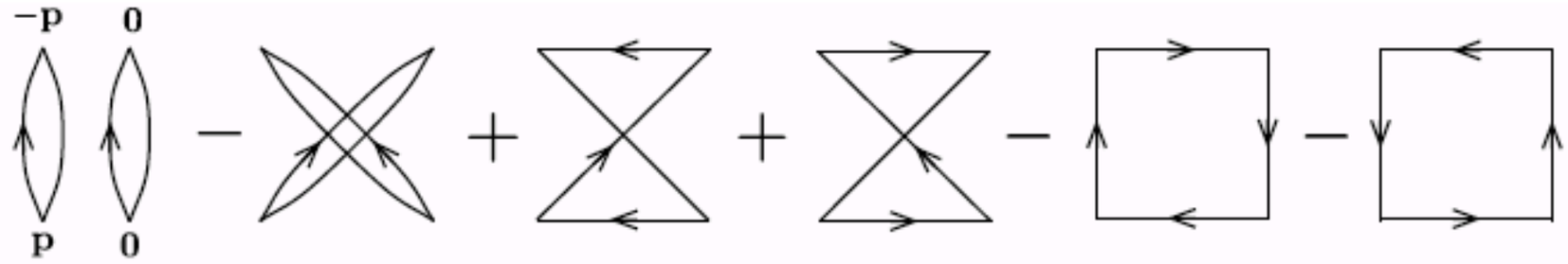
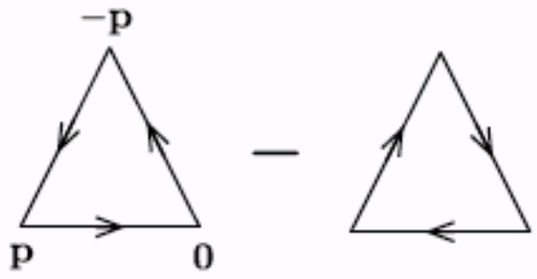
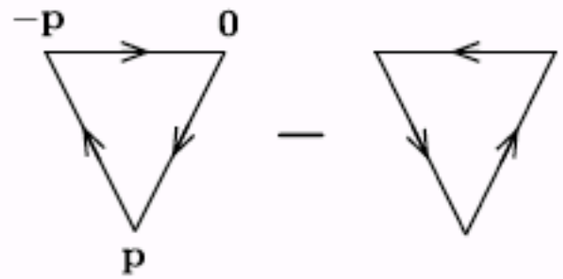
$G_{\pi\pi\to\pi\pi}(t) =$ 

$G_{\pi\pi\to\rho}(t) =$ 

$G_{\rho\to\pi\pi}(t) =$ 

Identified two states near evergy level of 2 non-interaction pions

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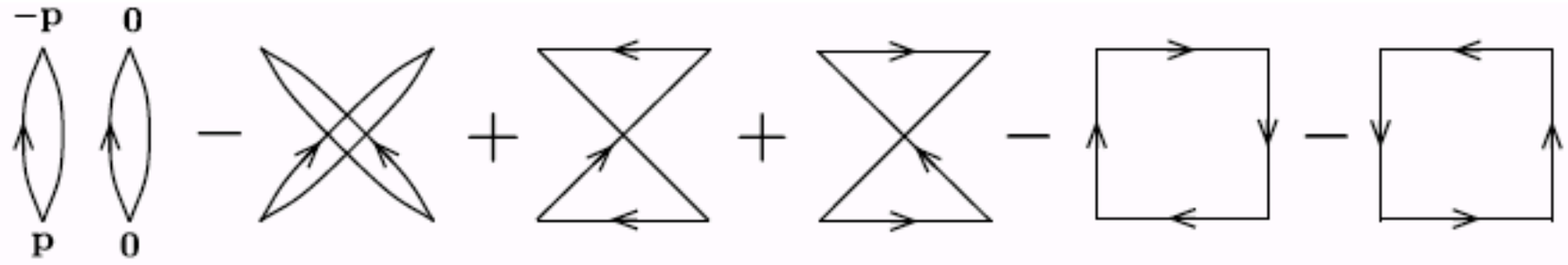

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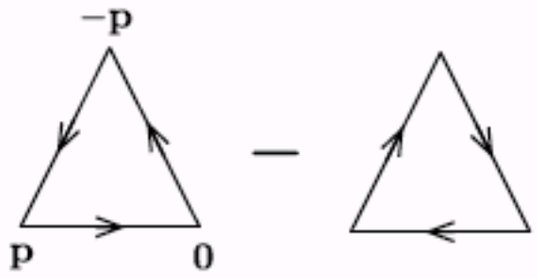

