Baryons (and Mesons) on the Lattice

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EBAC
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Spectroscopy

Spectroscopy reveals fundamental aspects of hadronic physics
- Essential degrees of freedom?
- Gluonic excitations in mesons - exotic states of matter?

- Status
  - Can extract excited hadron energies & identify spins,
  - Pursuing full QCD calculations with realistic quark masses.

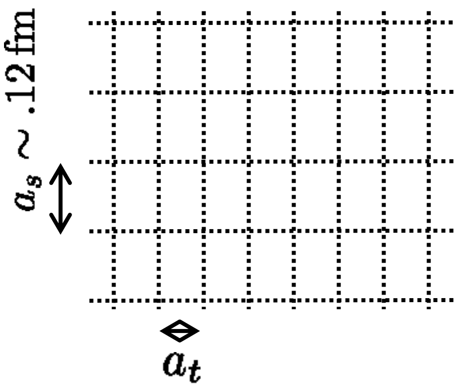
- New spectroscopy programs world-wide
  - E.g., BES III, GSI/Panda
  - Crucial complement to 12 GeV program at JLab.
    - Excited nucleon spectroscopy (JLab)
    - JLab GlueX: search for gluonic excitations.

Regularization of QCD on a lattice

$$Z_{\rm QCD} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, e^{iS[\psi,\bar{\psi},A_\mu]}$$

quark field gluon field

Discretize on a finite grid - in Euclidean space-time (t→i t)



$\psi(x) \to \psi_x$ quark fields on the sites

$A_\mu(x) \to U_{x,\mu} = e^{-aA_{x\mu}}$ gauge-fields on the links

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \, f(\psi, \bar{\psi}, U) \, e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

do the fermion integral

$$\int \mathcal{D}U \, f(\mathcal{Q}^{-1}[U], U) \, \det \mathcal{Q}[U] e^{-\tilde{S}_G[U]}$$

probability, $\mathcal{P}[U]$

Monte Carlo

$$\sum_{\{U\}} f(\mathcal{Q}^{-1}[U], U)$$

$a_t^{-1} \sim 5.6\,\mathrm{GeV}$

$\begin{Bmatrix} 16^3 \\ 20^3 \\ 24^3 \end{Bmatrix} \times 128$

$L \gtrsim 2\,\mathrm{fm}$

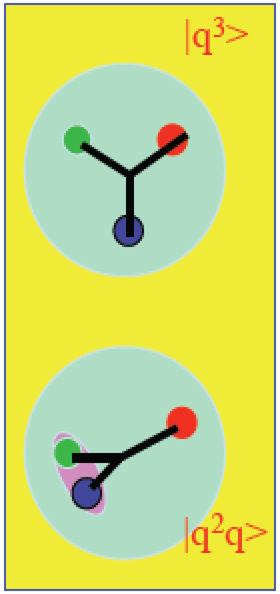
$N_F = 2 \oplus 1$
$(u, d \oplus s)$

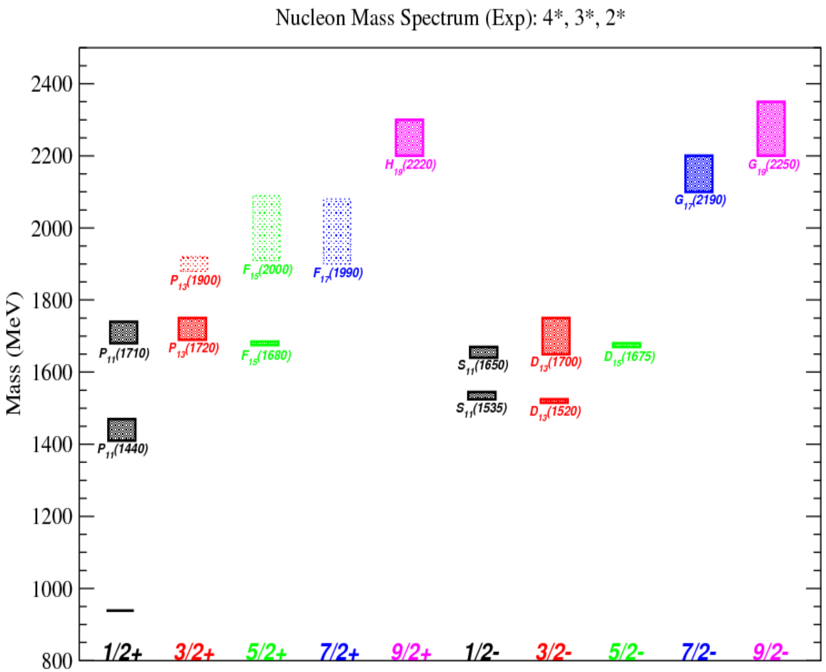
$230\,\mathrm{MeV} < m_\pi < 700\,\mathrm{MeV}$

Baryon Spectrum

"Missing resonance problem"

- What are collective modes?
- What is the structure of the states?
  - Major focus of (and motivation for) JLab Hall B
  - Not resolved experimentally @ 6GeV





Variational Method

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

Matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

Variational solution = generalized eigenvalue problem

$$C(t) v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t) C(t_0) v^{\mathfrak{n}}$$

Eigenvalue $\rightarrow$ spectrum

$$\lambda_{\mathfrak{n}}(t) \rightarrow e^{-E_{\mathfrak{n}}(t-t_0)}$$

Eigenvectors $\rightarrow$ 'optimal' operators

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}} \Phi_1 + v_2^{\mathfrak{n}} \Phi_2 + \dots$$

Orthogonality needed for near degenerate states

Light quark baryons in SU(6)

Conventional non-relativistic construction:

6 quark states in SU(6)

$$u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow$$

$$SU(6) \subseteq SU(3)_{\mathrm{Flavor}} \otimes SU(2)_{\mathrm{Spin}}$$

Baryons

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_{MS} \oplus \mathbf{70}_{MA} \oplus \mathbf{20}_A$$

$$\begin{aligned}
\text{Symmetric :}&(\mathbf{10},\mathbf{4}) \qquad\qquad +(\mathbf{8},\mathbf{2}) &&= \mathbf{56}\\
\text{Mixed :}&(\mathbf{10},\mathbf{2})+(\mathbf{8},\mathbf{4})+(\mathbf{8},\mathbf{2})+(\mathbf{1},\mathbf{2}) &&= \mathbf{70}\\
\text{Antisymmetric :}&\qquad\qquad\qquad (\mathbf{8},\mathbf{2}) \qquad +(\mathbf{1},\mathbf{4}) &&= \mathbf{20}
\end{aligned}$$

Relativistic operator construction: SU(12)

Relativistic construction: 3 Flavors with upper/lower components

$$\underbrace{SU(12) \subseteq SU(3)_{\text{Flavor}} \otimes \left[SU(2)_{\text{Upper/lower}} \otimes SU(2)_{\text{Spin}}\right]}$$

Dirac

Times space (derivatives)

$$\text{Op}_S \leftarrow \text{Derivative}_M \otimes \left[\text{Flavor}_M \otimes \text{Dirac}_M\right]_M$$

Color contraction is Antisymmetric

More operators than SU(6): mixes orbital ang. momentum & Dirac spin

$$\text{Symmetric}: 2 * (\mathbf{10}, \mathbf{4}) + (\mathbf{10}, \mathbf{2}) + (\mathbf{8}, \mathbf{4}) + 2 * (\mathbf{8}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}) = \mathbf{182}$$

Symmetric: 182 positive parity + 182 negative parity

Orbital angular momentum via derivatives

Derivatives in ladders:

$$\begin{aligned}\vec{D}^{(q)}_{l=+1} &= \frac{i}{2}\left(\vec{D}_x + i\vec{D}_y\right)\\ \vec{D}^{(q)}_{l=0} &= -\frac{i}{\sqrt{2}}\vec{D}_z\\ \vec{D}^{(q)}_{l=-1} &= -\frac{i}{2}\left(\vec{D}_x - i\vec{D}_y\right)\end{aligned}$$

Couple derivatives onto single-site spinors:

$$\left(D^{[1]}\Psi^{[S]}\right)^{J,M} = \sum_{l,s}\langle 1,l;S,s|J,M\rangle \vec{D}^{[1]}_{L=1,l}\Psi^{S,s}$$

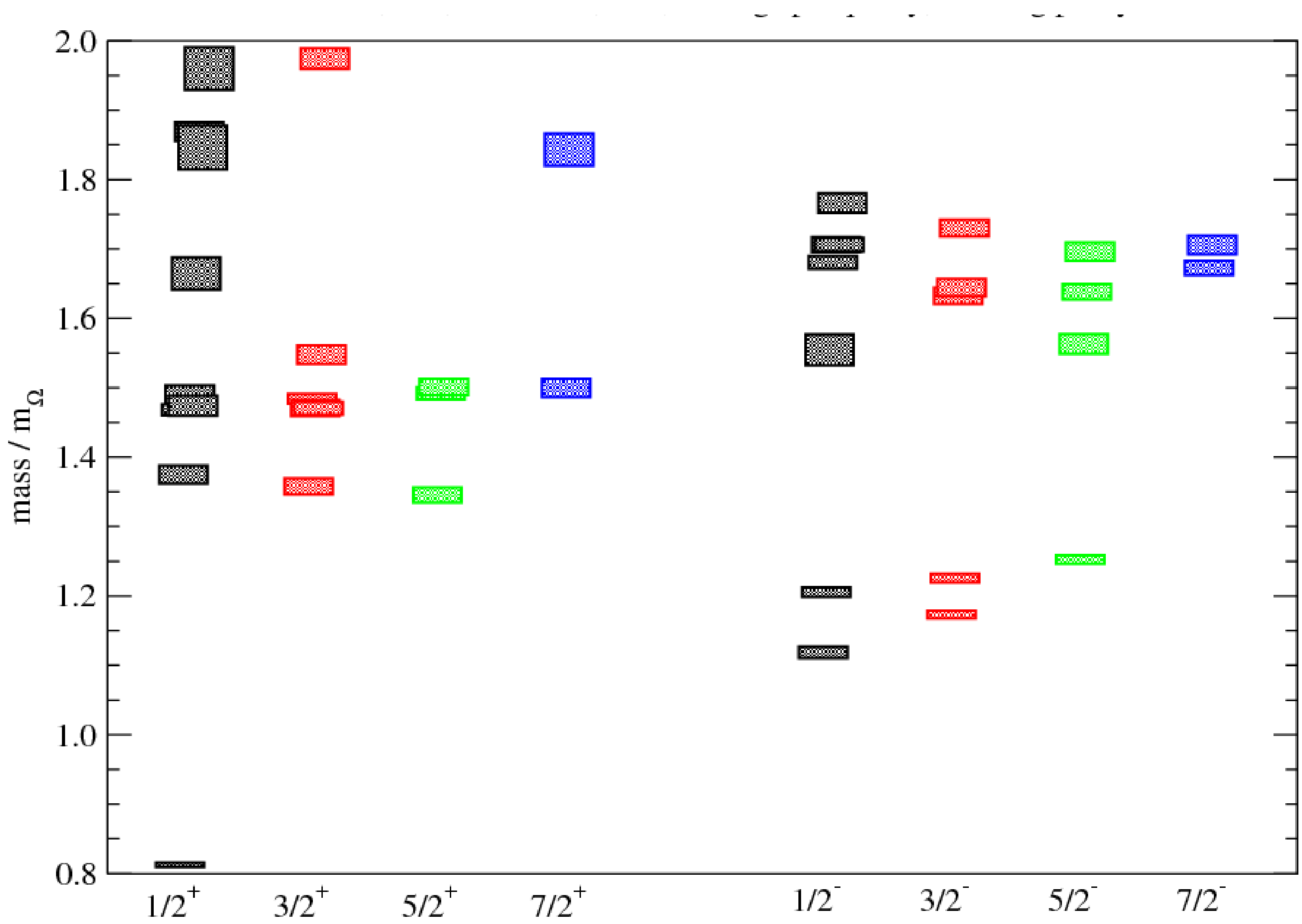
Project onto lattice irreducible representations

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} = \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda}\mathcal{O}^{J,M}$$

