Extraction of Resonances from

EBAC-DCC (Dynamical Coupled-Channel) Model

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$\boxed{\textbf{Introduction}}$

Extraction of N^* information from πN data is important !

- $*$ Understanding spectrum and structure of N^* within QCD and hadron structure models

Steps to extract N^*

1. Construct a reaction model through analysis of data

2. From the constructed model, resonance properties (pole position, vertex form factor) are extracted with analytic continuation

$\underline{N^* \text{ information}}$

πN scattering amplitude near a pole ($E \sim M_R$)

$$F_{\pi N}(E) \ \sim \ \frac{\bar{\Gamma}\,(M_R)\,\bar{\Gamma}\,(M_R)}{E - M_R} \ + \ \text{(regular terms)}$$

Parameters characterizing Resonance

$*$ Pole position of amplitude : M_R

$*$ $N^* \to MB$ decay vertex : $\bar{\Gamma}\,(M_R)$

POLE SEARCH !

Suzuki, Sato, Lee, PRC **79**, 025205 (2009)

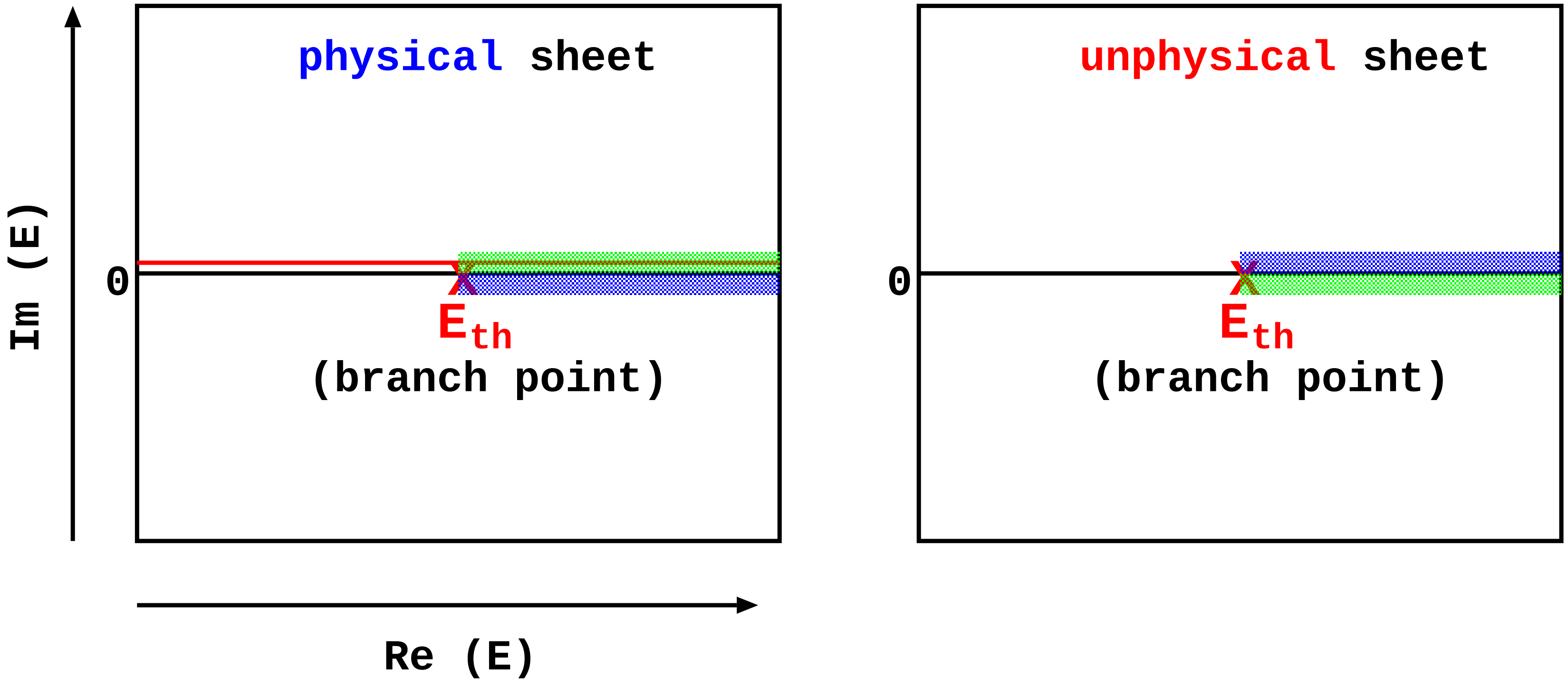
arXiv:0910.1742

Multi-layered structure of complex energy plane

e.g., single-channel meson-baryon scattering

$$T(p', p; E) \;=\; V(p', p) \;+\; \int dq q^2 \; V(p', q) \; G(q, E) \; T(q, p; E)$$

Scattering amplitude is a double-valued function of E !



