Meson-Baryon model JM for the Nucleon Resonance Studies in Charged Double Pion Electroproduction

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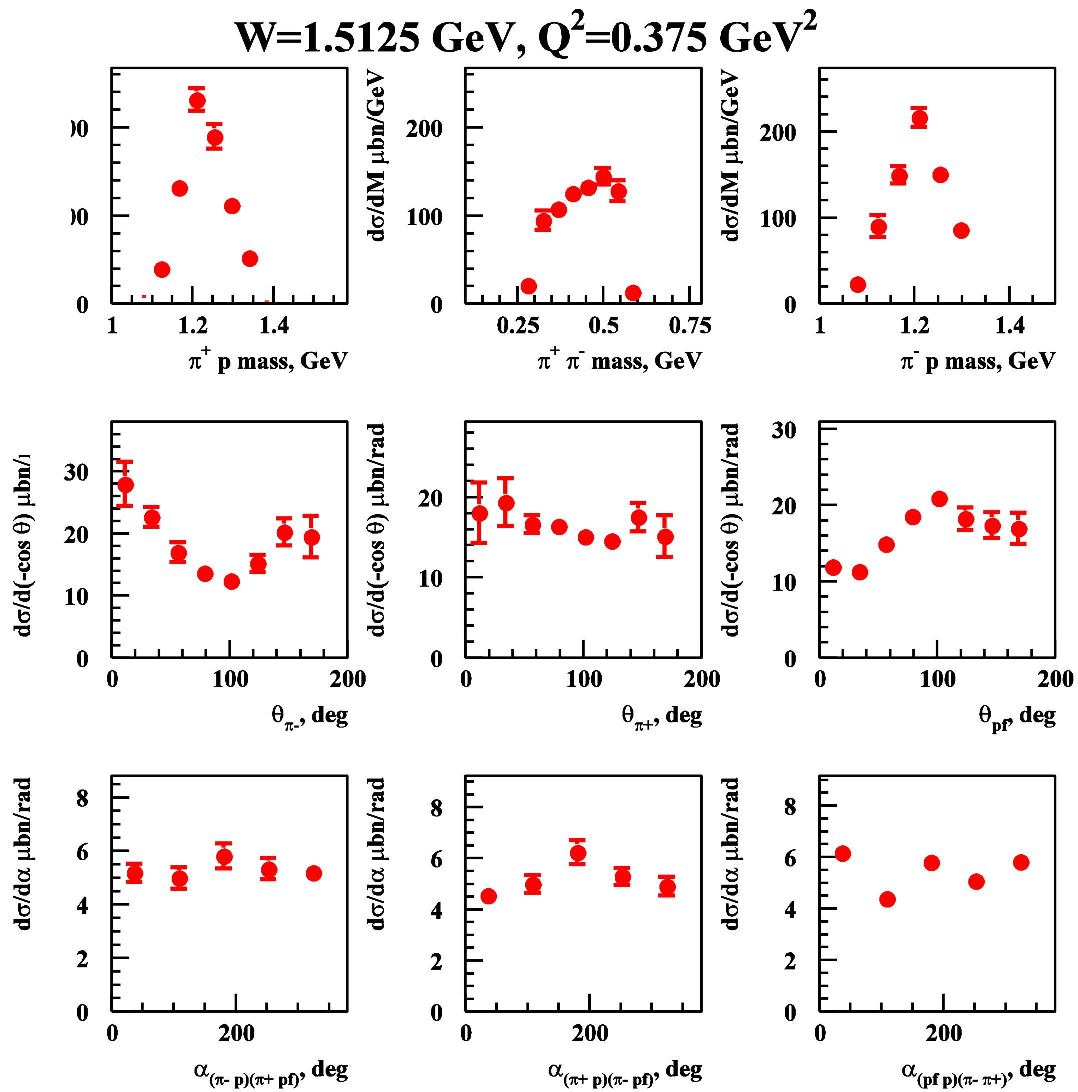
- CLAS $\pi^+\pi^-p$ electroproduction data and JM model basic.
- Definition of resonance parameters.
- Extraction of non-resonant contributions and N* parameters from the fit of CLAS data on $\pi^+\pi^-p$ electroproduction
- Conclusions and outlook

$N\pi\pi$ electroproduction data from CLAS

The measurements with an unpolarized e^- beam onto a proton target offer nine differential cross sections in each (W,Q^2) bin.



Number data points > 17500

$1.3 < W < 2.1$ GeV ;