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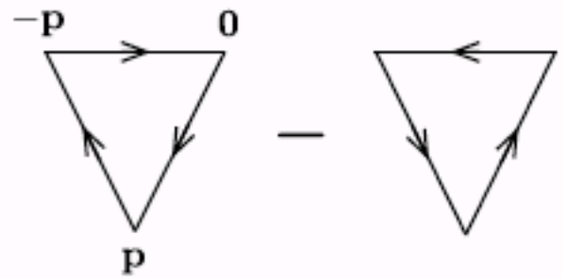

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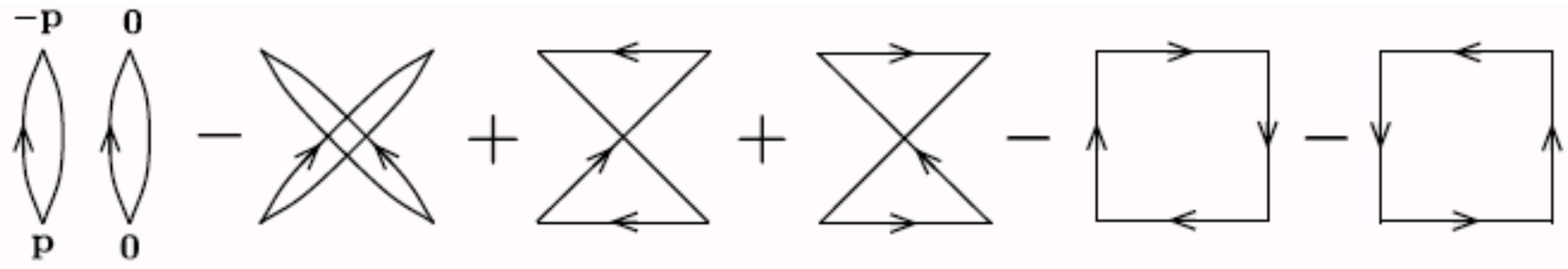

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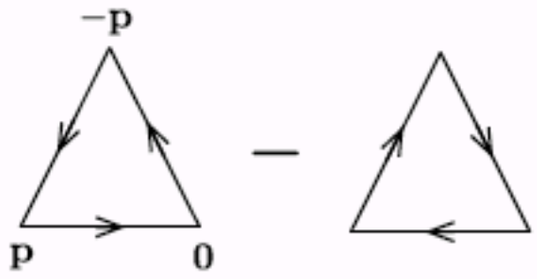

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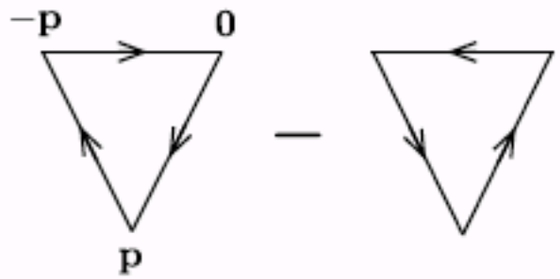