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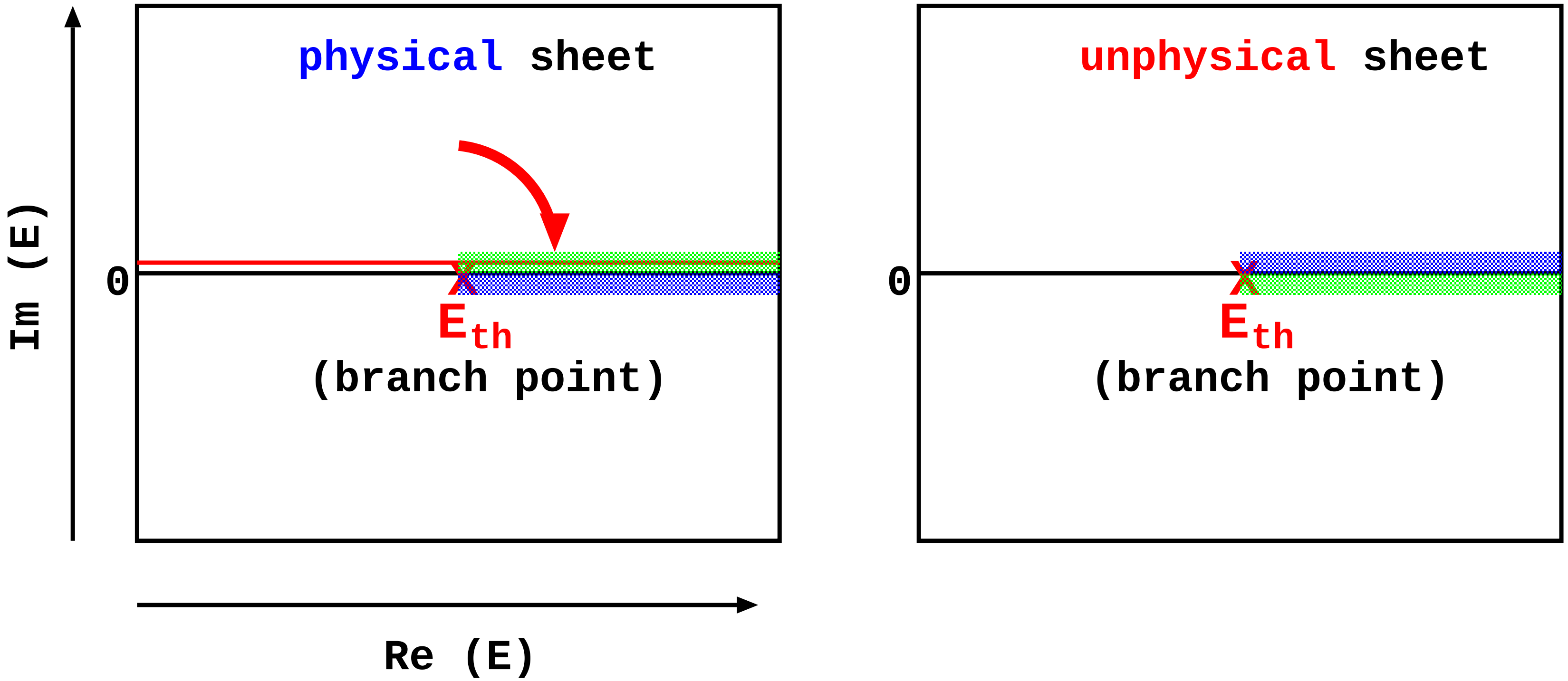

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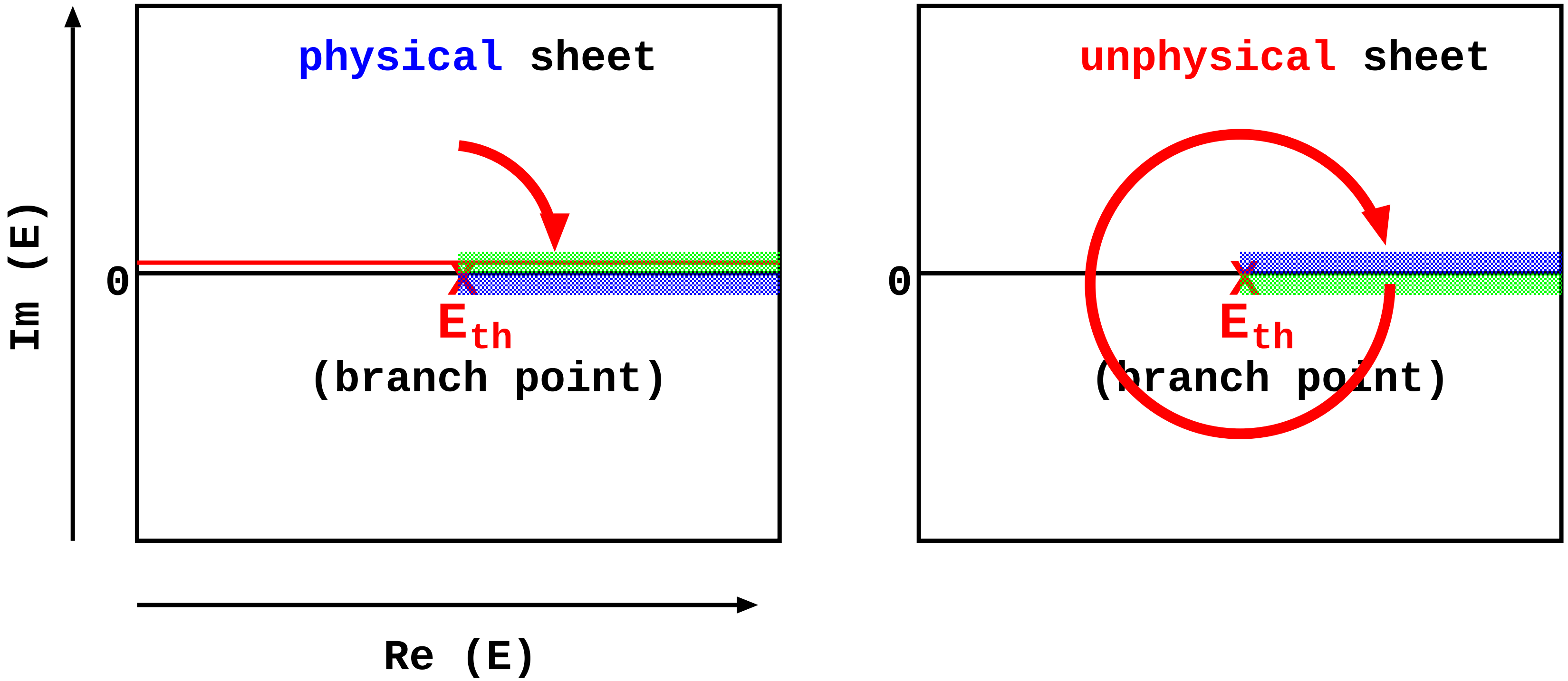