0905.2160 (PRD), 0909.0200 (PRL), 1004.4930

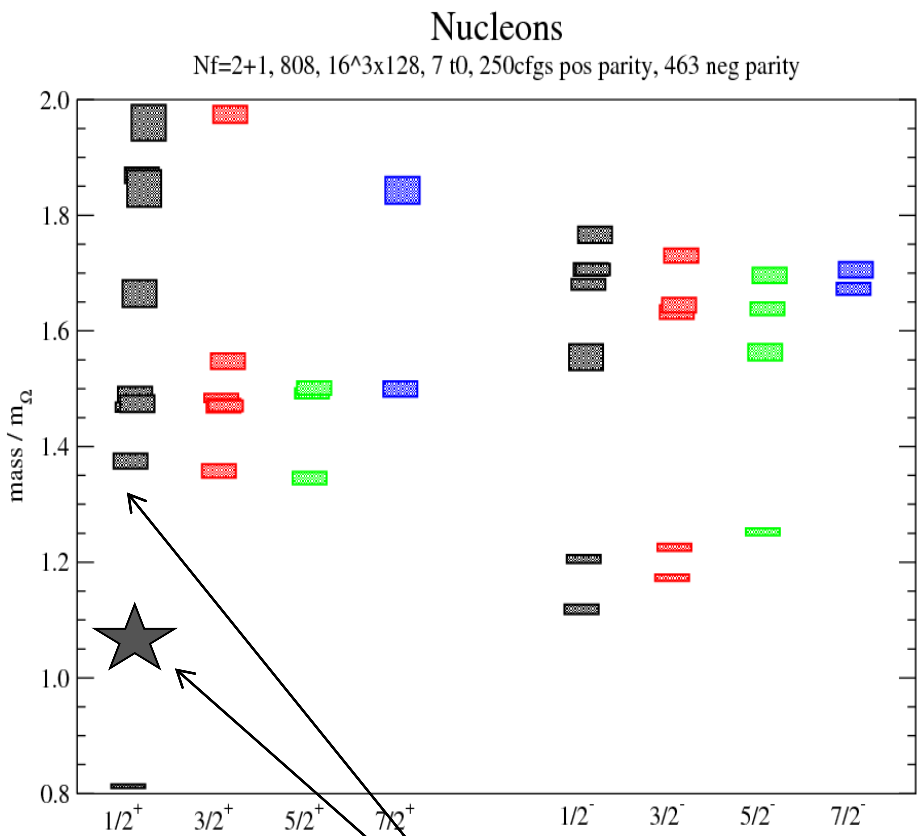
Spin identified Nucleon spectrum

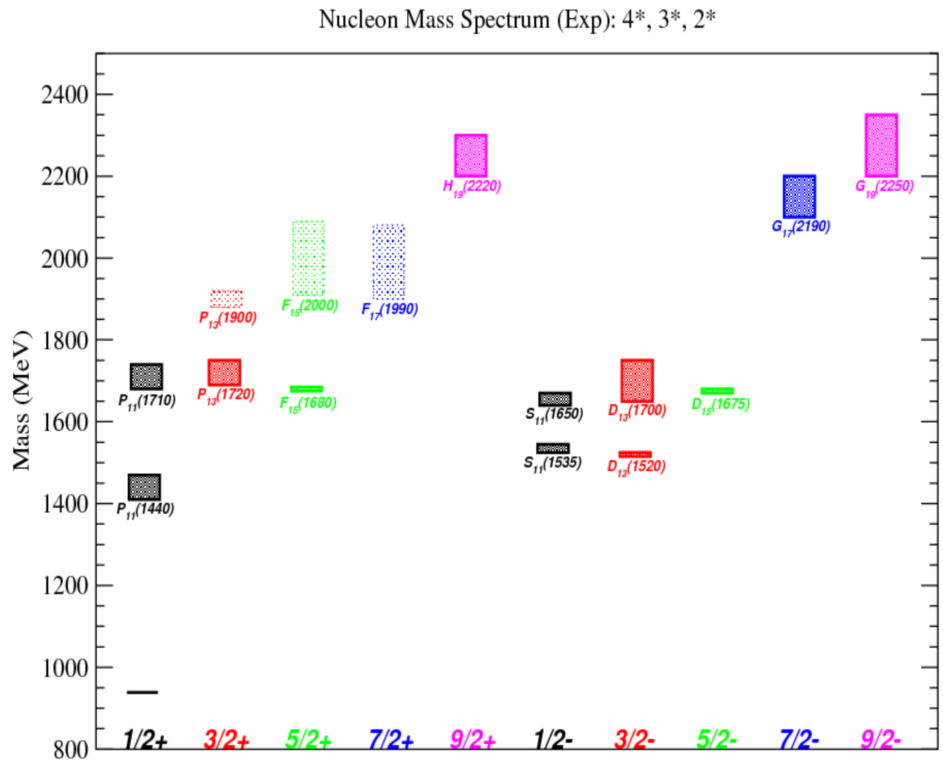
$m_\pi \sim 520\text{MeV}$



Experimental comparison

Pattern of states very similar





Where is the “Roper”?

Thresholds & decays: need multi-particle ops

Towards resonance determinations

- Augment with multi-particle operators
- Heavy masses: some elastic scattering
  - Finite volume (scattering) techniques (Lüscher)
  - Phase shifts $\rightarrow$ width

- Elastic & inelastic scattering:
  - Overlapping resonances
  - Will need/extend to finite-volume multi-channel
  - E.g., work by Bonn group
  - R. Young (next talk!)

Phenomenology: Nucleon spectrum

Looks like quark model?