$0.25 < Q^2 < 1.5$ GeV^2

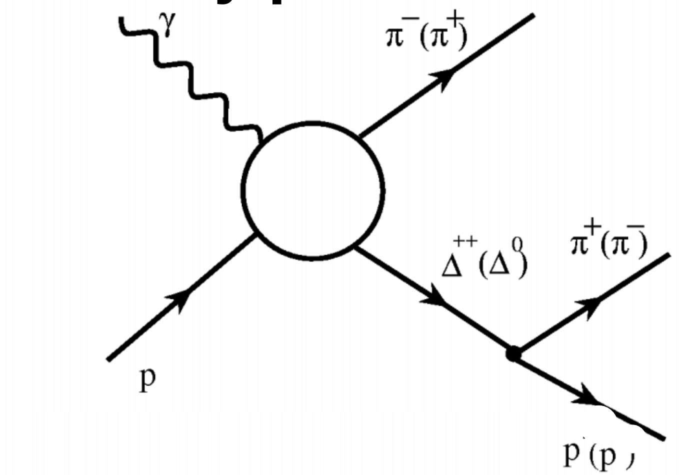
prelim. $2.0 < Q^2 < 5.0$ GeV^2

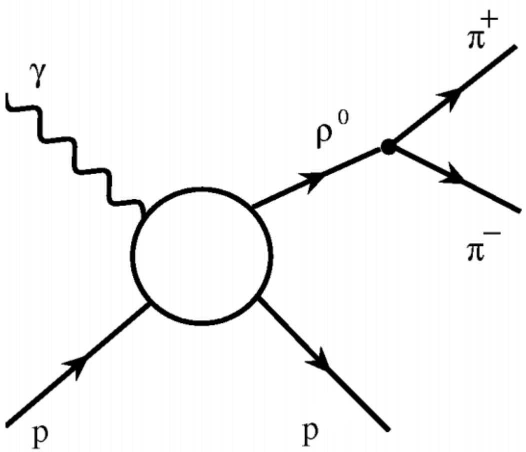
M. Ripani *et al*., PRL,91, 022002 (2003);

G. Fedotov *et al*., PRC 79, 015204 (2009).

Nππ electroproduction mechanisms incorporated into JM model.

3-body processes:





Isobar channels included:

$\pi^-\Delta^{++}$

- **All well established N*s with πΔ decays and 3/2⁺(1720) candidate, seen in CLAS Nππ data.**
- **Reggeized Born terms with effective FSI & ISI treatment .**
- **Extra πΔ contact term.**

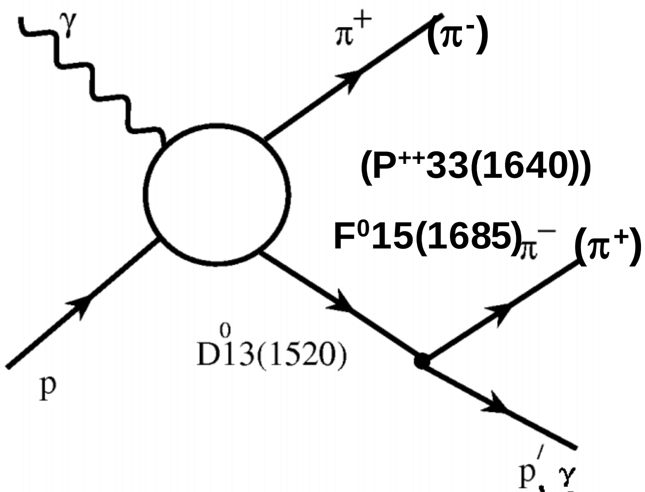
ρp

•All well established N*s with ρp decays and 3/2⁺(1725) candidate.

•Diffractive ansatz for non-resonant part and ρ-line shrinkage in N* region.

continued

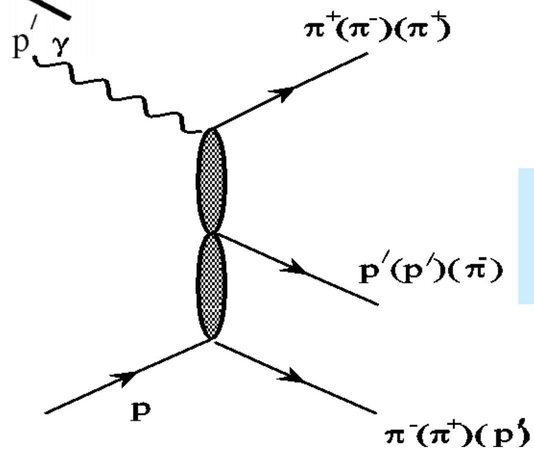
3-body processes:



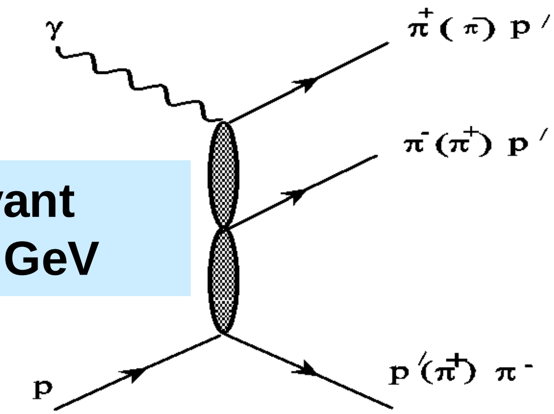
Isobar channels included:

• $\pi^+D^0_{13}(1520)$, $\pi^+F^0_{15}(1685)$, $\pi^-P^{++}_{33}(1640)$ isobar channels; observed for the first time in the CLAS data at *W* > 1.5 GeV.

Direct 2π production



Most relevant at W<1.65 GeV



Full model description with all expressions for contributing amplitudes can be found in:

- V. Mokeev, V.Burkert, T-S.H.Lee *et al*., Phys. Rev. C80, 045212 (2009).
- https://www.jlab.org/~mokeev/jm/mokeev_pipip.pdf.