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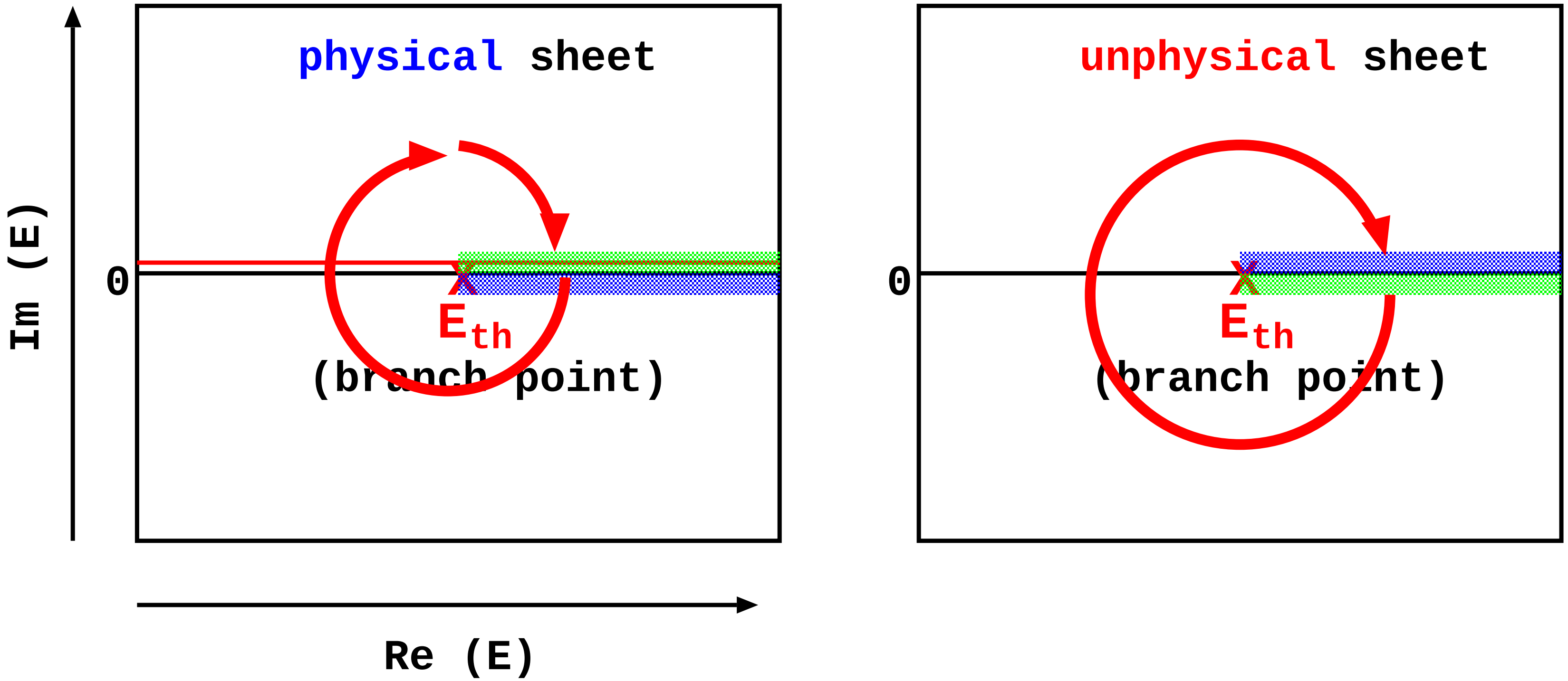


$$\text{N-channel case} \Longrightarrow 2^N \text{ Riemann sheets}$$

$$\text{2-channel case} \Longrightarrow 4 \text{ Riemann sheets}$$

$$(\text{channel 1, channel 2}) = (p,p),\ (u,p),\ (p,u),\ (u,u)$$

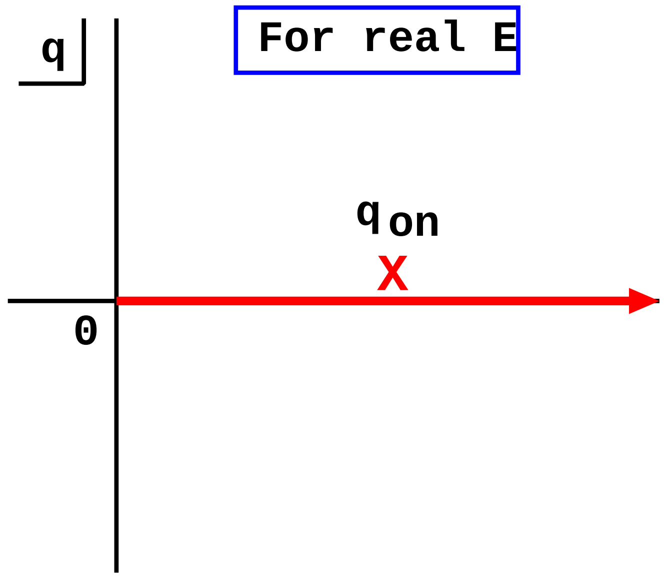
$$p: \text{ physical sheet}$$

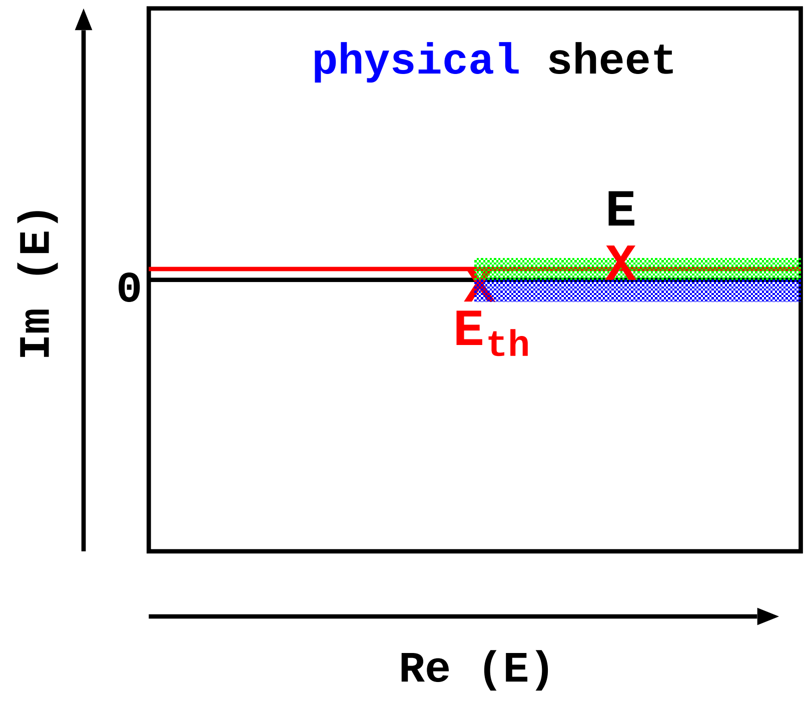
$$u: \text{ unphysical sheet}$$

How to choose Riemann sheet of complex E-plane

$$T(p', p; E) \;=\; V(p', p) \;+\; \int_C dq q^2 \; V(p', q) \; G_{MB}(q, E) \; T(q, p; E)$$

$$G_{MB}(q, E) = \frac{1}{E - E_M(q) - E_B(q) + i\epsilon} \quad , \qquad E_X(q) = \sqrt{q^2 + m_X^2}$$