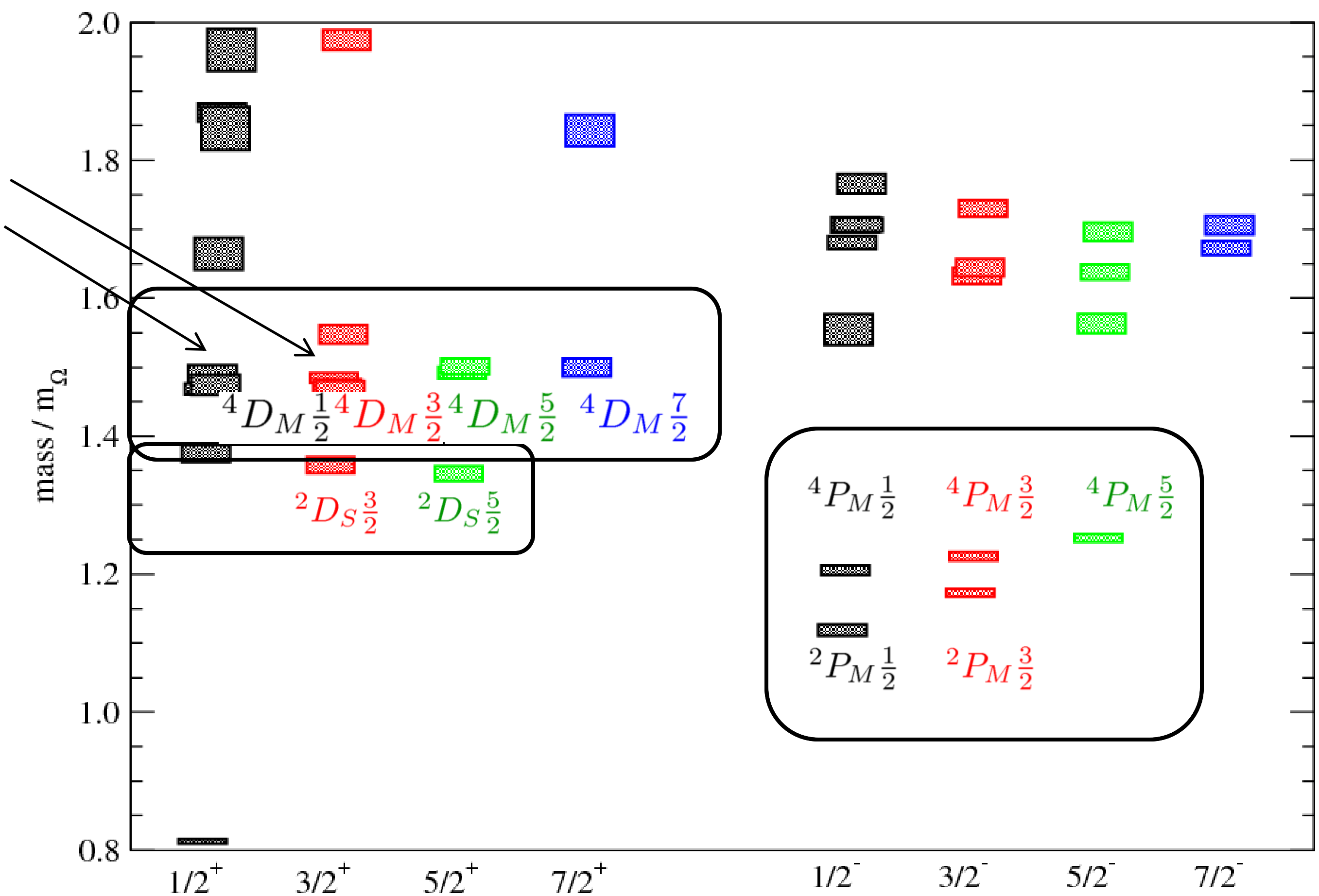
Compare overlaps & QM mixings

$m_\pi \sim 520$MeV

[20,1+]
P-wave

[70,2+]
D-wave

[56,2+]
D-wave

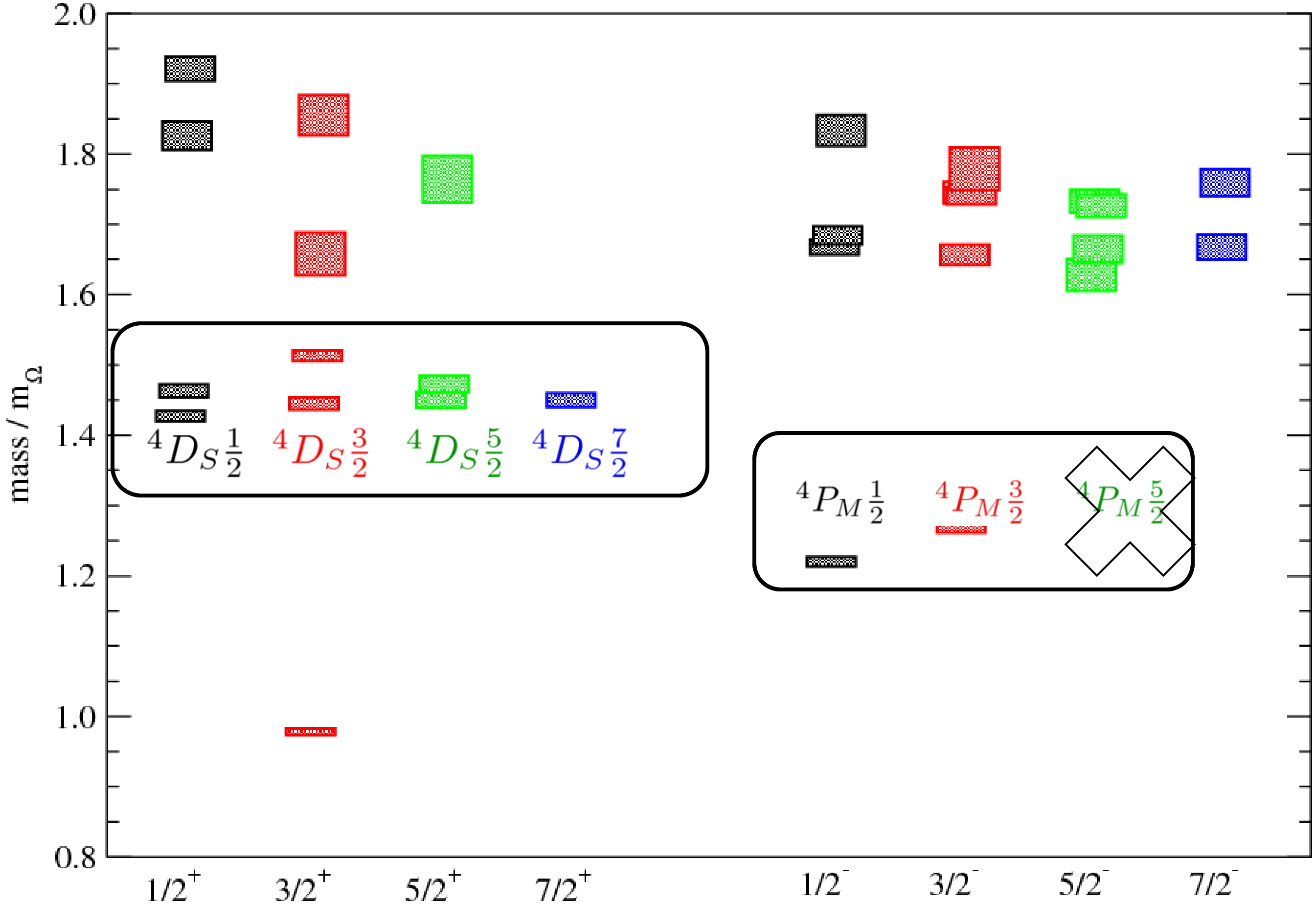


[70,1-]
P-wave

Spin identified Δ spectrum

Spectrum slightly higher than nucleon

[56,2+]
D-wave



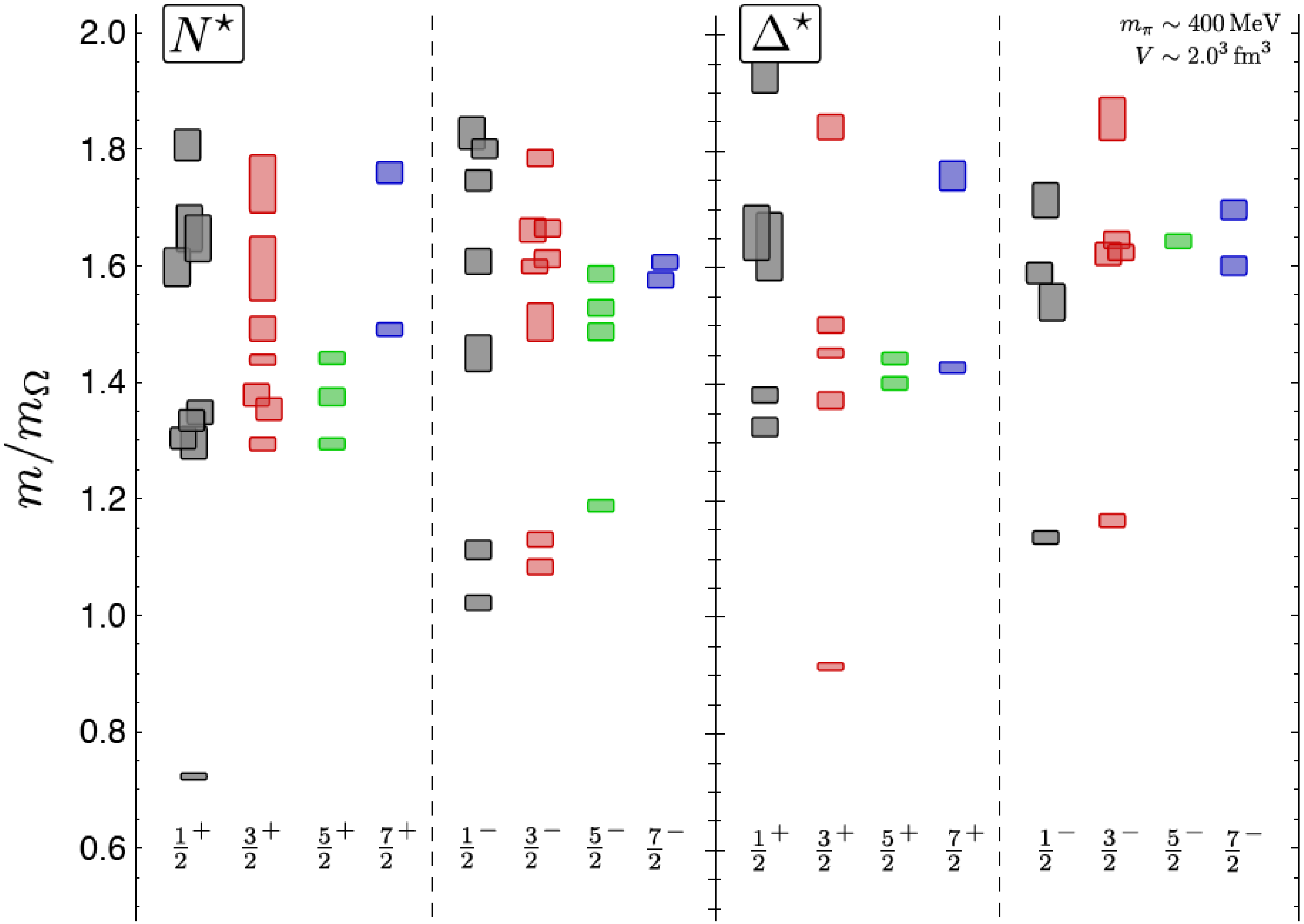
[70,1-]
P-wave

Nucleon & Delta Spectrum

Lighter mass: states spreading/interspersing

m_π ~ 400 MeV

< 2% error bars



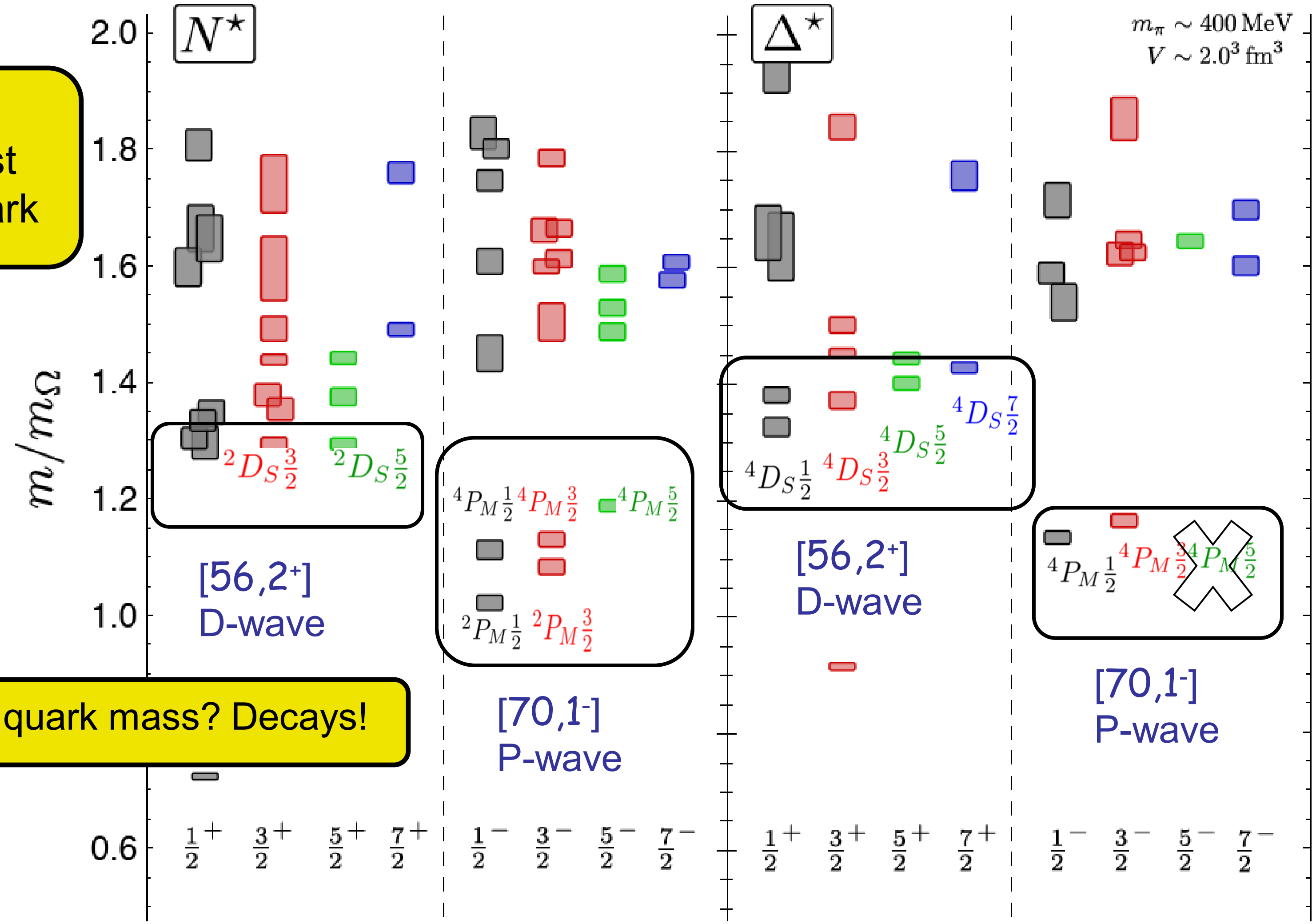
Nucleon & Delta Spectrum

First results from GPU-s

Discern structure: wave-function overlaps

Suggests spectrum at least as dense as quark model

< 2% error bars

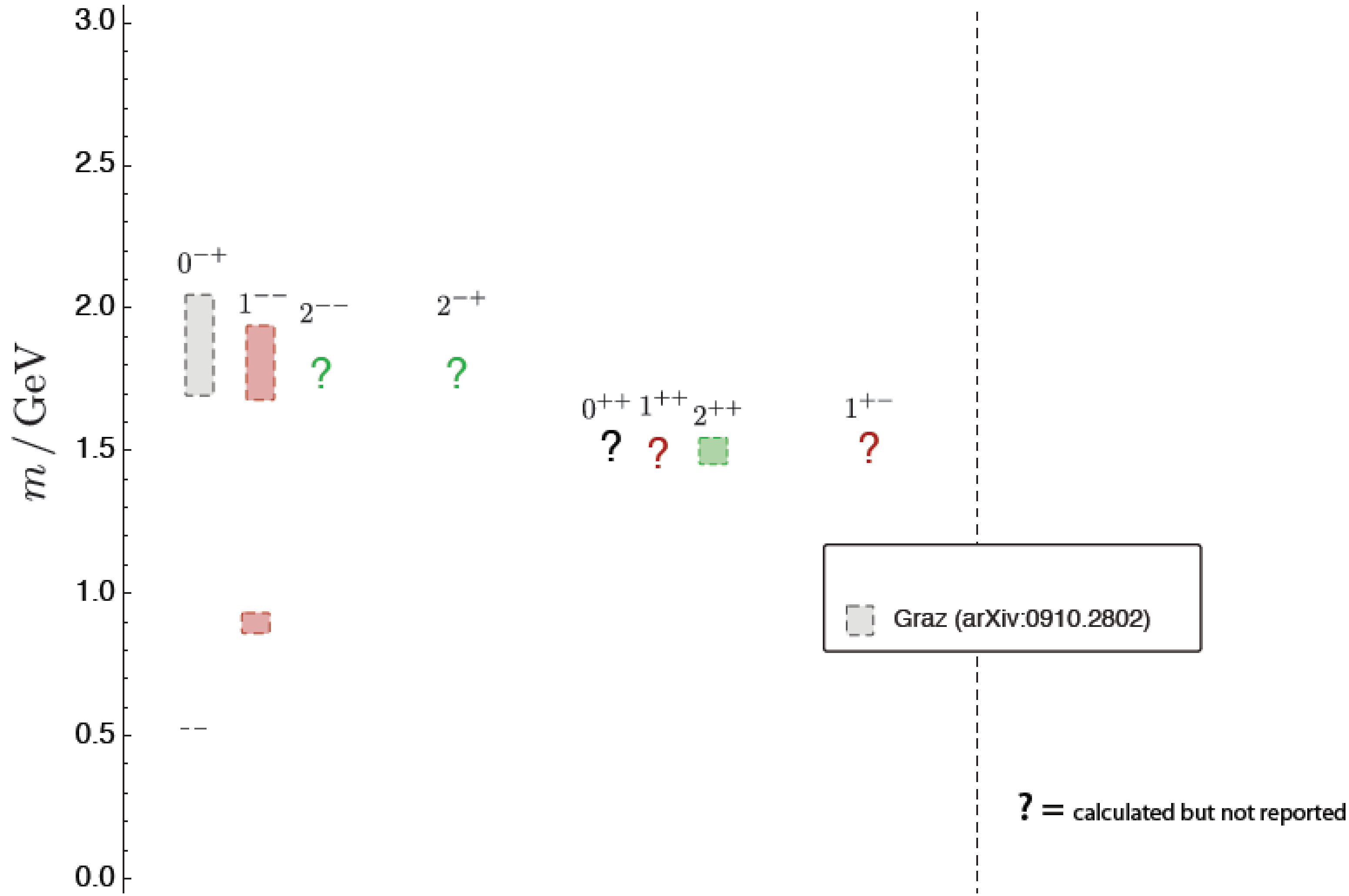


Change at lighter quark mass? Decays!