Definition of N* parameters in JM

Regular BW ansatz for resonant amplitudes

contain only phases described in the slides #11,12

$$\langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle = \sum_{N^*} \frac{\langle \lambda_f | T_{dec} | \lambda_R \rangle \langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle}{M_r^2 - W^2 - i\Gamma_r(W) M_r}$$

where M_r, Γ_r are resonance mass and energy-dependent total width and $\langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle$, $\langle \pi \lambda_f | T_{dec} | \lambda_R \rangle$ are electromagnetic production and strong decay amplitudes of N^* with helicity λ_R, respectively. Indicies λ_f stand for the final states populated in N^* decays in helicity representation.

Hadronic parameters:

The $\langle \lambda_f | T_{dec} | \lambda_R \rangle$ hadr. dec. amplitudes were estimated from $\Gamma_{f,LS}(W)$ partial hadronic decay widths, that were transformed to $\Gamma_{\lambda_f}(W)$:

$$\langle \lambda_f | T | \lambda_R \rangle = \langle \lambda_f | T_{dec}^{J_r} | \lambda_R \rangle d_{\mu\nu}^{J_r}(cos\theta^*) e^{i\mu\phi^*},$$
$$\mu = \lambda_R,$$
$$\nu = -\lambda_\Delta \ for \ \pi\Delta$$
$$\nu = \lambda_{p'} - \lambda_\rho \ for \ \rho p,$$

where J^r is N^* spin; θ^*, ϕ^* spherical angles for stable fin. particle emission in the N^* rest frame

real

$$\langle \lambda_f | T_{dec0}^{J_r} | \lambda_R \rangle = \frac{2\sqrt{2\pi}\sqrt{2J_r + 1} M_{N^*} \sqrt{\Gamma_{\lambda_f}}}{\sqrt{\langle P_i^0 \rangle}},$$

where index 0 stands for the amplitudes or variables evaluated at the resonant point $W = M_{res}$. $\langle P_i^0 \rangle$ is absolute value of 3 momentum of the final stable particle, averaged over running mass of the final unstable hadron.

W- evolution of decay amplitude:

$$\langle f : (ls) | T_{dec} | \lambda_R \rangle (W) = \langle f : (ls) | T_{dec0} | \lambda_R \rangle$$

$$\left[\frac{M_{res}}{W} \frac{J_l^2(p^0 R) + N_l^2(p^0 R)}{J_l^2(pR) + N_l^2(pR)} \right]^{1/2} \sqrt{\frac{\langle p^0 \rangle}{\langle p \rangle}}$$

where J_l, N_l are Bessel's and Neumann's functions, R=1f

$\langle p^0 \rangle$ and $\langle p \rangle$ are averaged momenta of the stable particle calculated at the resonant W_{res}=M_{res} ($\langle p^0 \rangle$) and at the current W, respectively.

Cont'd

Electrocouplings:

The $A_{1/2}$, $A_{3/2}$ were defined as:

$$\Gamma_\gamma = \frac{q^2_{\gamma,0}}{\pi}\frac{2M_N}{(2J_{res}+1)M_{N^*}}\left\{A^2_{1/2}+A^2_{3/2}\right\},$$

where $q_{\gamma,0}$ is photon 3-momentum abs. value at $W=M_{N^*}$

The relations between electrocouplings $A_{1/2}$, $A_{3/2}$, $S_{1/2}$ and $\langle\lambda_R|T_{em}|\lambda_\gamma\lambda_p\rangle$ amplitudes were determined from requirement to reproduce BW cross section for single N^* state:

$$\sigma(W) = \frac{\pi}{q^2_\gamma}(2J_r+1)\frac{M^2_{N^*}\Gamma_i(W)\Gamma_\gamma}{(M^2_{N^*}-W^2)^2 - M^2_{N^*}\Gamma^2_{tot}(W)},$$

N* electromagnetic production amplitudes and electrocouplings:

real

$$\langle\lambda_R|T_{em}|\lambda_\gamma\lambda_p\rangle = \frac{W}{M_r}\sqrt{\frac{8M_N M_r q_{\gamma_0}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_0}}{q_\gamma}}A_{1/2,3/2}(Q^2);$$

$$|\lambda_\gamma - \lambda_p| = \tfrac{1}{2},\ \tfrac{3}{2}$$

-for transverse photons

real

$$\langle\lambda_R|T_{em}|\lambda_\gamma\lambda_p\rangle = \frac{W}{M_r}\sqrt{\frac{8M_N M_r q_{\gamma_0}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_0}}{q_\gamma}}\sqrt{2}S_{1/2}(Q^2)$$

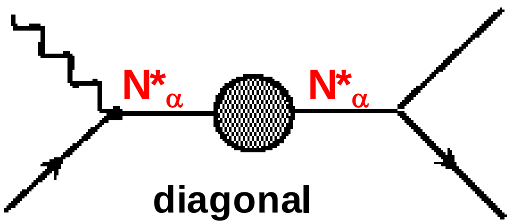
-for longitudinal photons

Unitarized BW anstaz

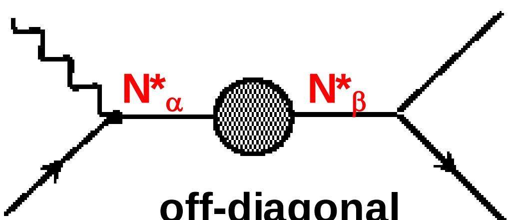
We apply unitarization procedure proposed in I.J.R.Aitchison NP A189 (1972), 417:

$$T_{\gamma p\to MB} = f_{\beta MB} S_{\alpha\beta} f_{\alpha\gamma p}$$

where $f_{\alpha\gamma p}$, $f_{\beta MB}$ are the α-th N* electroproduction and β-th N* hadronic decay amplitude to the meson-baryon (MB) final state; $S_{\alpha\beta}$ is the operator for resonance propagation, taking into account all transitions between α and β N* states, allowed by conservation laws in the strong interactions. No extra phases were used.