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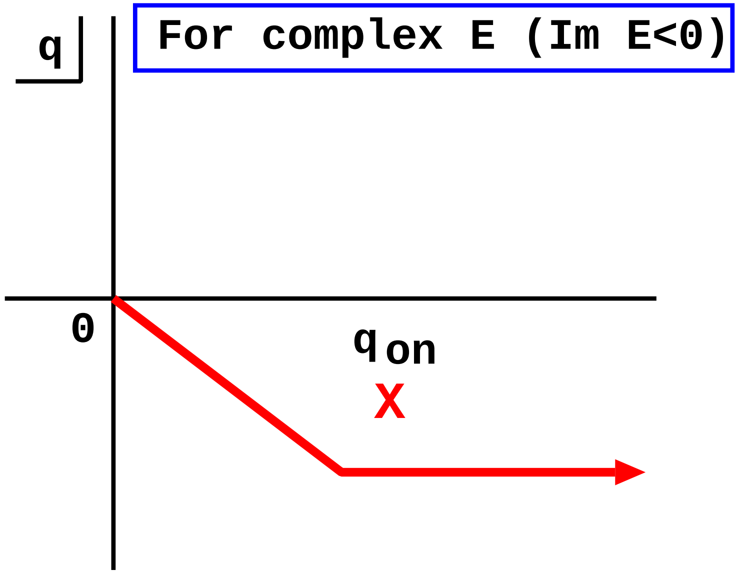

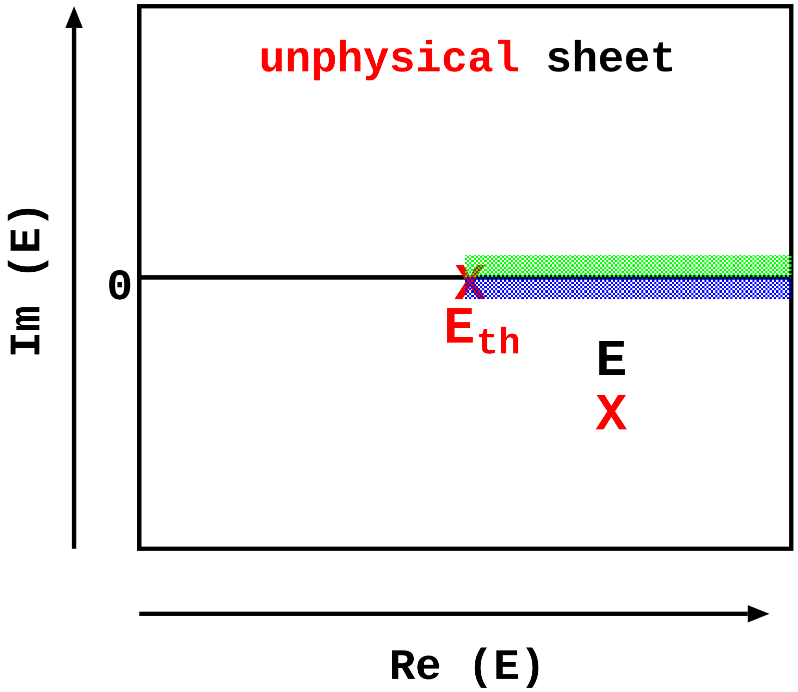

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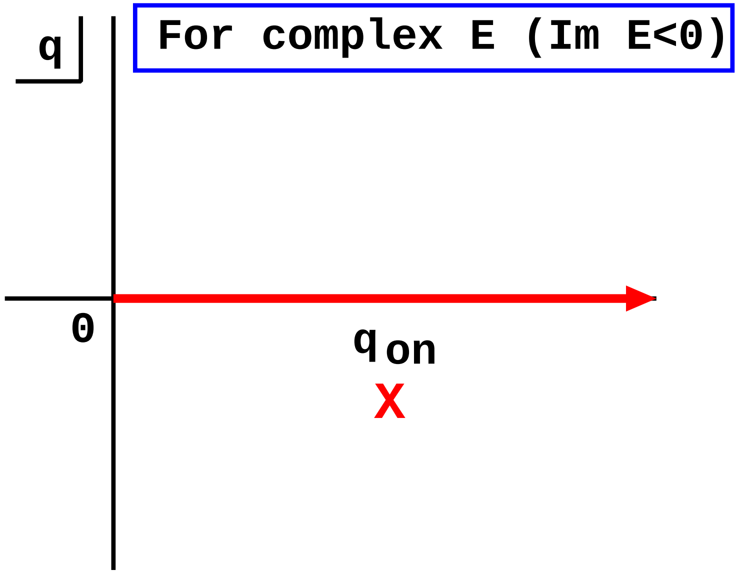
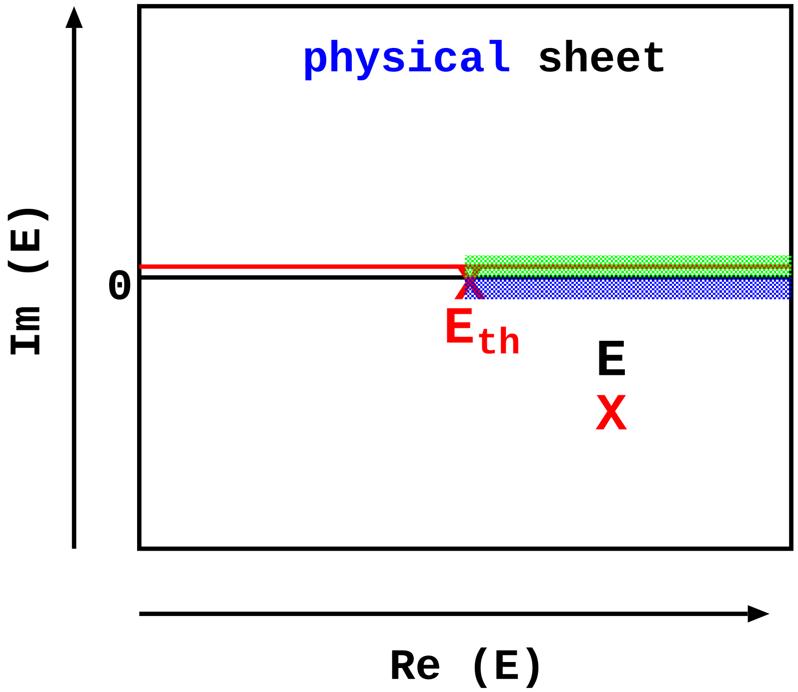
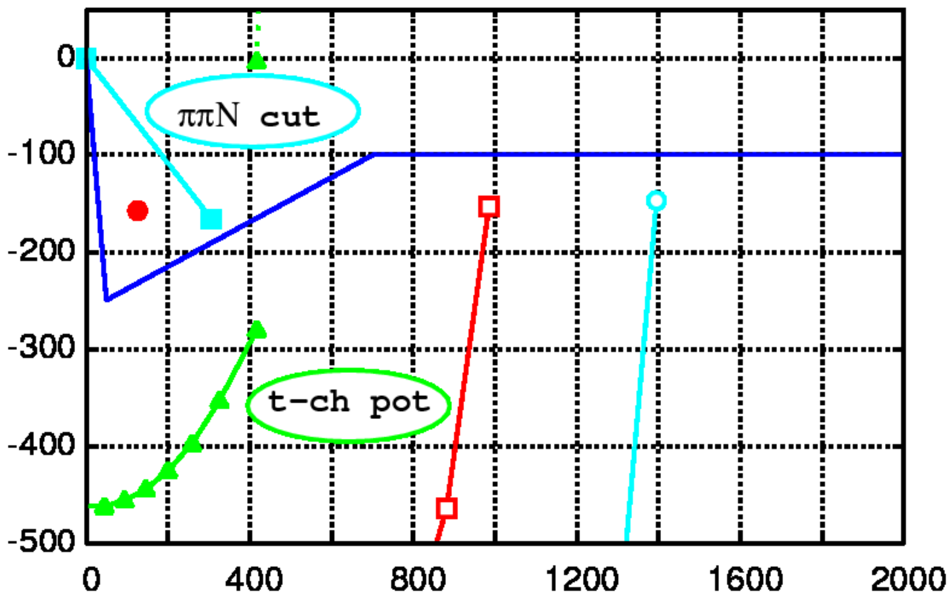