Isovector Meson Spectrum

the competition

$N_F = 2$ (u,d) $m_\pi \sim 500$ MeV

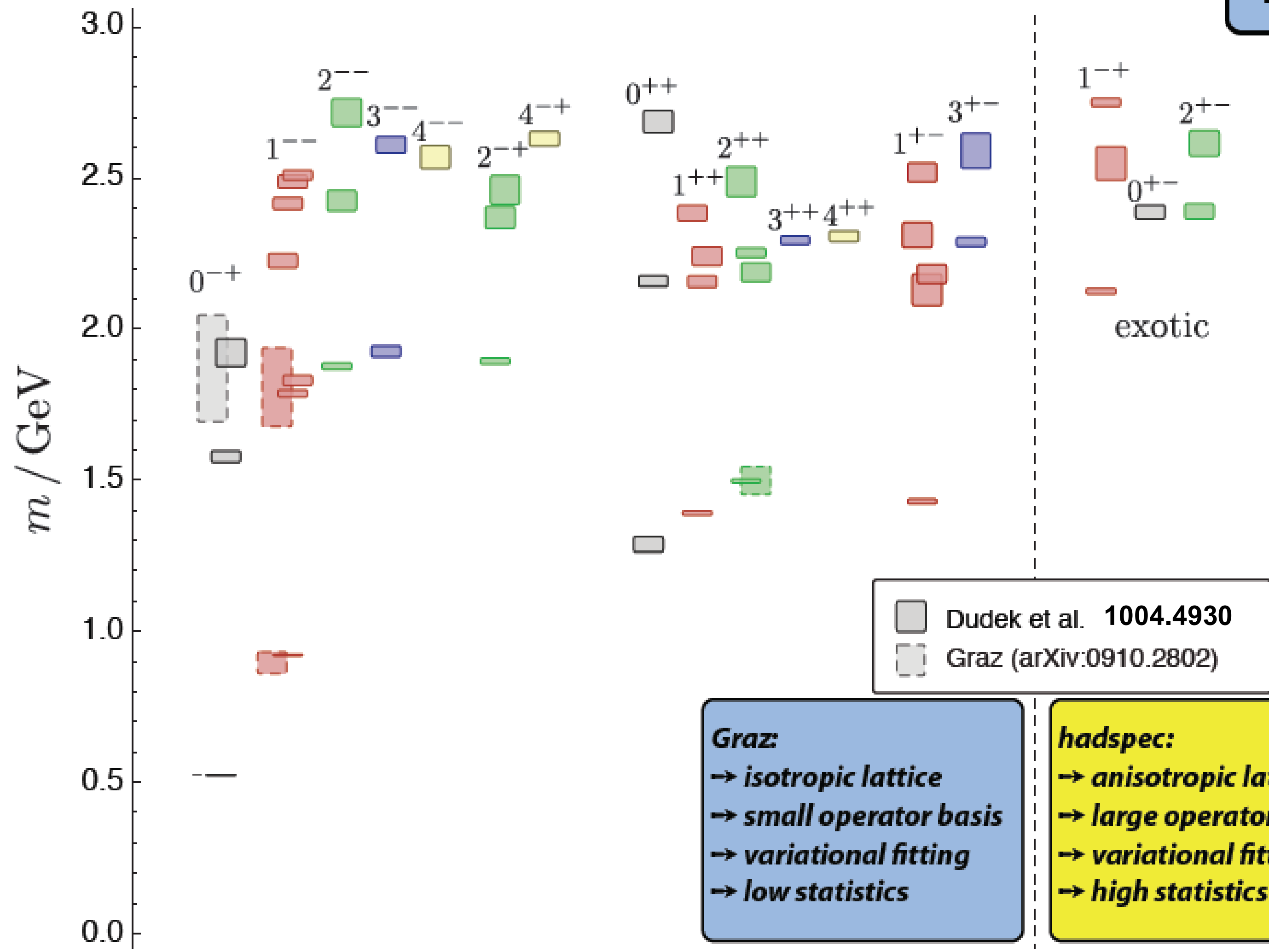


Isovector Meson Spectrum

the competition

$N_F = 2+1$ (u,d,s) $m_\pi \sim 520$ MeV $V = 16^3$

PRL 103:262001,2009

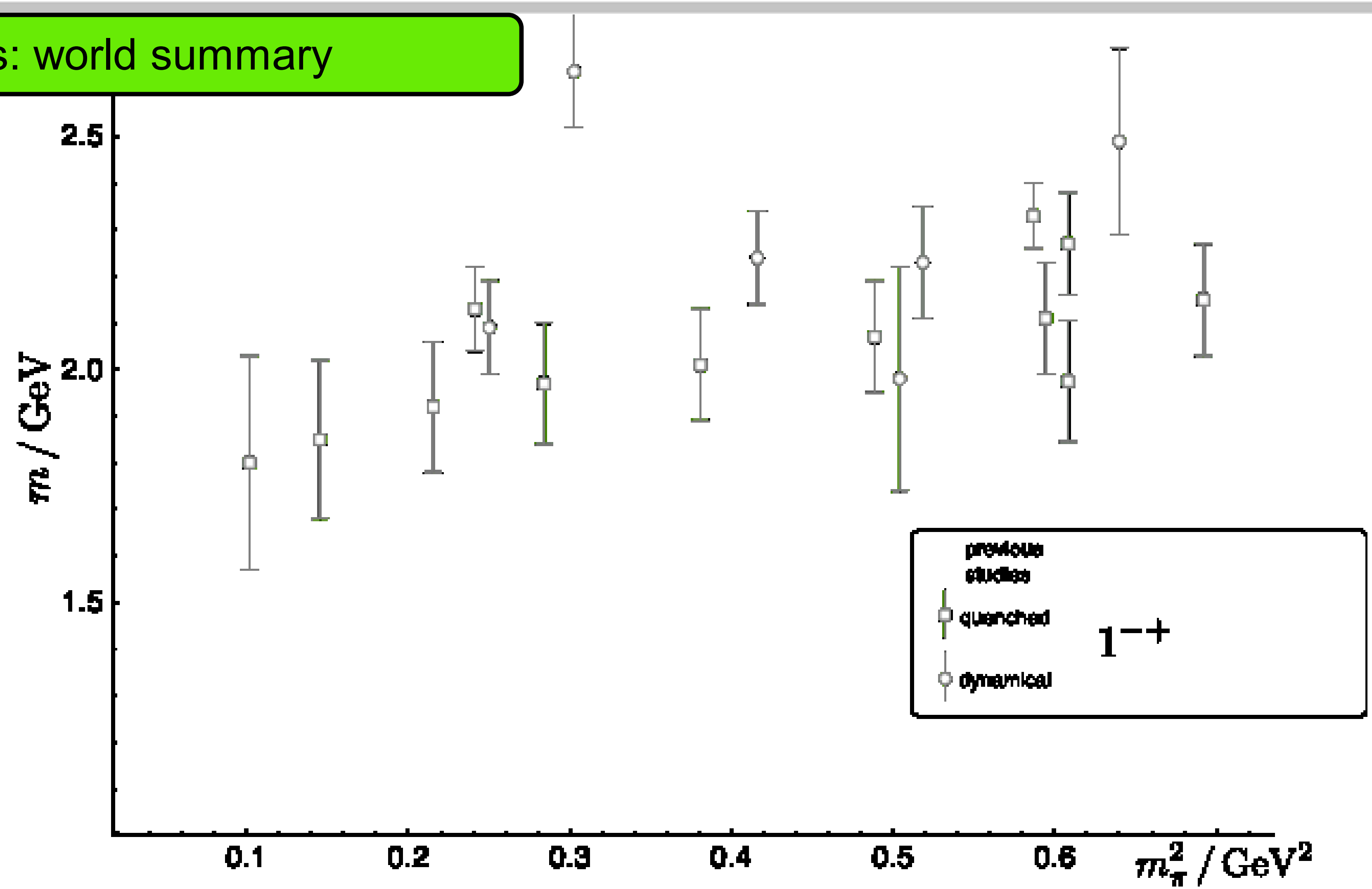


Graz:
- ***isotropic lattice***
- ***small operator basis***
- ***variational fitting***
- ***low statistics***

hadspec:
- ***anisotropic lattice***
- ***large operator basis***
- ***variational fitting***
- ***high statistics***