diagonal regular BW



off-diagonal

Off-diagonal transitions incorporated into JM:

$S_{11}(1535) \leftrightarrow S_{11}(1650)$
$D_{13}(1520) \leftrightarrow D_{13}(1700)$
$3/2^{+}(1720) \leftrightarrow P_{13}(1700)$

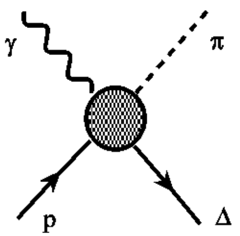
Inverse of JM unitarized N* propagator:

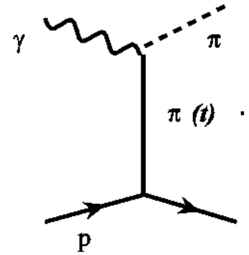
$$S^{-1}_{\alpha\beta} = M^2_{N*}\delta_{\alpha\beta} - i\Big(\sum_i \sqrt{\Gamma_{\alpha i}}\sqrt{\Gamma_{\beta i}}\Big)\sqrt{M_{N*\alpha}}\sqrt{M_{N*\beta}} - W^2\delta_{\alpha\beta}$$

Non-resonant contributions to πΔ channels

Minimal set of current conserving Born terms

Analytical expression for amplitudes



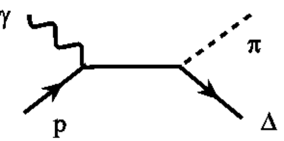


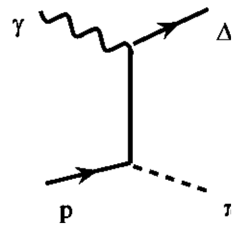
contact term:

$$t^{c}_{\lambda_\Delta\lambda_\gamma\lambda_p} = g_c(Q^2,t)\overline{u}_\mu(p_2,\lambda_\Delta)u(p_1,\lambda_p)\varepsilon_\mu(q,\lambda_\gamma)$$

pion in flight term:

$$t^{pif}_{\lambda_\Delta\lambda_\gamma\lambda_p} = g_\pi(Q^2,t)\frac{(2p_\pi^\mu - q^\mu)\varepsilon_\mu(q,\lambda_\gamma)}{t-m_\pi^2}\cdot \overline{u}_\nu(p_2,\lambda_\Delta)u(p_1,\lambda_p)(q^\nu - p_\pi^\nu)$$





nucleon term:

$$t^{N}_{\lambda_\Delta\lambda_\gamma\lambda_p} = g_N(Q^2)g_0\frac{(2p_1^\mu + q^\mu)\varepsilon_\mu(q,\lambda_\gamma)}{s-m_N^2}\cdot \overline{u}_\nu(p_2,\lambda_\Delta)u(p_1,\lambda_p)p_\pi^\nu$$

Δ in flight:

$$t^{\Delta}_{\lambda_\Delta\lambda_\gamma\lambda_p} = 2g_\Delta(Q^2,t)\frac{(2p_2^\mu - q^\mu)\varepsilon_\mu(q,\lambda_\gamma)}{u-m_\Delta^2}\cdot \overline{u}_\nu(p_2,\lambda_\Delta)p_\pi^\nu u(p_1,\lambda_p),$$

current conservation.

$$g_\pi(Q^2,t) = g_c(Q^2,t)$$

$$g_\Delta(Q^2,t) = \frac{g_\pi(Q^2,t)+g_N(Q^2)}{2}$$

con'd

Vertex functions.

$$g_\pi(Q^2,t) = G_{\pi,em}(Q^2)G_{\pi N\Delta}(t).$$

$$G_{\pi,em}(Q^2) = \frac{1}{\left(1+\frac{Q^2(GeV^2)}{\Lambda_\pi^2}\right)}\frac{1}{G_{\pi N\Delta}(t_{min})}$$

$\Lambda_\pi^2 = 0.462$ C.J. Bebek et al., Phys. Rev. **D17**, 1693 (1978).

$$G_{\pi N\Delta}(t) = g_0\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t}$$

The interaction constant g_0 and cut-off parameter Λ are: $g_0 = 2.1/m_\pi$ and $\Lambda = 0.75$ GeV. R. Machleidt, in Advances in Nuclear Physics **19**, (1979).

Regge trajectory implementation.