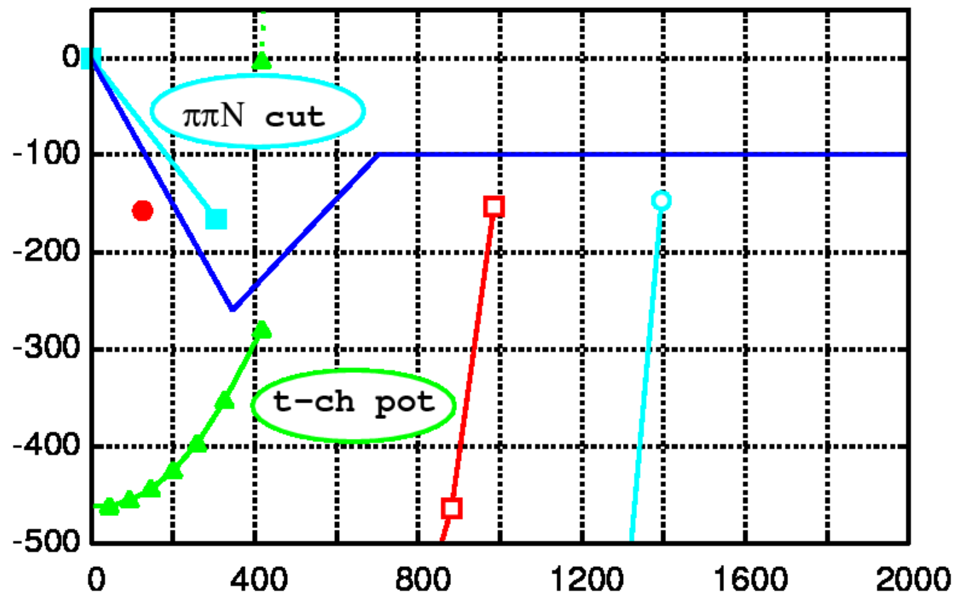


Choose momentum integral path to avoid singularities

Complex Momentum planes



Path to see unphysical sheet



Path to see physical sheet

Procedure to find a pole position

1. Choose a complex energy

2. For the chosen energy, choose an appropriate momentum path

   (avoid singularity, select sheet)