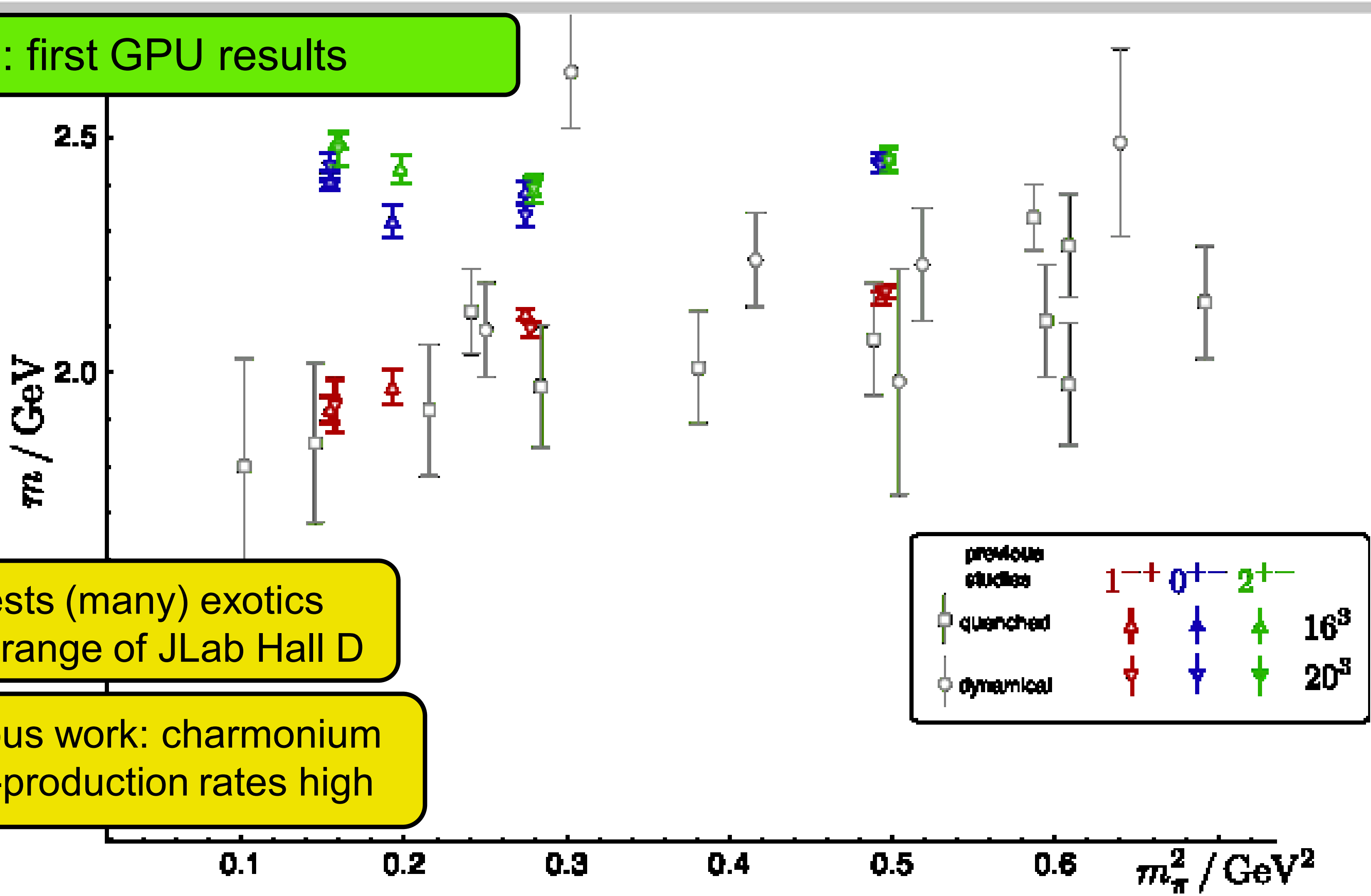
Exotic matter

Exotics: world summary



Exotic matter

Exotics: first GPU results

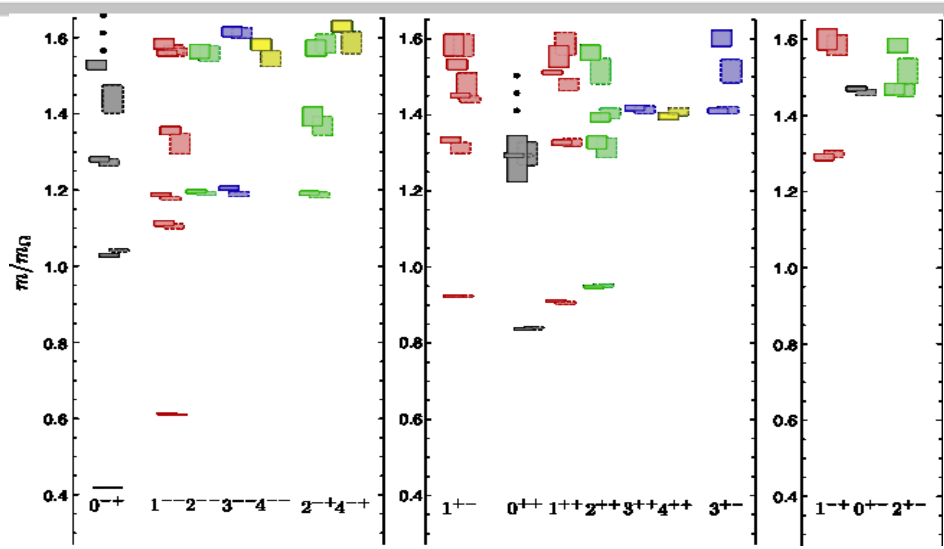


Suggests (many) exotics within range of JLab Hall D

Previous work: charmonium photo-production rates high

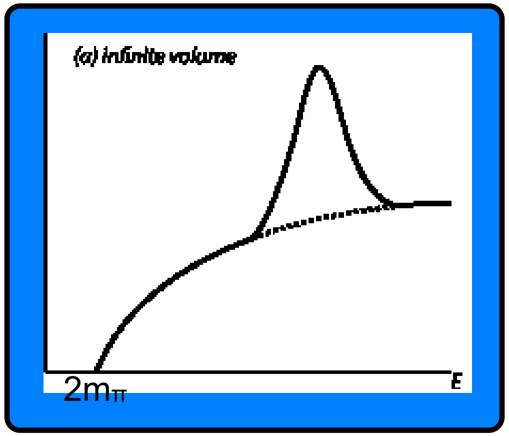
Current GPU work: (strong) decays - important experimental input

Spectrum of finite volume field theory



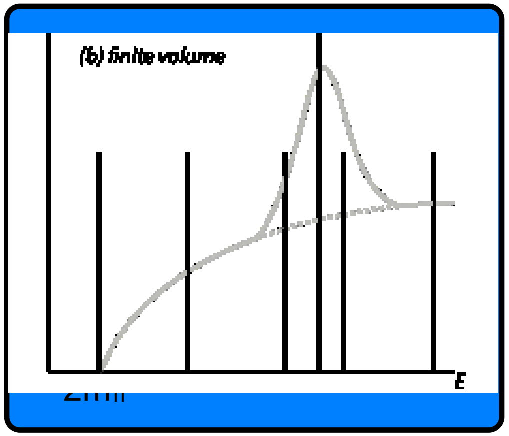
There are states missing that we know should be there

The 'continuum' of meson-meson (or meson-baryon) scattering states !



Infinite volume, a continuous spectrum of ππ states

$$E(p) = 2\sqrt{m_\pi^2 + p^2}$$



Finite volume, a discrete spectrum of states

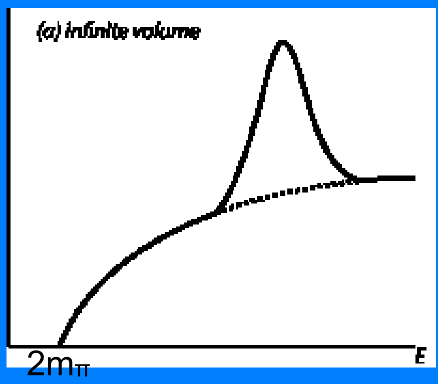
e.g. a free particle $\varphi(x) = e^{ipx}$

periodic boundary conditions

$$\varphi(x+L) = \varphi(x)$$

quantized momenta $p = n\dfrac{2\pi}{L}$

Spectrum of finite volume field theory



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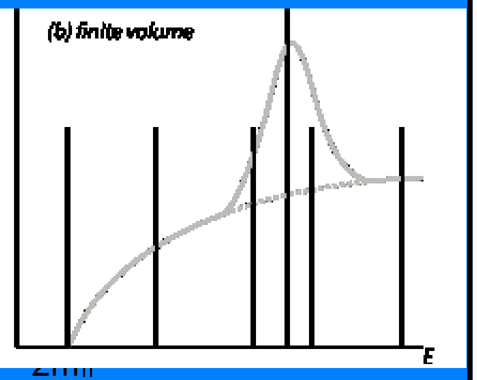

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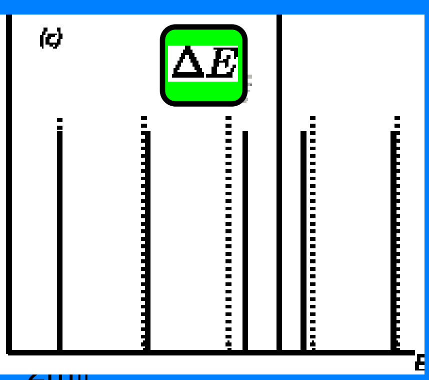


Non-interacting two-particle states have known energies

$$E(p) = 2\sqrt{m_\pi^2 + n\left(\frac{2\pi}{L}\right)^2}$$

ΔE(L) ↔ δ(E) : Lüscher method

Deviation from free energies depends upon the interaction and contains information about the scattering phase shift

Finite volume scattering

Reverse engineer

Use known phase shift → anticipate spectrum

E.g. just a single elastic resonance

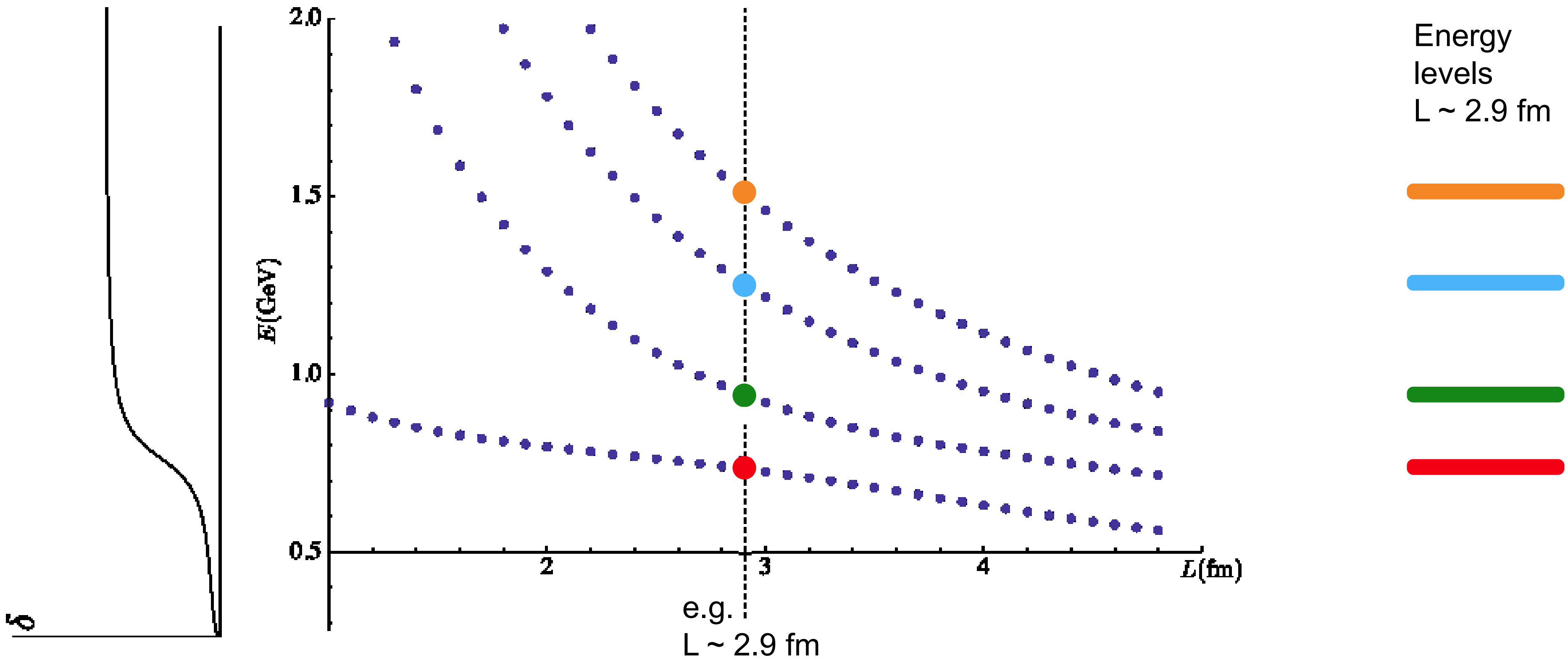
e.g.

$$\pi\pi \longrightarrow \rho \longrightarrow \pi\pi$$

$$\pi N \longrightarrow \Delta \longrightarrow \pi N$$

Lüscher method
- essentially scattering in a periodic cubic box (length **L**)
- finite volume energy levels **E(δ,L)**

Using the Lüscher method

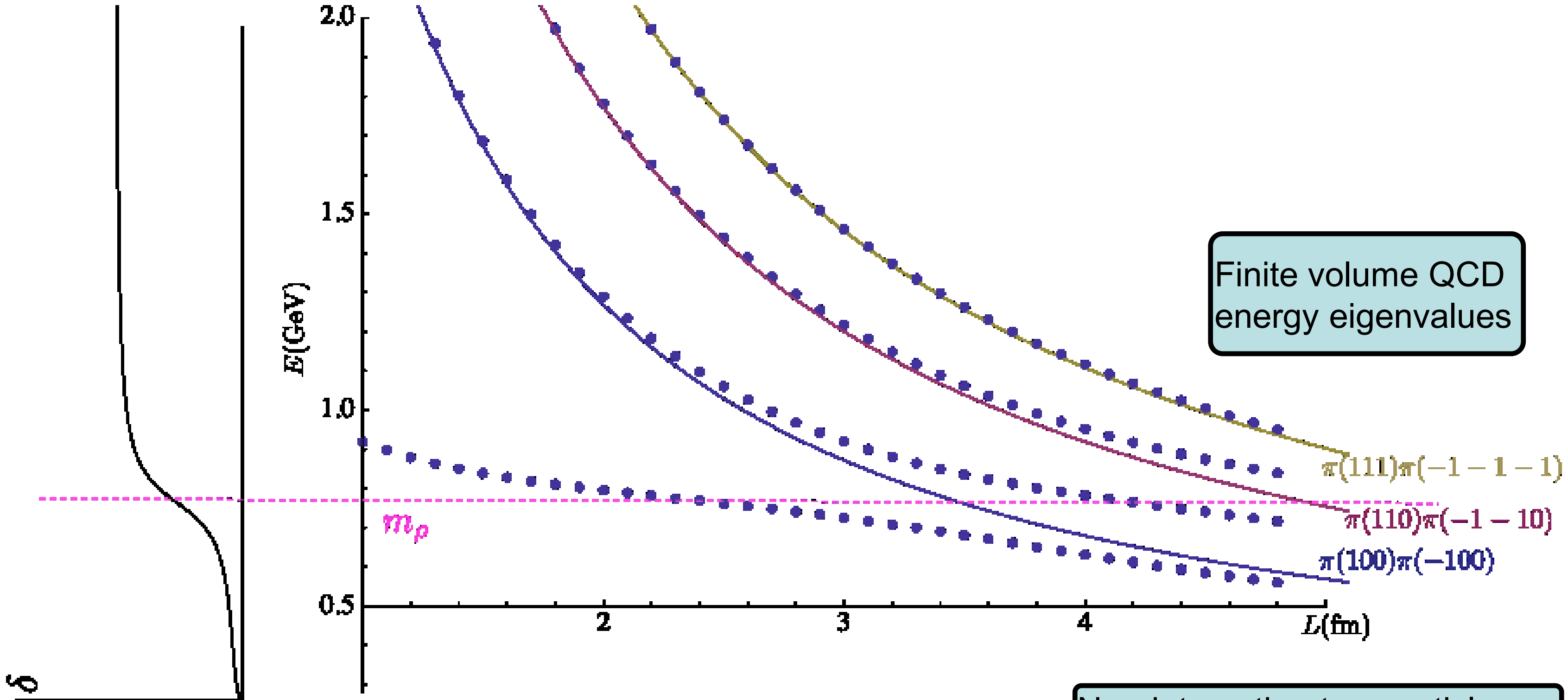


Excited state spectrum at a single volume

Discrete points on the phase shift curve

Do more volumes, get more points

The interpretation



Finite volume QCD energy eigenvalues

Non-interacting two-particle states have known energies

$$E(p) = 2\sqrt{m_\pi^2 + n\left(\frac{2\pi}{L}\right)^2}$$

"non-interacting basis states"