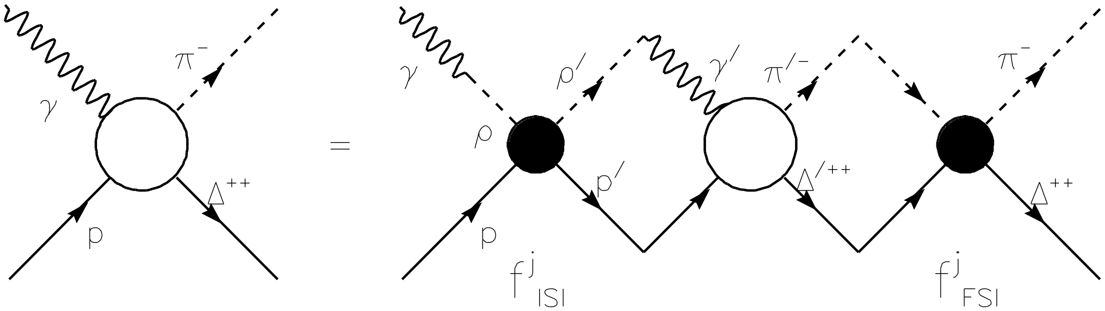
$$\frac{1}{t - m_\pi^2} \rightarrow R_\pi(t) = \left(\frac{S}{S_0}\right)^{\alpha_\pi(t)}$$

$$\frac{\pi\alpha_\pi^{'}}{\sin(\pi\alpha_\pi(t))} \cdot \frac{1+e^{-i\pi\alpha_\pi(t)}}{2} \cdot \frac{1}{\Gamma(1+\alpha_\pi(t))}$$

proposed in M.Vanderhaeghen, M.Guidal and J.M.Laget, Phys. Rev. **C57**, 1454 (1998).
multiplicative factor:

$$\alpha_R(t - m_\pi^2)R_\pi(t)$$

Absorptive ansatz for phenomenological treatment of ISI & FSI in $\pi\Delta$ channels



K.Gottfried, J.D.Jackson, Nuovo Cimento 34 (1964) 736. M.Ripani et al., Nucl Phys. A672, 220 (2000).

Applied to non resonant parts of $\pi\Delta$ amplitudes decomposed over PW's of total angular momenta J.

$$f^{jcorr}_{\lambda\mu} = f^{j}_{ISI} f^{j}_{\lambda\mu} f^{j}_{FSI}$$
$$\lambda = \lambda_\gamma - \lambda_p, \mu = -\lambda_\Delta$$

Absorptive factors are related to S-matrix elements of ρp and $\pi\Delta$ elastic scattering amplitudes, respectively:

$$f^j_{ISI} = \langle \lambda_\rho \lambda_p | S^j | \lambda_\rho \lambda_p \rangle^{1/2}$$
$$f^j_{FSI} = \langle \pi \lambda_\Delta | S^j | \lambda_\Delta \pi \rangle^{1/2}$$

$$S = 1 + 2iT$$

Re-normalization of $\pi\Delta$ ρp amplitudes

Consider ISI & FSI limit, that incorporates just single N^* with only one open elastic channel. Abs. values of $\gamma_v p \to \pi\Delta$ amplitudes should remain unchanged at the resonant point $\to$ Resonant $\pi\Delta$, ρp elastic amplitudes at resonant point should be equal to $i \to S = -1$.

Determined in this way normalization factor:

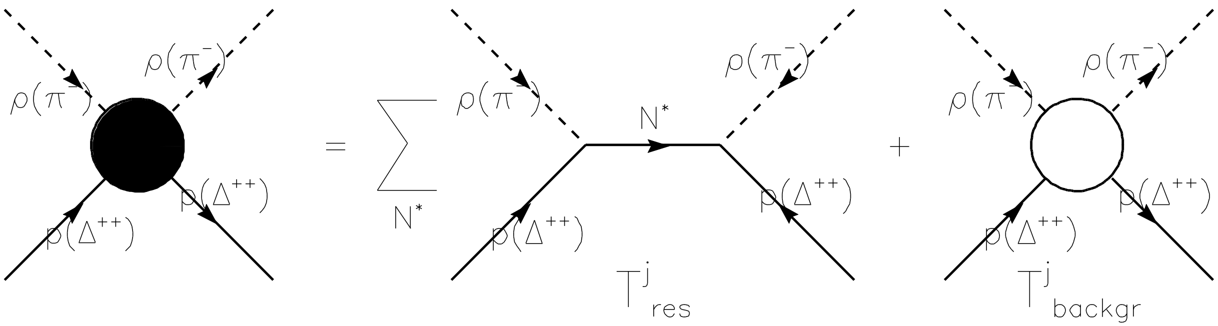
$$N_{isifsi} = \frac{P_{\pi(\rho)}}{8\pi(2j+1)W} \qquad (1)$$

P_π, P_ρ are three − momenta moduli of pion and ρ.

The normalization factor N_{isifsi} of (Eq.1) was applied to full $\pi\Delta$, ρp elastic amplitudes in the PW of total angular momentum j.