3. With the chosen path, solve Lippmann-Schwinger equation

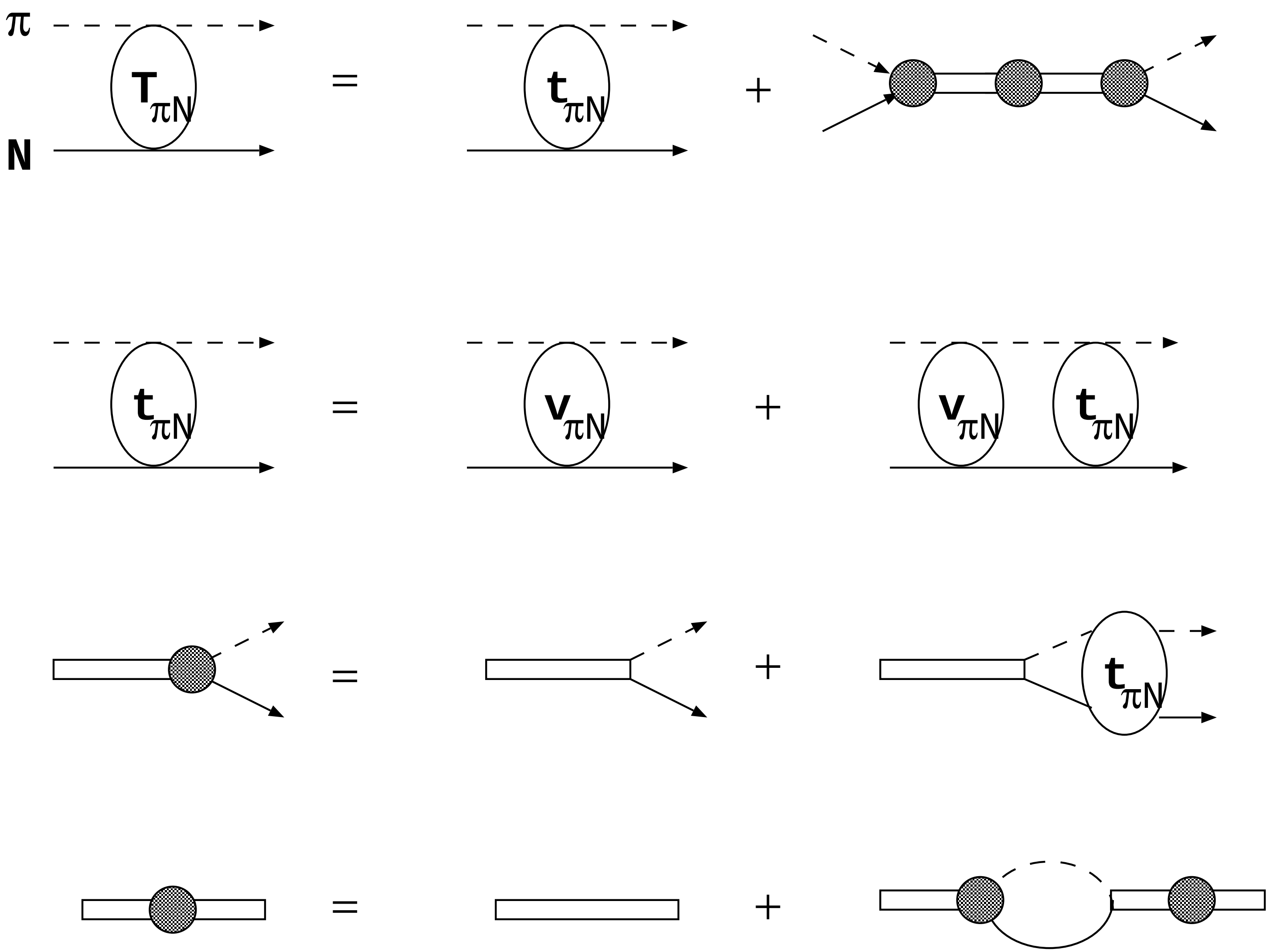
   to obtain T-matrix

4. Repeat the above steps 1-3, until a complex energy,

   which gives singular T-matrix, is found.

   The energy is the pole position.

More Practical ! More Detail !



$$T_{\alpha\beta} \;=\; t^{NR}_{\alpha\beta} + t^{R}_{\alpha\beta} \qquad (\alpha,\ \beta \;=\; \pi N,\ \eta N,\ \pi\Delta,\ \rho N,\ \sigma N)$$

$$t^{R}_{\alpha\beta} \;=\; \sum_{i,j} \bar{\Gamma}_{\alpha,i}\ [G_{N^*}]_{ij}\ \bar{\Gamma}_{\beta,j}$$

$$\bar{\Gamma}_{\alpha,j} \;=\; \Gamma_{\alpha,j} + \sum_{\gamma} \int_C dq q^2\ t_{\alpha\gamma}\ G_{\gamma}\ \Gamma_{\gamma,j}$$

$$[G^{-1}_{N^*}]_{ij} \;=\; (E - m_{N^*_i})\delta_{i,j} - \Sigma_{i,j}(E)$$

$$\Sigma_{i,j}(E) \;=\; \sum_{\gamma} \int_C dq q^2 \Gamma_{\gamma,i}\ G_{\gamma}\ \bar{\Gamma}_{\gamma,j}$$

At pole , $\det[G^{-1}_{N^*}] = 0$ because $[G_{N^*}]_{ij} \;=\; \dfrac{C_{ij}}{\det[G^{-1}_{N^*}]}$

Table 1: The resonance pole positions M_R [listed as (Re $M_R, -$Im M_R)] Re$(E) \leq 2000$ MeV and $-$Im$(E) \leq 250$ MeV [PRL **104**, 042302 (2010)]

	$M^0_{N^*}$	M_R (JLMS)	Location	PDG
S_{11}	1800	(1540, 191)	($uuuupp$)	(1490 - 1530, 45 - 125)
	1880	(1642, 41)	($uuuupp$)	(1640 - 1670, 75 - 90)
P_{11}	1763	(1357, 76)	($upuupp$)	(1350 - 1380, 80 - 110)
	1763	(1364, 105)	($upuppp$)	
	1763	(1820, 248)	($uuuuup$)	(1670 - 1770, 40 - 190)
P_{13}	1711	—		(1660 - 1690, 57 - 138)
D_{13}	1899	(1521, 58)	($uuuupp$)	(1505 - 1515, 52 - 60)
D_{15}	1898	(1654, 77)	($uuuupp$)	(1655 - 1665, 62 - 75)
F_{15}	2187	(1674, 53)	($uuuupp$)	(1665 - 1680, 55 - 68)
S_{31}	1850	(1563, 95)	(u–uup–)	(1590 - 1610, 57 - 60)
P_{31}	1900	—		(1830 - 1880, 100 - 250)
P_{33}	1391	(1211, 50)	(u–ppp–)	(1209 - 1211, 49 - 51)
	1600	—		(1500 - 1700, 200 - 400)
D_{33}	1976	(1604, 106)	(u–uup–)	(1620 - 1680, 80 - 120)

Evaluation of Residue !

Suzuki, Sato, Lee, arXiv:0910.1742

πN residue

$$t^R_{\alpha\beta} \quad = \quad \sum_{i,j} \bar{\Gamma}_{\alpha,i} \; [G_{N^*}]_{ij} \; \bar{\Gamma}_{\beta,j} \qquad\qquad (\alpha = \beta = \pi N)$$

$$[G^{-1}_{N^*}]_{ij} \quad = \quad (E - m_{N^*_i})\delta_{i,j} - \Sigma_{i,j}(E)$$

For $E \to M_R$, $\qquad [G_{N^*}(E)]_{ij} \;=\; \dfrac{\chi_i \chi_j}{E - M_R}$

$$\sum_j (G_{N^*}(M_R)^{-1})_{ij} \; \chi_j \;=\; 0 \qquad \Longrightarrow \qquad \sum_j [m_{N^*_i}\delta_{ij} + \Sigma(M_R)_{ij}] \; \chi_j \;=\; M_R \; \chi_i$$

M_R : eigenvalue , $\quad \chi_i$: eigenvector

$$t^R(E \to M_R) \quad \sim \quad \frac{\bar{\Gamma}^R \; \bar{\Gamma}^R}{E - M_R} \quad \propto \quad \frac{Re^{i\phi}}{E - M_R}$$

$$\bar{\Gamma}^R \quad = \quad \sum_j \chi_j \bar{\Gamma}_j(p = p^0, E = M_R)$$

	JLMS		GWU-VPI		Cutkosky		Jülich	
	R	ϕ	R	ϕ	R	ϕ	R	ϕ
$P_{33}(1210)$	52	-46	52	-47	53	-47	47	-37
$P_{11}(1356)$	37	-111	38	-98	52	-100	48	-64
(1364)	64	-99	86	-46	-	-		
(1820)	20	-168	-	-	9	-167	-	-