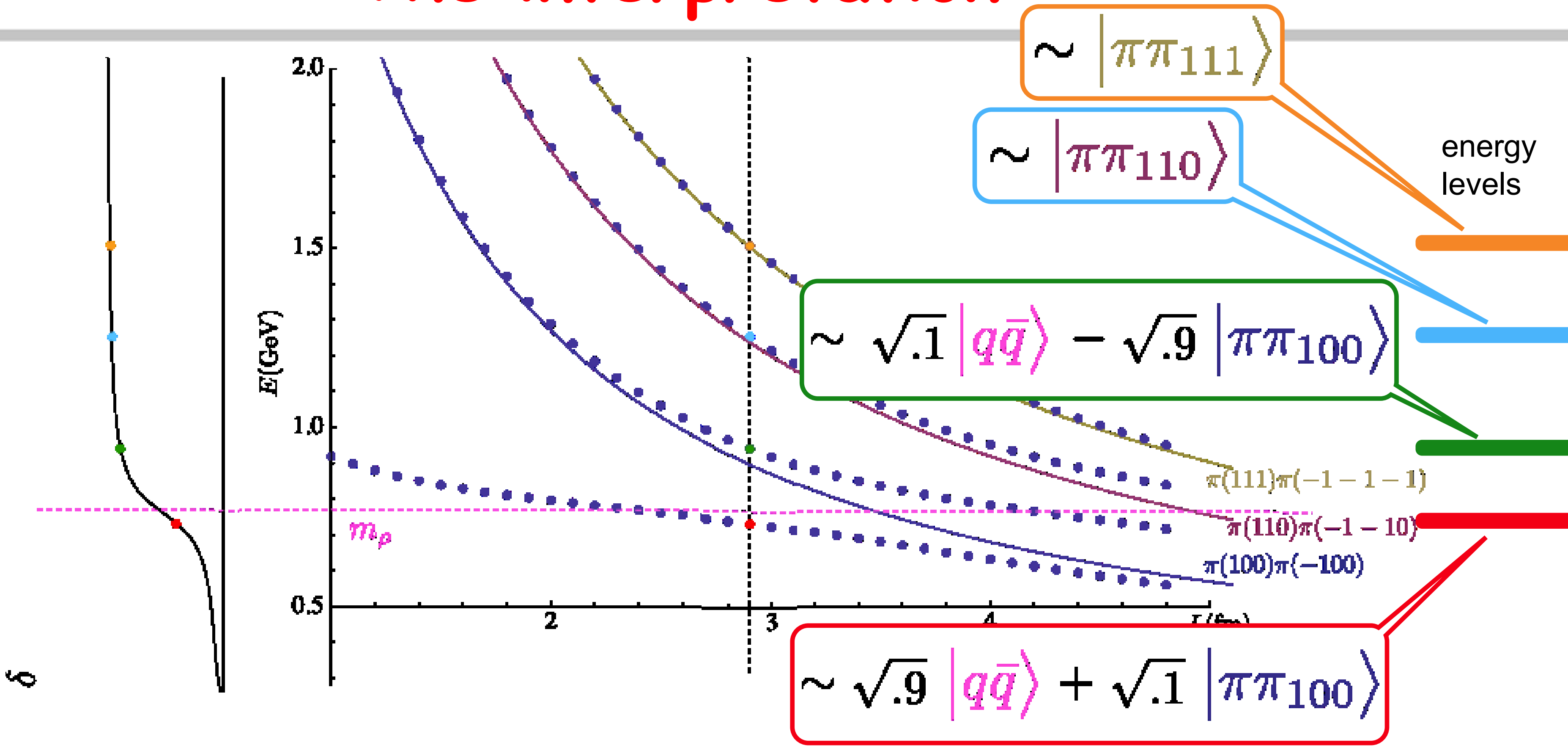
$$|\pi\pi_{100}\rangle$$

$$|q\bar{q}\rangle \qquad |\pi\pi_{110}\rangle$$

$$|\pi\pi_{111}\rangle$$

Level repulsion - just like quantum mechanical pert. theory

The interpretation



Multi-particle states

We don't see the meson-meson (or meson-baryon) states in the spectrum

Should we ? where will they be (roughly) ?

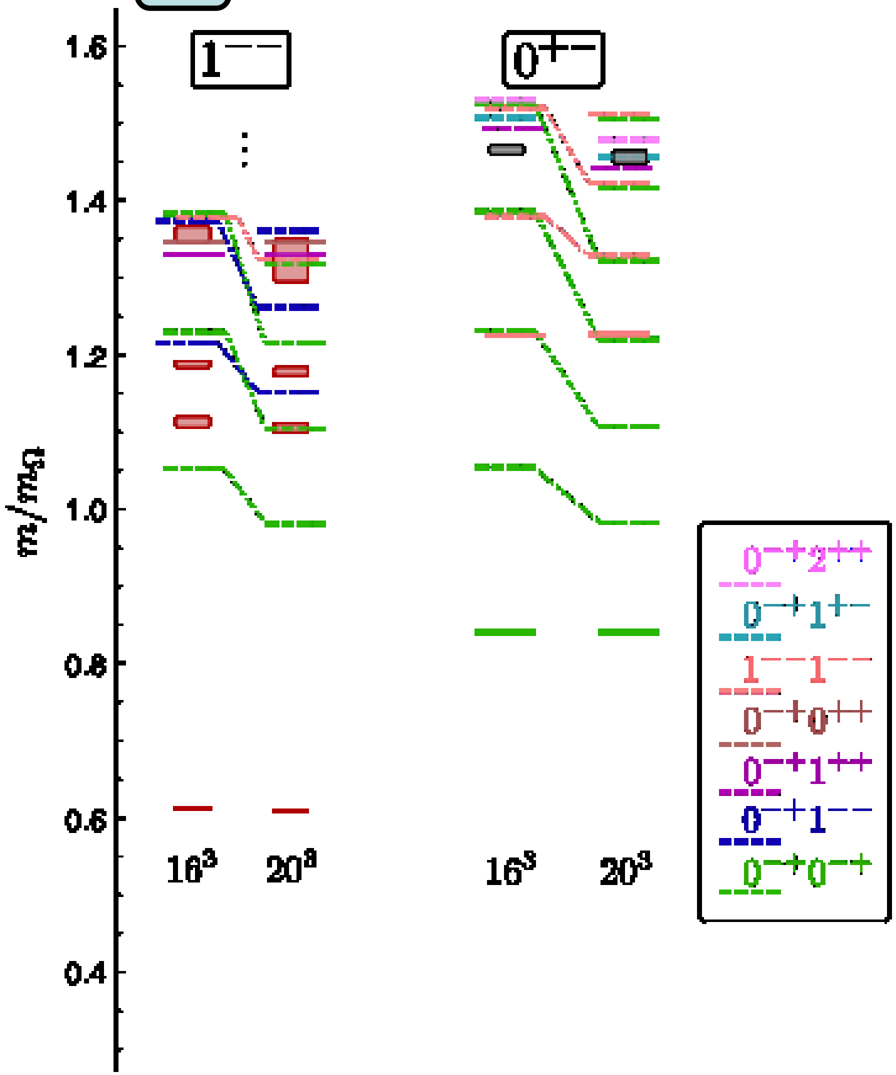
Plot the non-interacting meson levels as a guide

$|A(\vec{p})B(-\vec{p})\rangle$ $\quad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$

Need multi-particle operators for overlap

Annihilation diagrams. GPU-s to the rescue!

e.g.