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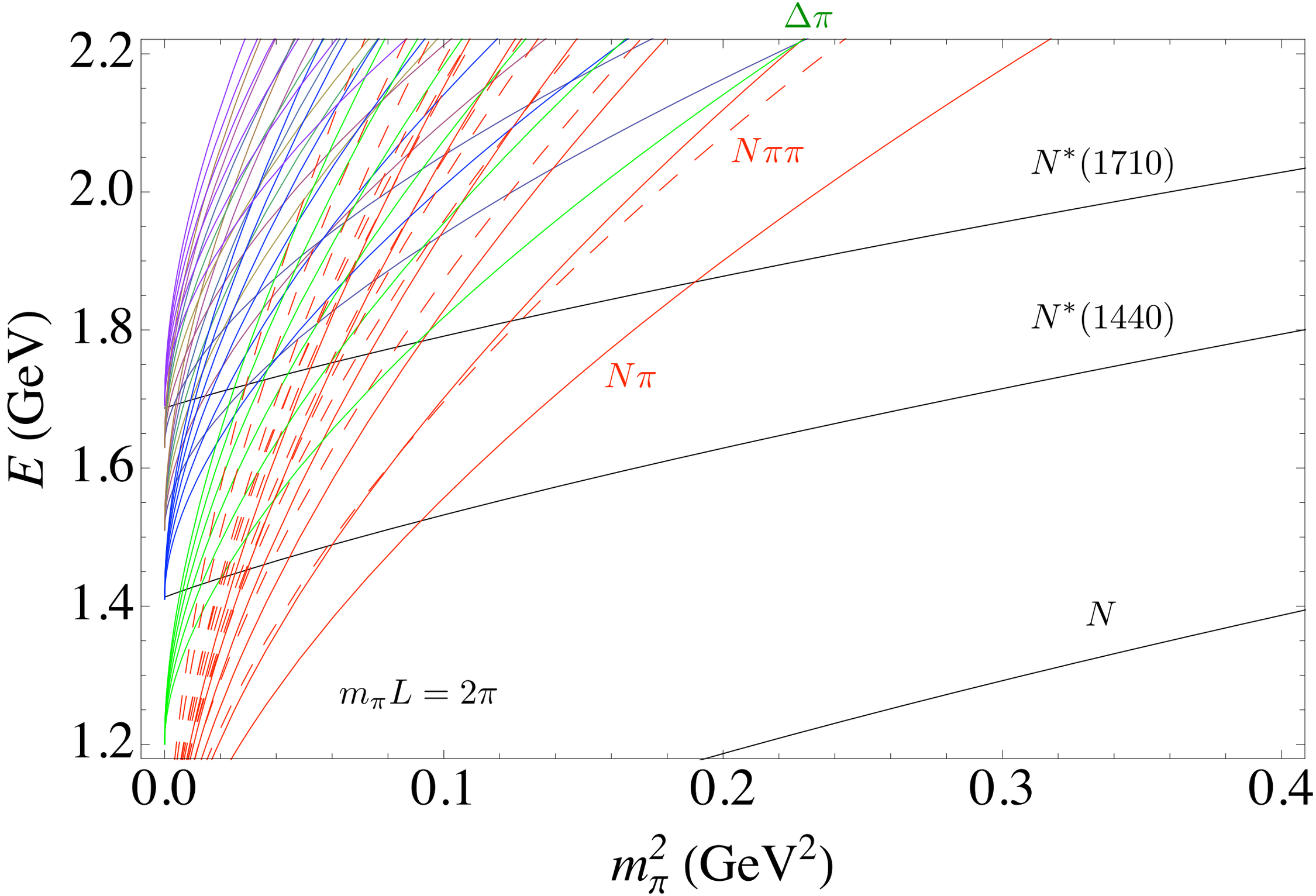
Determined phase-shift to extract $g_{\rho\pi\pi}$

Rescale phase-space to physical kinematics to estimate width

$\Gamma^{\rm lat} \simeq 162 \pm 35\,{\rm MeV}$ $\qquad \Gamma^{\rm phys} \simeq 150\,{\rm MeV}$

Coupled channel resonances



Multi-channel *S*-matrix on finite volume

- Extension of Lüscher by He, Feng & Liu JHEP(2005)

  Two-channel *S*-matrix

$$S^{(l)}(E) = \begin{pmatrix} \eta_l e^{2i\delta_1^l} & i\sqrt{1-\eta_l^2}\, e^{i(\delta_1^l+\delta_2^l)} \\ i\sqrt{1-\eta_l^2}\, e^{i(\delta_1^l+\delta_2^l)} & \eta_l e^{2i\delta_2^l} \end{pmatrix}$$