- Larger difference in P_{11} resonance

Analysis	P11 poles (MeV)	
JLMS	(1357, 76)	(1364, 105)
CMB	(1370, 114)	(1360, 120)
GWU/VPI	(1359, 82)	(1388, 83)
Jülich	(1387, 74)	(1387, 71)

$\Rightarrow$ Simultaneous fit to inelastic channels $(\pi N \rightarrow \pi N, \pi\Delta, \rho N, \sigma N)$ could improve the agreement   cf S. Ceci et al., PRL **97**, 062002 (2006)

* $\underline{\gamma^{(*)}N \rightarrow N^* \text{ transition form factor}}$

$$t^R_{\alpha\beta} = \sum_{i,j} \bar{\Gamma}_{\alpha,i} \, [G_{N^*}]_{ij} \, \bar{\Gamma}_{\beta,j} \qquad (\alpha = \pi N, \quad \beta = \gamma^{(*)}N)$$

$$= \sum_{i,j} \bar{\Gamma}_{\alpha,i} \, \frac{\chi_i \chi_j}{E - M_R} \bar{\Gamma}_{\beta,j} \qquad \text{for } E \rightarrow M_R$$

$$= \frac{\bar{\Gamma}^R_\alpha \, \bar{\Gamma}^R_\beta}{E - M_R}$$

Definition of helicity amplitude in EBAC-DCC model

$$A_{3/2}(Q^2) = X < N^*, s_z = 3/2| - \vec{J}(Q^2) \cdot \vec{\epsilon}_{+1} | N, s_N = 1/2 >$$

$$= X X' \, \bar{\Gamma}^R_{\gamma^{(*)}N}(Q^2, M_R, \lambda_\gamma = 1, \lambda_N = -1/2)$$

$A_{3/2}$ is complex

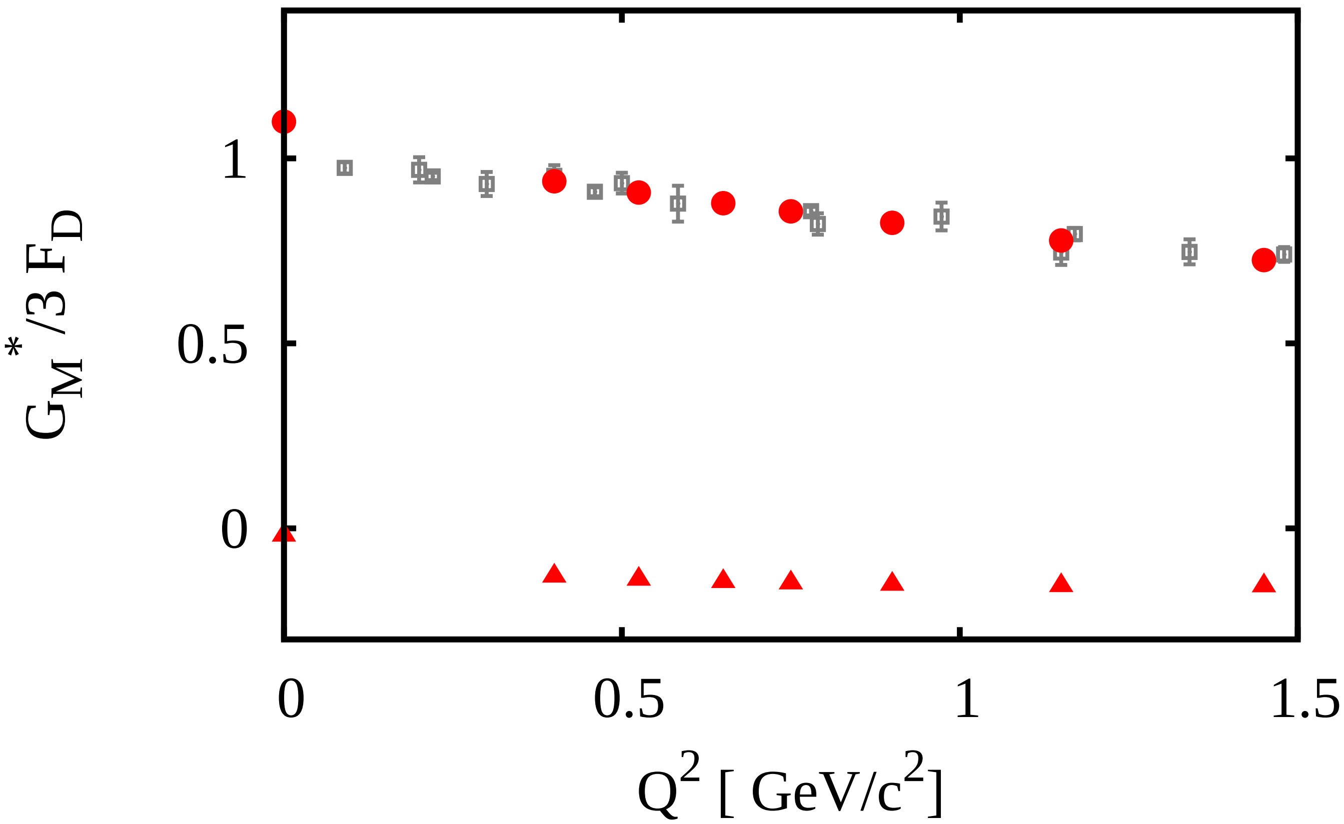
Definition based on Breit-Wigner parameterization

$$
\begin{aligned}
A_{3/2} &= \frac{[l(l+2)]^{1/2}}{2}(\mathcal{E}_{l+} - \mathcal{M}_{l+}) \\
\mathcal{M}_{l\pm} &\equiv \mathrm{Im}\left[M_{l\pm}(W = M_{BW})\right]/c_{kin}
\end{aligned}
$$