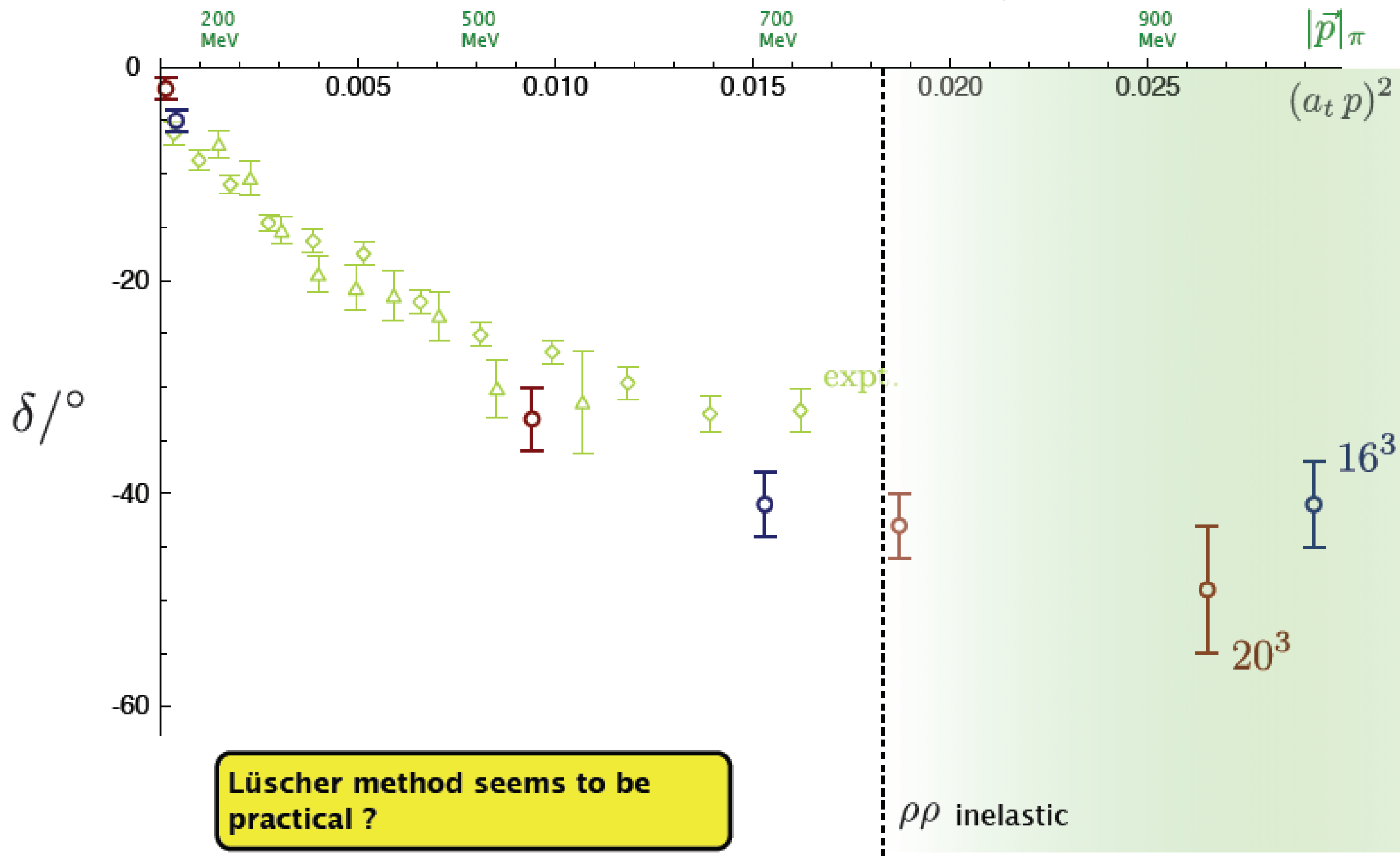


Phase Shifts: demonstration

ππ isospin=2

N_F = 2+1 (u,d,s) $m_\pi \sim$ 400 MeV

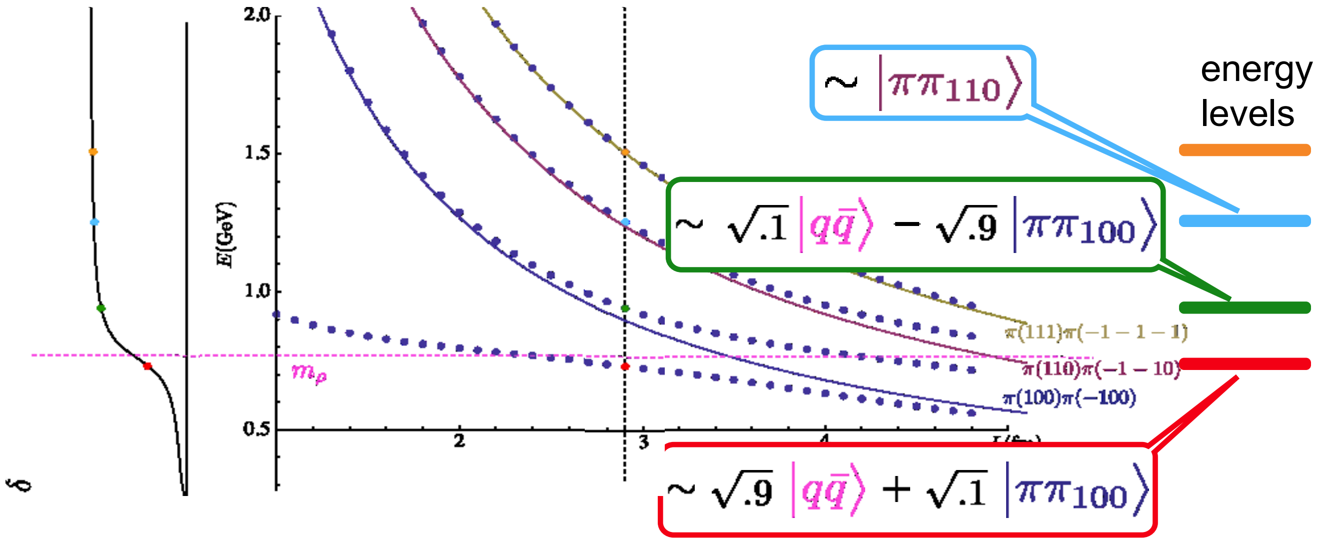
$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}[\bar{\psi}\gamma_5\psi](\vec{x}) \cdot \sum_{\vec{y}} e^{-i(-\vec{p})\cdot\vec{y}}[\bar{\psi}\gamma_5\psi](\vec{y})$$



Lüscher method seems to be practical ?

Where are the Form Factors??

- Previous efforts
  - Charmonium: excited state E&M transition FF-s (1004.4930)
  - Nucleon: 1st attempt: E&M Roper->N FF-s (0803.3020)
- Spectrum first!
  - Basically have to address "what is a resonance" up front
  - (Simplistic example): FF for a strongly decaying state: linear combination of states



Summary

- Strong effort in excited state spectroscopy
  - New operator & correlator constructions $\rightarrow$ high lying states
  - Finite volume extraction of resonance parameters – promising
  - Significant progress in last year, but still early stages
- Initial results for excited state spectrum:
  - Suggests baryon spectrum at least as dense as quark model
  - Suggests multiple exotic mesons within range of Hall D

- Resonance determination:
  - Start at heavy masses: have some "elastic scattering"
  - Already have smaller masses: move there + larger volumes (m_π~230MeV, L= 3 & 4fm)
  - Now: multi-particle operators & annihilation diagrams (gpu-s)
  - Starting physical limit gauge generation
  - Will need multi-channel finite-volume analysis for (in)elastic scattering

Backup slides

- The end

Towards resonance determinations

- Augment with multi-particle operators
  - Needs “annihilation diagrams” – provided by *Distillation*
    Ideally suited for (GPU-s)

arxiv:0905.2160

- Resonance determination
  - Scattering in a finite box – discrete energy levels
  - Lüscher finite volume techniques
  - Phase shifts $\rightarrow$ Width

- First results (partially from GPU-s)
  - Seems practical

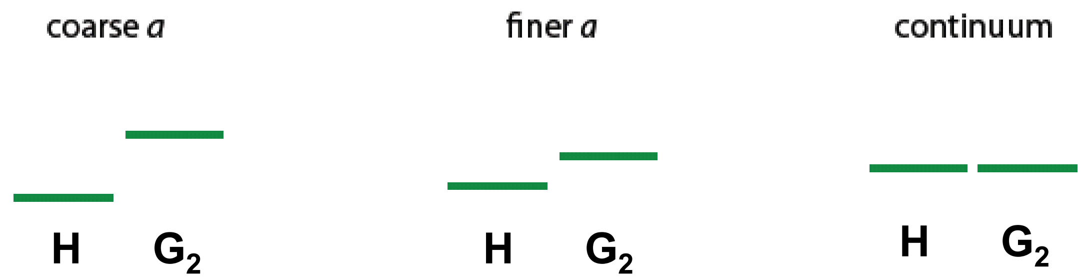
Backup slides

- The end

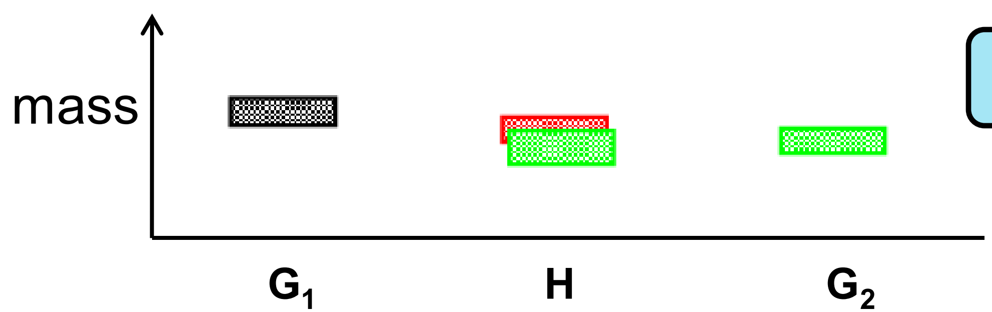
Determining spin on a cubic lattice?

Spin reducible on lattice

$$\mathrm{spin}\ \frac{5}{2}\ _{(6)} \longrightarrow H\ _{(4)} + G2\ _{(2)}$$



Might be dynamical degeneracies



Spin 1/2, 3/2, 5/2, or 7/2 ?

Spin reduction & (re)identification

Variational solution:

$$C_{ij}(t) = \langle 0|\phi_i(t)\phi_j(0)|0\rangle \rightarrow \sum_\alpha Z_i^{(\alpha)} Z_j^{(\alpha)*} e^{-m_\alpha t}$$

Continuum

$$\langle 0 \mid \mathcal{O} \mid \tfrac{5}{2}^- (\vec{0}, r)\rangle = Z$$

$$\downarrow$$

Lattice

$$\langle 0 \mid \mathcal{O}_H \mid \frac{5}{2}^- (\vec{0}, r)\rangle = Z_H$$

$$\langle 0 \mid \mathcal{O}_{G2} \mid \frac{5}{2}^- (\vec{0}, r)\rangle = Z_{G2}$$

Method: Check if converse is true

Correlator matrix: near orthogonality

Normalized Nucleon correlator matrix

C(t=5)

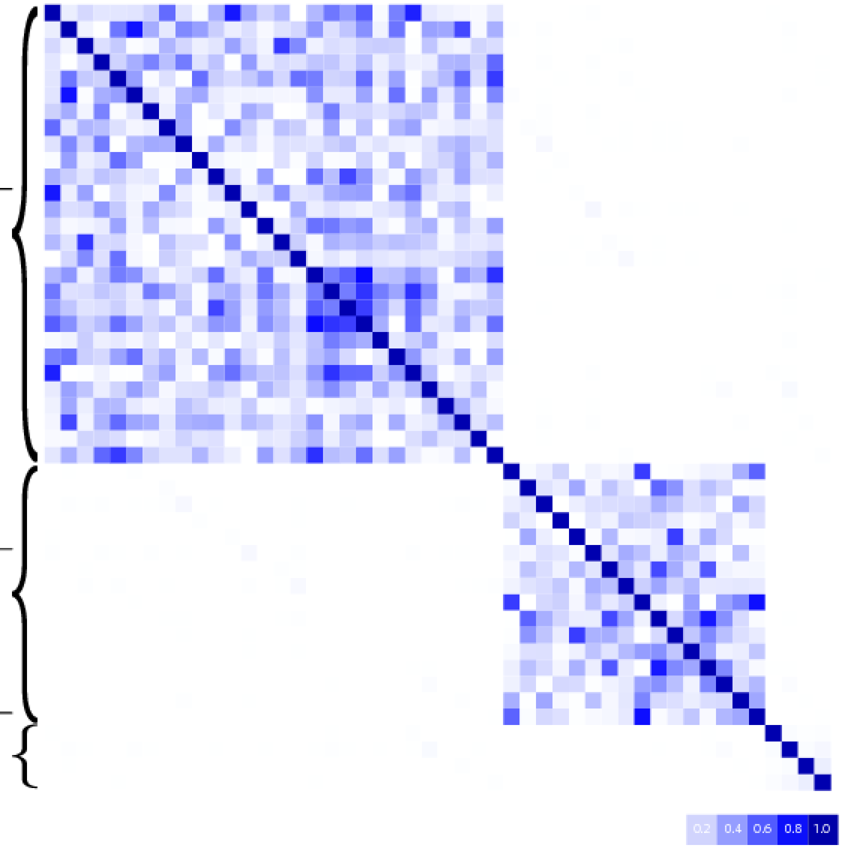
$$\frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

$$H_u$$

$J^P = \frac{3}{2}^-$

$J^P = \frac{5}{2}^-$

$J^P = \frac{7}{2}^-$



Near perfect factorization:

Continuum orthogonality

Small condition numbers ~ 200

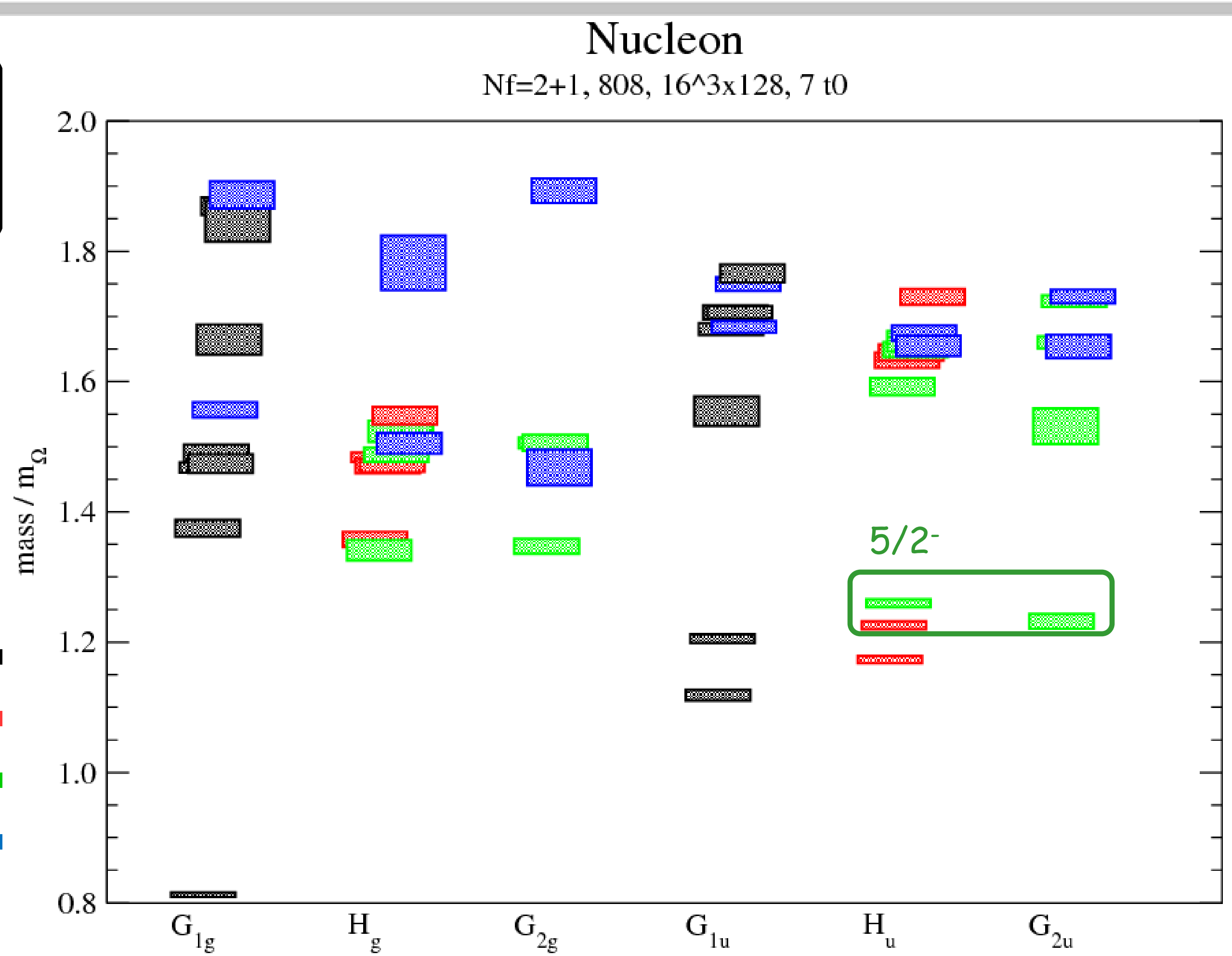
PRL (2007), arXiv:0707.4162 & 0902.2241 (PRD)