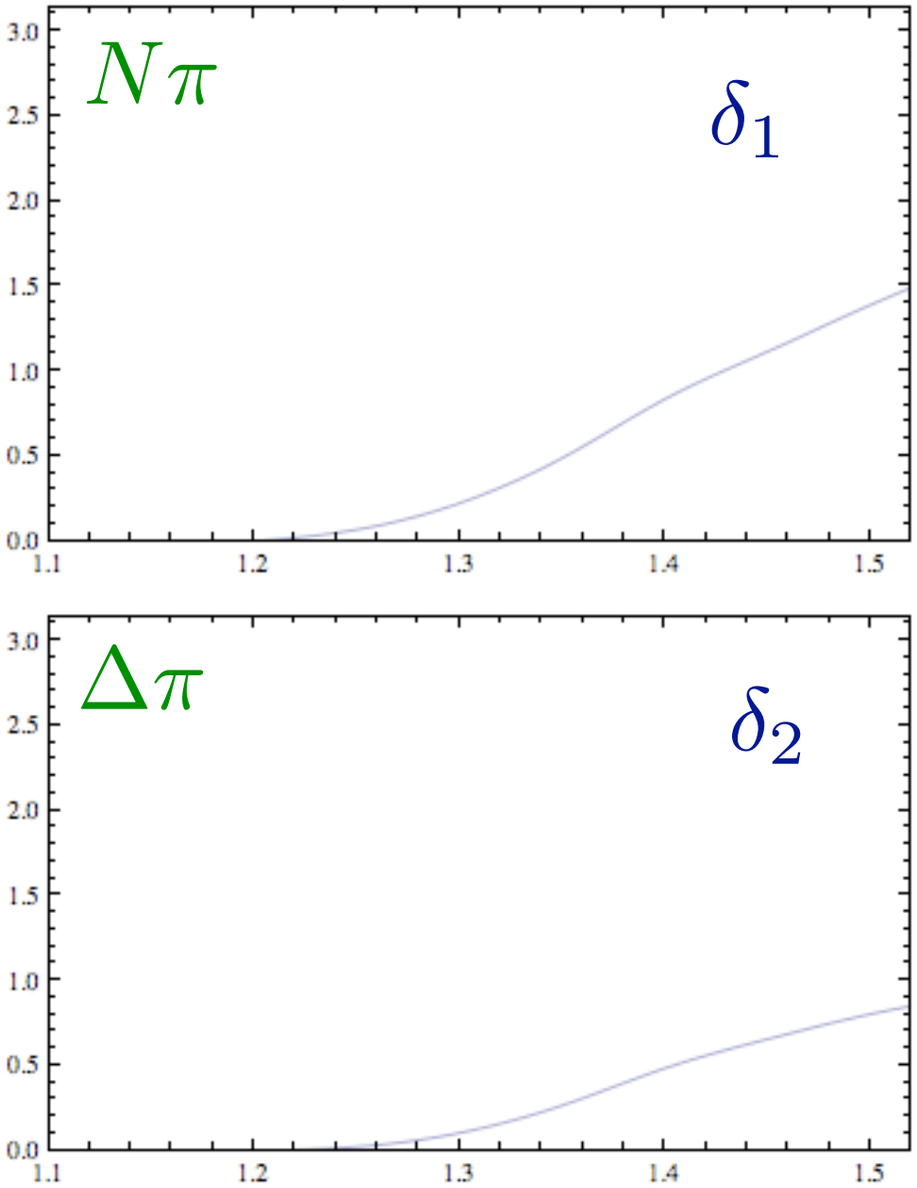
- Scattering parameters δ_1, δ_2, η each as function of *E*

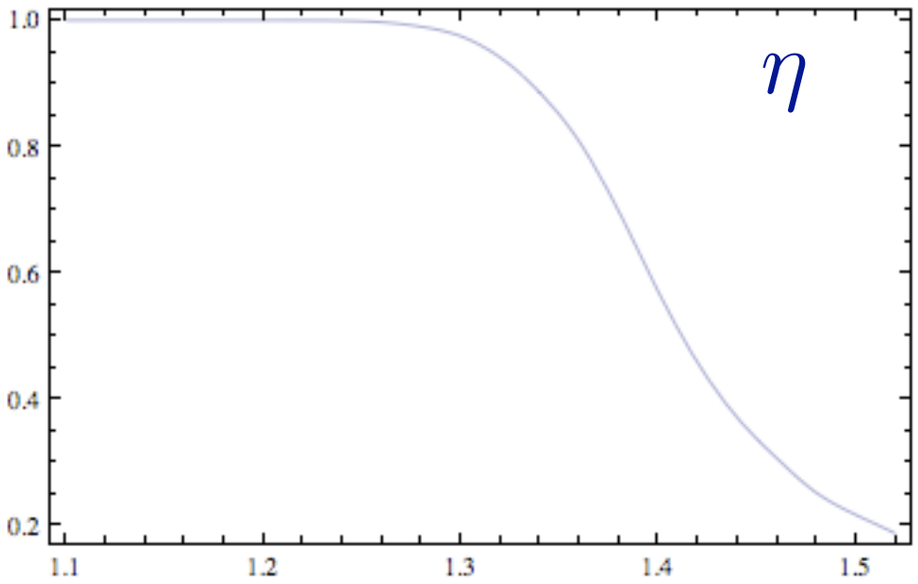
- Finite-volume eigenstates constrained by (*s*-wave)

$$\cos(\Delta_1 + \Delta_2 - \delta_1^0 - \delta_2^0) = \eta_0 \cos(\Delta_1 - \Delta_2 - \delta_1^0 + \delta_2^0)$$

$$\cot \Delta_i = \mathcal{M}_{00}(k_i^2) = \frac{\mathcal{Z}_{00}(1, q_i^2)}{\pi^{3/2} q_i} \qquad q_i = k_i \frac{L}{2\pi}$$