Cont'd

$\pi\Delta$, ρp elastic scattering amplitudes

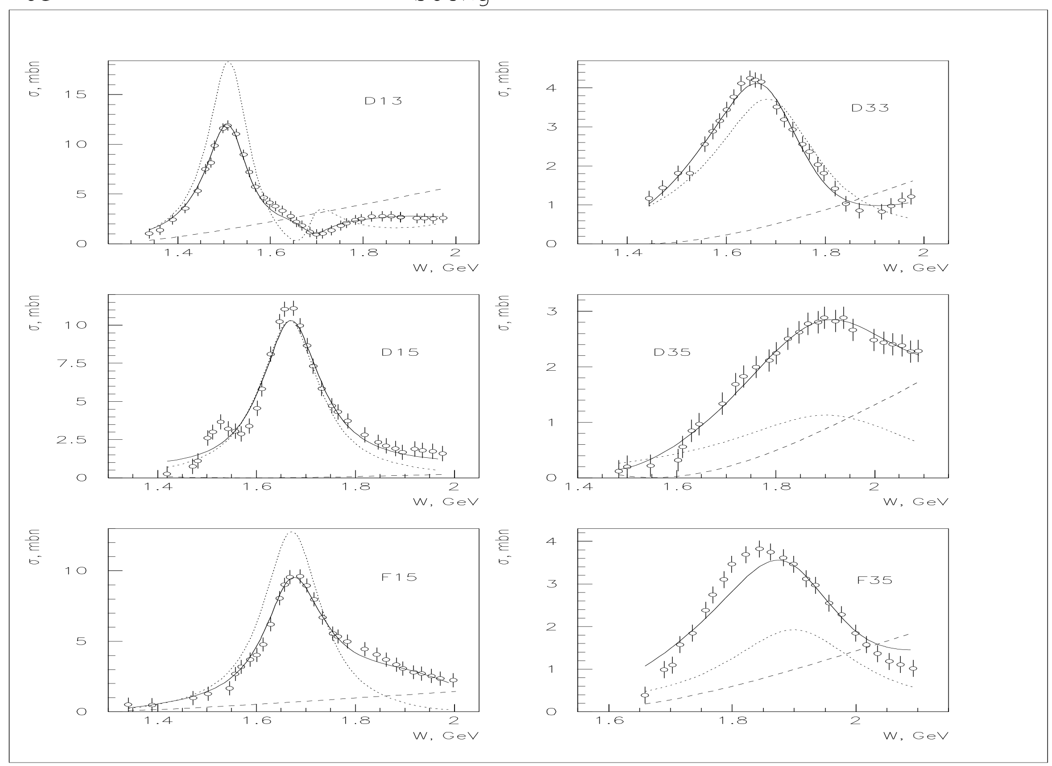


BW ansatz for resonant part

Exclude double counting: non-resonant ampl. &dressed gNN* verticies

$T^j res \rightarrow 0.5 T^j res$ ⇨ $f^j res=0$

$T^j backgr$ from πN data fit



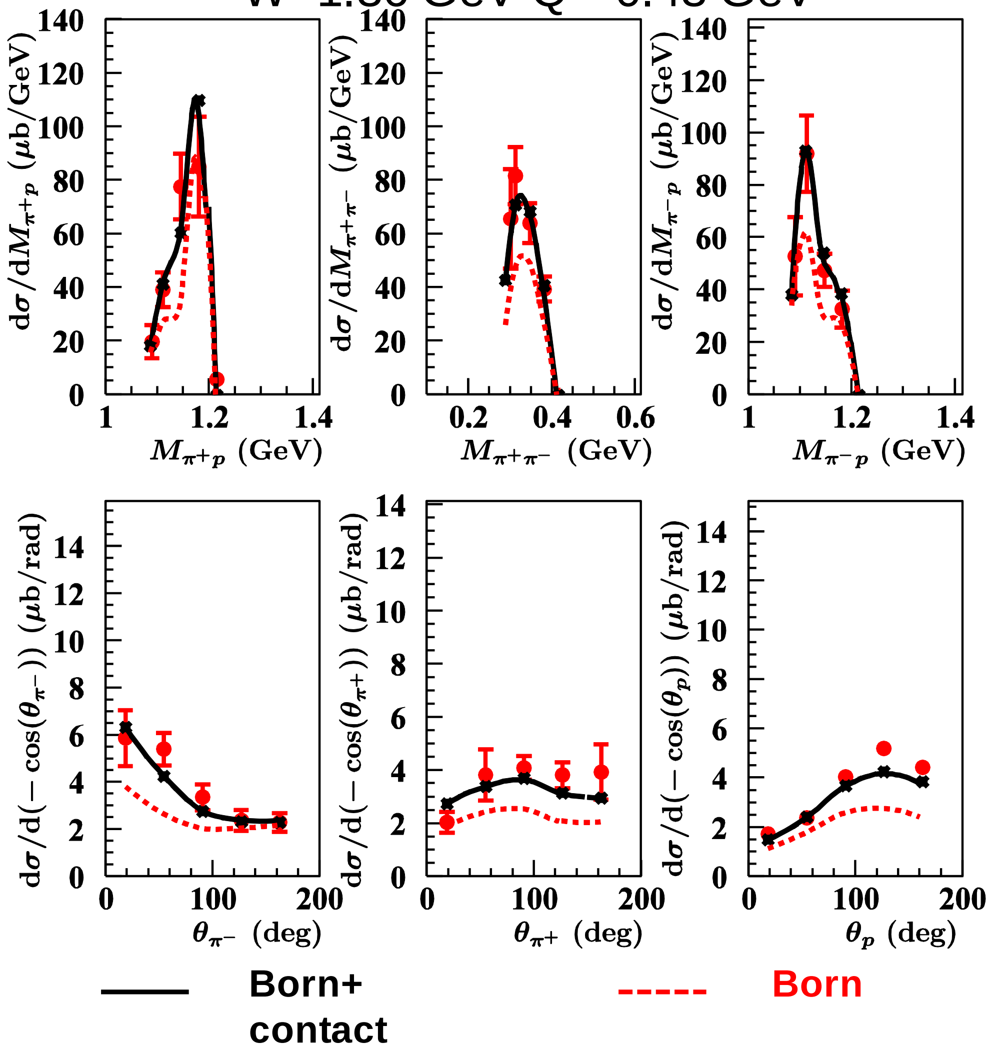
Potential improvements:
New results on $\pi\Delta$ & ρp elastic amplitudes