- $A_{3/2}$ is real

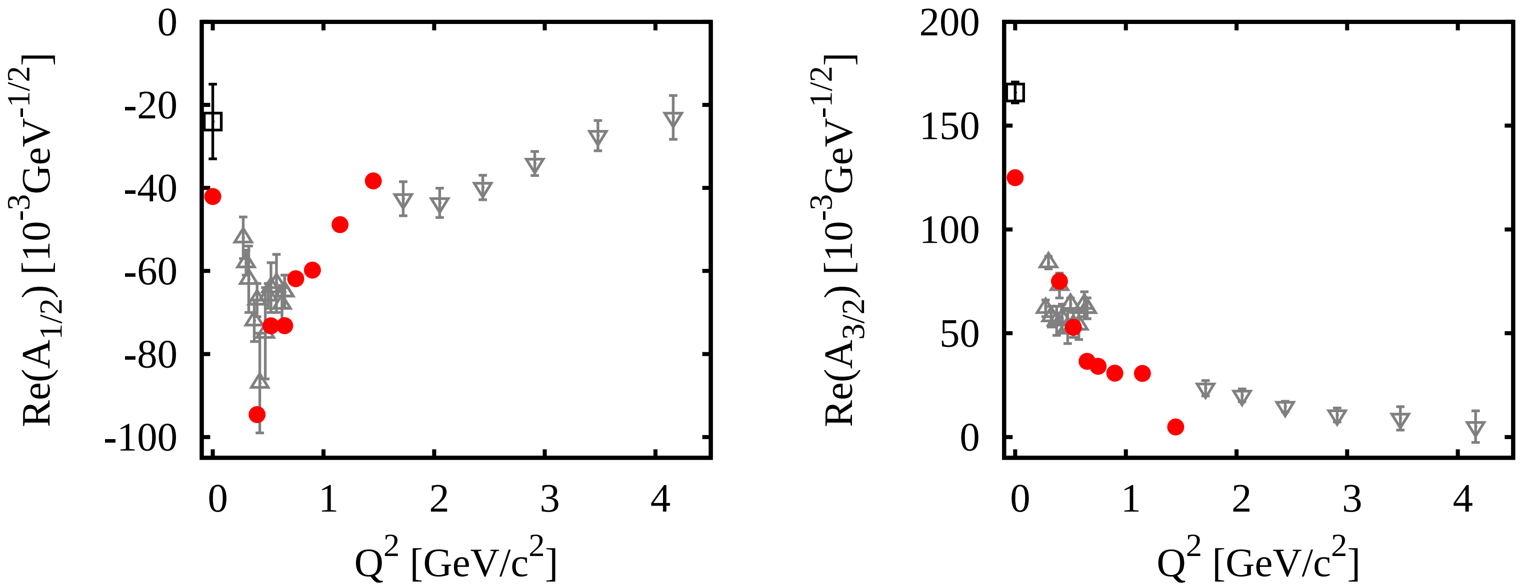
The magnetic N-Δ (1232) transition form factor $G^*_M(Q^2)$

Suzuki et al., arXiv:0910.1742



$\gamma N \to N^*(D_{13}(1520))$ form factors

Suzuki et al., arXiv:0910.1742



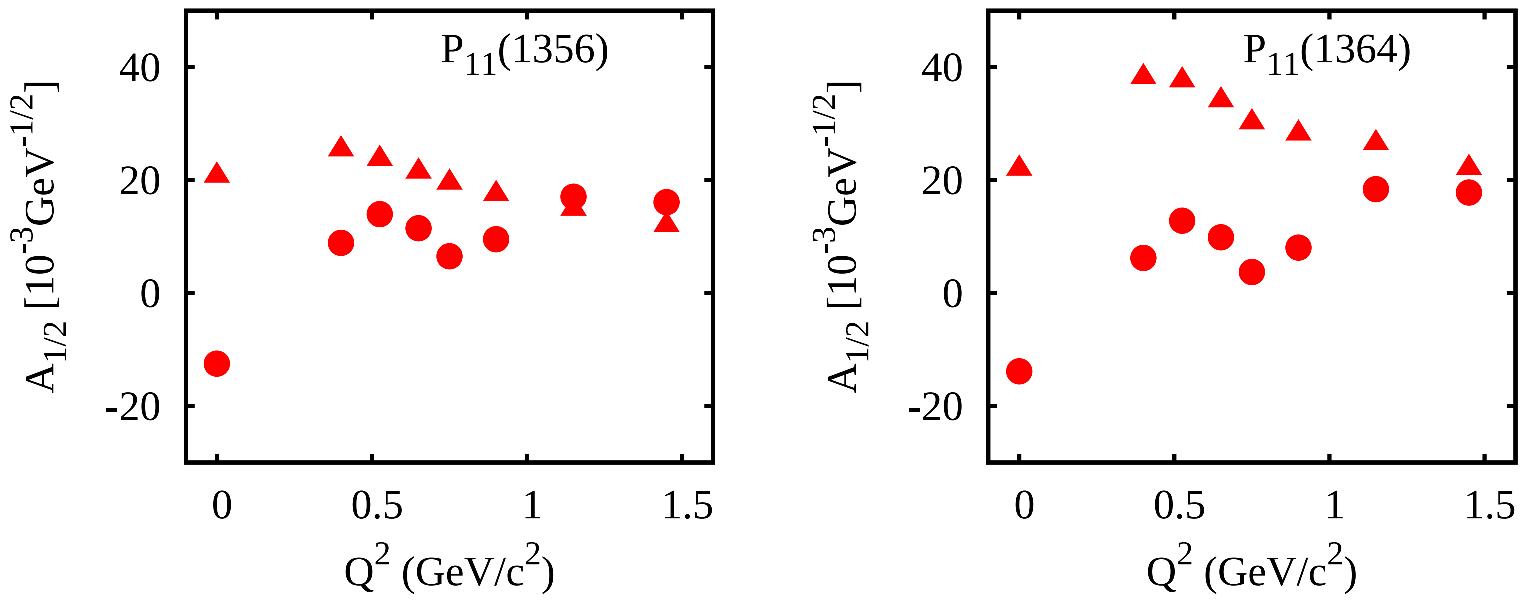
Data from CLAS collaboration arXiv:0909.2349; 0906.4081

Real part dominates

$\Rightarrow$ good agreement with previous analysis based on BW parameterization

$\gamma N \to N^*(1356), N^*(1364)$ form factors of P_{11}

Suzuki et al., arXiv:0910.1742



circles (triangles) are real (imaginary) parts

Large imaginary parts, two poles

$\Rightarrow$ direct comparison with previous analysis is not meaningful