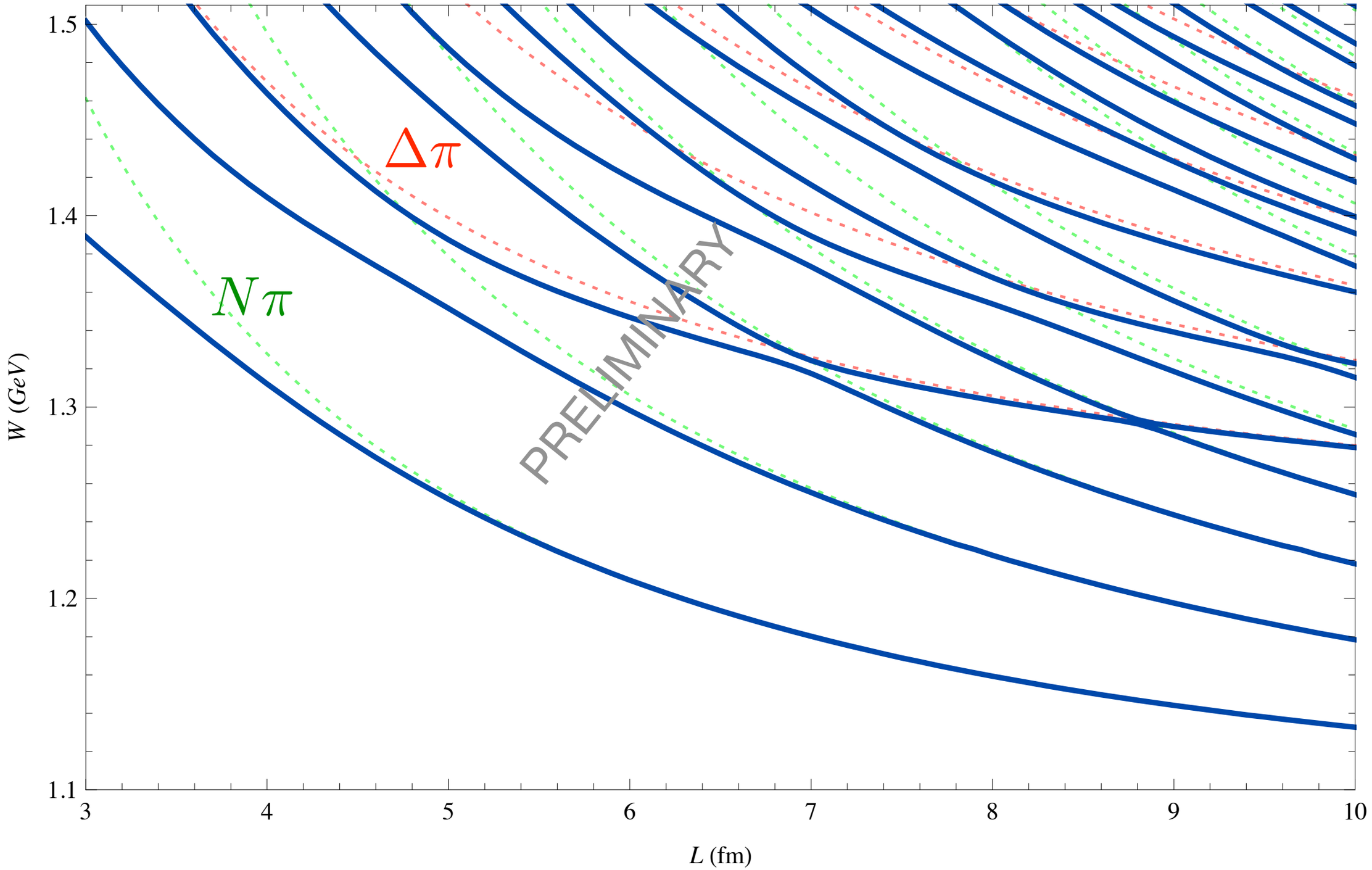
The “Roper”





Two-channel extraction of physical P11 scattering parameters

The “Roper” on a finite volume



Towards the Roper

- Dynamics of finite-volume effects

- N-sigma channel also quite strong in Roper

- Delta and sigma also “unstable”: $N\pi\pi$

- Finite-momentum modifications (even for rest Roper)
  Delta-pi has Delta at finite momentum (modified like rho)

- The main challenge: investigate inversion for lattice QCD

Scattering length

Lüscher

$$E_0 - 2m = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

Beane *et al.* (2005)

$$c_1 = \frac{1}{\pi}\sum_{\mathbf{j}\neq\mathbf{0}}^{|\mathbf{j}|<\Lambda}\frac{1}{|\mathbf{j}|^2} - 4\Lambda = -2.8372 \quad , \quad c_2 = c_1^2 - \frac{1}{\pi^2}\sum_{\mathbf{j}\neq\mathbf{0}}\frac{1}{|\mathbf{j}|^4} = 6.37518$$