Extra contact terms in πΔ isobar channels

$$t^c = \{A(W,Q^2)\,\varepsilon^{\gamma}_{\mu}\overline{U}^{\nu}_{\Delta}\gamma^{\mu}U\,p^{\pi}_{\upsilon} +$$
$$B(W,Q^2)\,\varepsilon^{\gamma}_{\upsilon}\overline{U}^{\nu}_{\Delta}\gamma^{\delta}U\,(2p^{\pi}_{\upsilon} - q_{\gamma})_{\delta}\}^*$$
$$\frac{1}{t-\Lambda^2} \qquad \Lambda^2 = 1.64 \quad GeV^2$$

Parameters A(W,Q²), B(W,Q²) were taken from the CLAS data fit.

W=1.36 GeV Q²=0.43 GeV²



Non-resonant contributions in ρp isobar channel

Diffractive ansatz from J. D. Bjorken, PRD3, 1382 (1971).

$$T_{diff} = iAe^{bt}$$

$$t = (P_{p'} - P_{\gamma'})^2$$

Good approximation for t<1.0 GeV²; at larger t full ρp amplitudes are dominated by N*

b=b(L_{fluct}) from D.G.Cassel et al, PRD24, 2878 (1981).

JM model improvement A=A(W,Q²), essential in N* area at W<1.8 GeV:

$$A(W,Q^2) = A(Q^2)(1 - e^{(-(W-1.41)/D)})$$

$$A(Q^2) = \frac{A_0}{(1+Q^2/\Lambda^2(W))}$$

$$\Lambda(W) = \Lambda_0(1 - e^{(-(W-1.41)/D_\lambda)})$$

L=0.77 GeV A=12
D_λ=0.30 GeV D=0.25 GeV

All details in:
N.V.Shvedunov et al, Phys of Atom. Nucl. 70, 427 (2007).