Nucleon spectrum in (lattice) group theory

$N_f = 2 + 1$, $m_\pi \sim 580$MeV

Units of Ω baryon

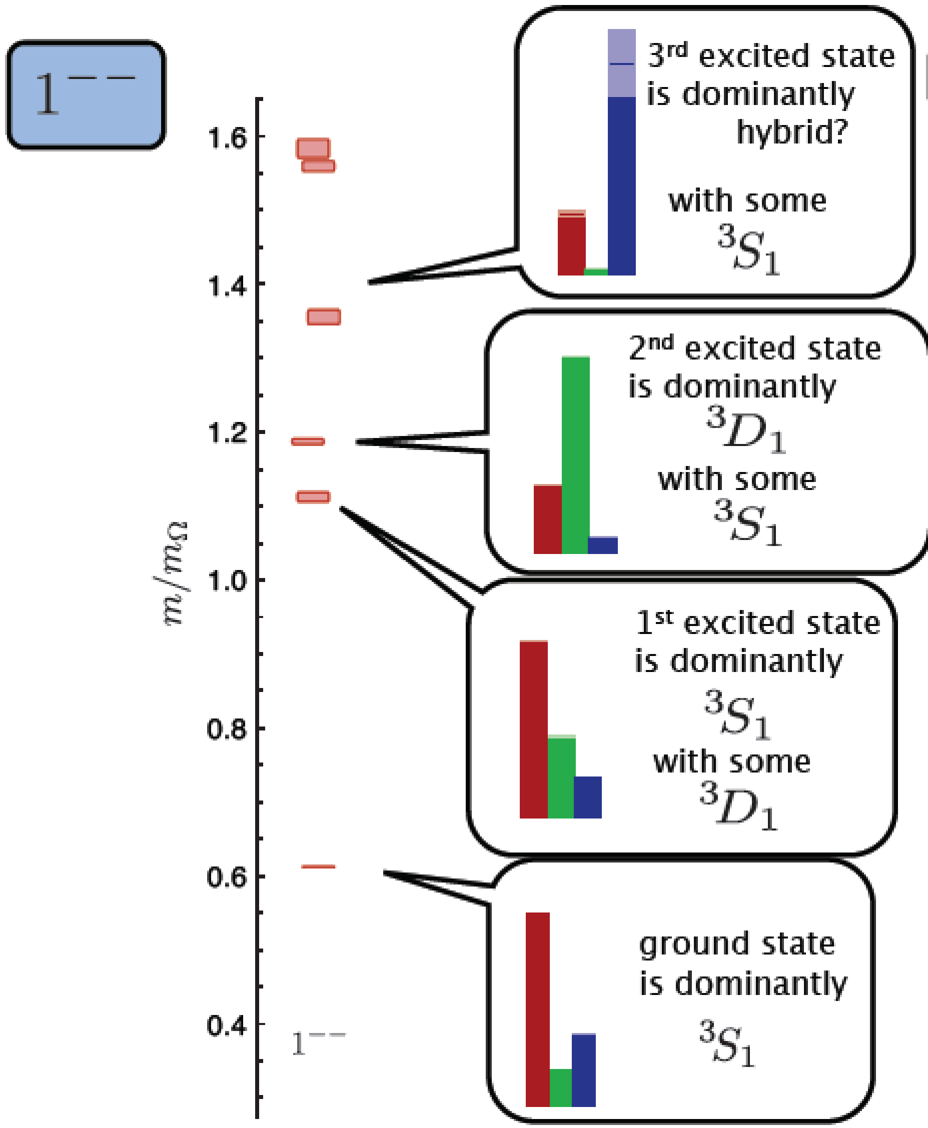
J = 1/2
J = 3/2
J = 5/2
J = 7/2



G_{1g}	H_g	G_{2g}	G_{1u}	H_u	G_{2u}
$1/2^+$	$3/2^+$	$5/2^+$	$1/2^-$	$3/2^-$	$5/2^-$
$7/2^+$	$5/2^+$	$7/2^+$	$7/2^-$	$5/2^-$	$7/2^-$
	$7/2^+$			$7/2^-$	

PRD 79(2009), PRD 80 (2009), 0909.0200 (PRL)

Interpretation of Meson Spectrum



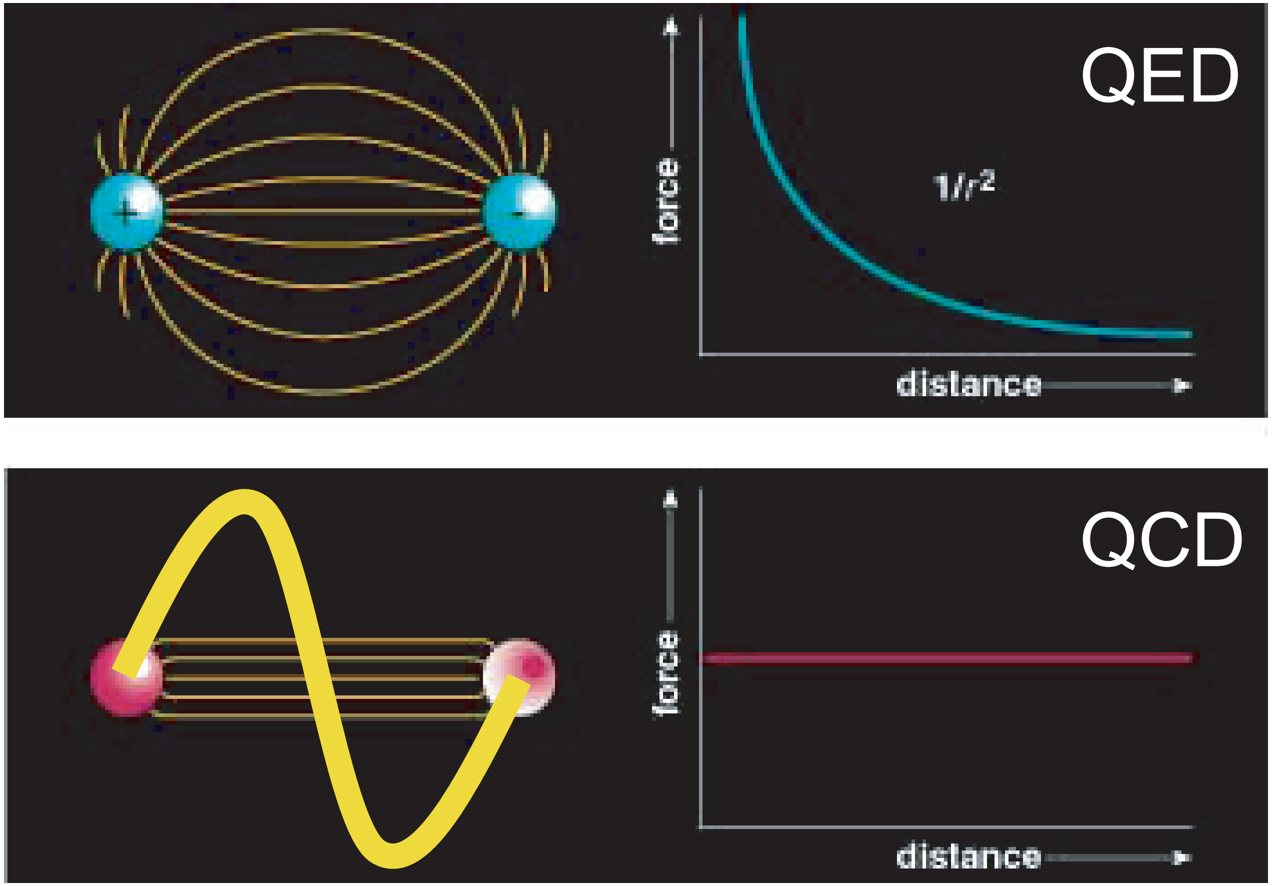
look at the 'overlaps' $Z_n^{\Gamma} = \langle n | \bar{\psi} \Gamma \psi | 0 \rangle$

ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

Future: incorporate in bound-state model phenomenology

Future: probe with photon decays

Exotic matter?



Can we observe exotic matter? *Excited string*

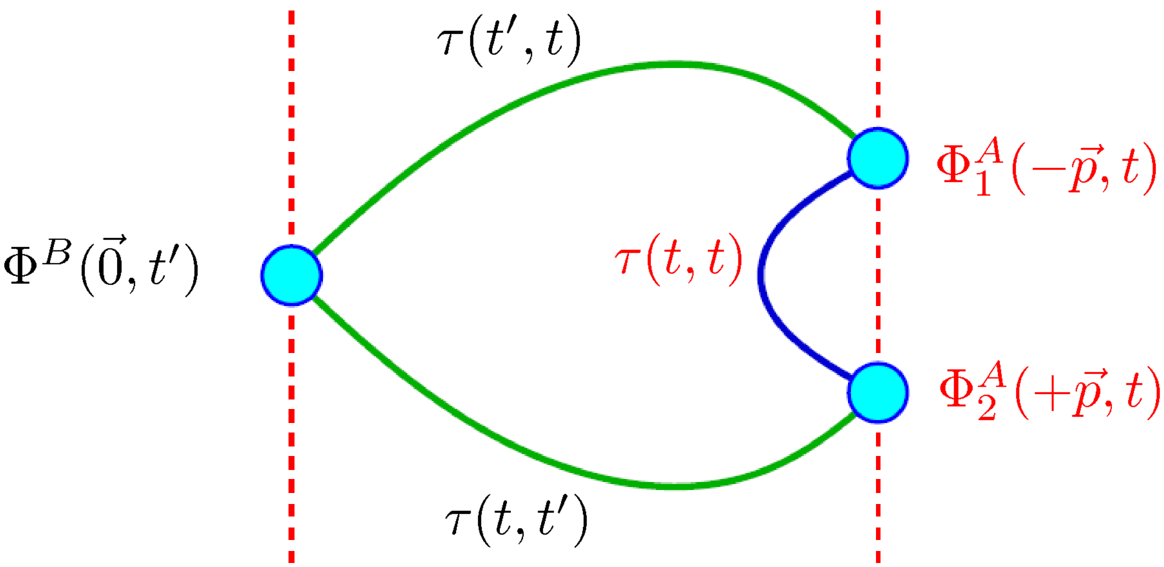
Distillation: annihilation diagrams

- Two-meson creation op

$$C(t',t) = \left\langle \chi^B(t') \left(\chi_1^A(-\vec{p})\chi_2^A(\vec{p})\right)^\dagger(t)\right\rangle$$

- Correlator

$$C_M^{(2)}(t',t) = \mathrm{Tr}\Big[\Phi^B(t')\,\tau(t',t)\,\left\{\Phi_1^A(t)\cdot\tau(t,t)\cdot\Phi_2^A(t)\right\}\,\tau(t,t')\Big]$$



arxiv:0905.2160

Operators and contractions

- New operator technique: *Subduction* PRL 103 (2009)
  - Derivative-based continuum ops -> lattice irreps
  - Operators at rest or in-flight, mesons & baryons

- Large basis of operators -> lots of contractions
  - E.g., nucleon H_g 49 ops up through 2 derivs
  - Order 10000 two-point correlators

- Feed all this to variational method

$$C_{AB}(t)v_B^{(n)}(t) = \lambda^{(n)}(t)C_{AB}(t_0)v_B^{(n)}(t)$$

  - Diagonalization: handles near degeneracies