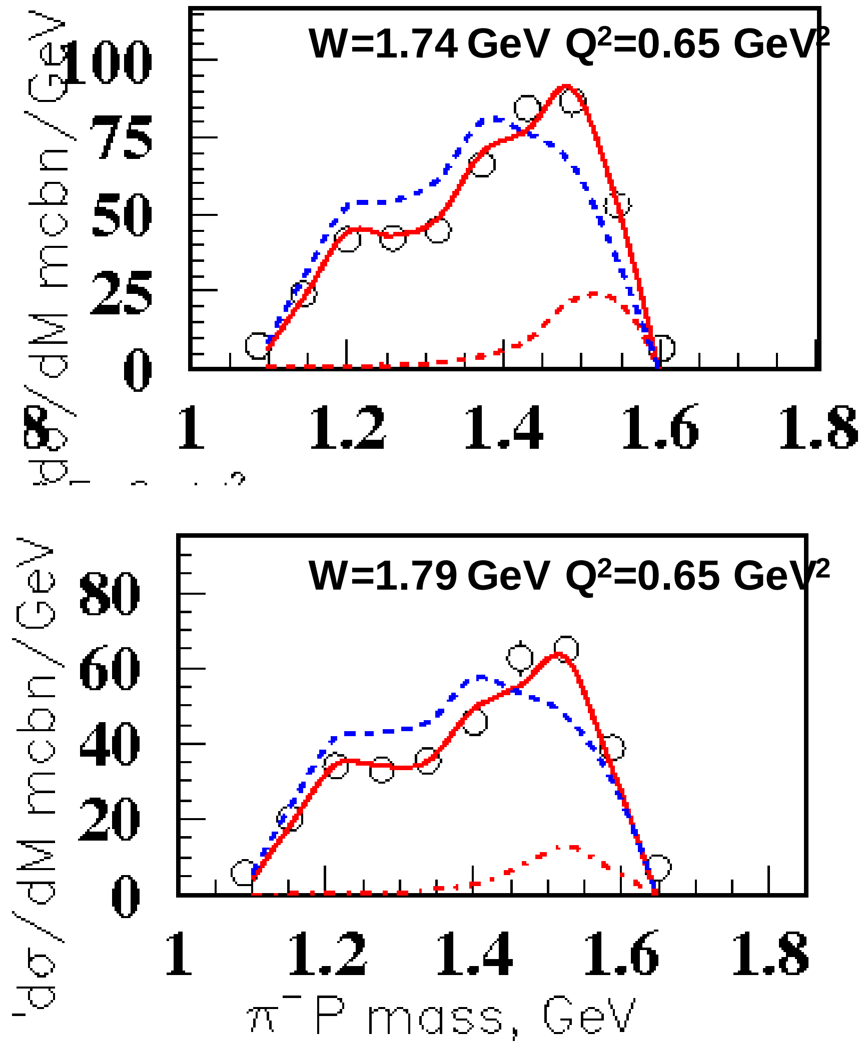
$\pi^+ D_{13}(1520)$ isobar channel

Evidence for $\pi^+ D_{13}(1520)$ isobar channel in the CLAS $\pi^+\pi^- p$ data

full JM results with $\pi^+ D_{13}(1520)$ implemented

full JM results without $\pi^+ D_{13}(1520)$ and adjusted direct 2π production

$\pi^+ D_{13}(1520)$ contribution



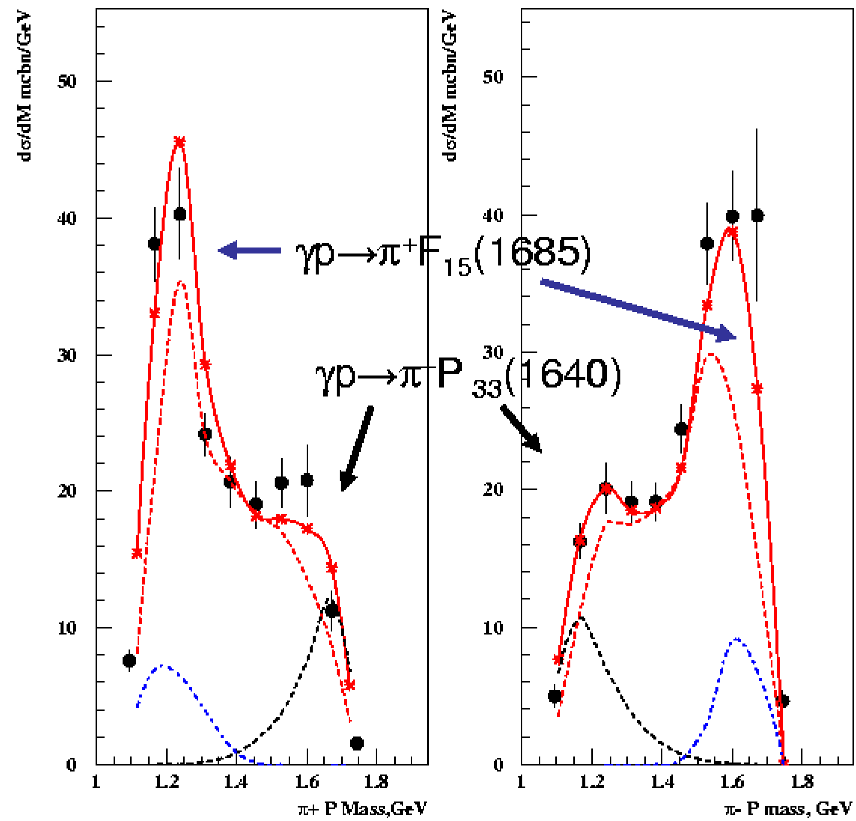
Born terms similar to employed in $\pi\Delta$ channels with additional γ_5 matrix

$\pi^+F_{15}(1685)$, $\pi^-P_{33}^{++}(1620)$ isobar channels

Evidence in the CLAS data

W=1.89 GeV
Q^2=0.95 GeV^2

- (solid red) **full JM results with $\pi^+F_{15}(1685)$ and $\pi^-P_{33}(1620)$ implemented**
- (dashed red) **full JM results without these channels**
- (dashed blue) **$\pi^+F_{15}(1685)$**
- (dashed black) **$\pi^-P_{33}(1620)$**



$\pi^+F_{15}(1685)$ amplitude:

$$M = A(W,Q^2)\bullet\left[\varepsilon^{\gamma}_{\mu}\overline{U}_{p'}\gamma^{\mu}U_p(P_{F15}\bullet P_{\pi^+})\bullet\exp\left\{-\frac{(M_{\pi^- p}-M_{F15})^2}{\Gamma^2_{F15}}\right\}\right]$$

$\pi^-P_{33}(1620)$ amplitude:

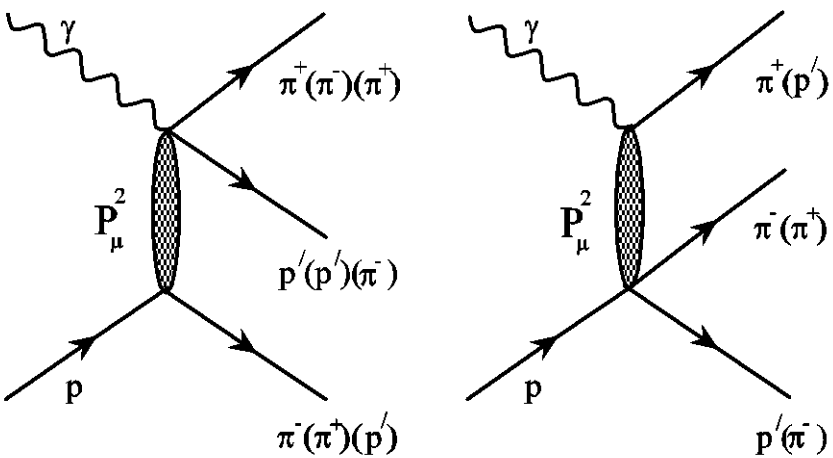
$$M = A(W,Q^2)\bullet\left[\varepsilon^{\gamma}_{\mu}\overline{U}_{p'}\gamma^{\mu}U_p\frac{1}{t-m^2_{\pi^*}}\bullet\exp\left\{-\frac{(M_{\pi^- p}-M_{P33})^2}{\Gamma^2_{P33}}\right\}\bullet\frac{1}{P_{p'}P_{\pi^-}}\right]$$

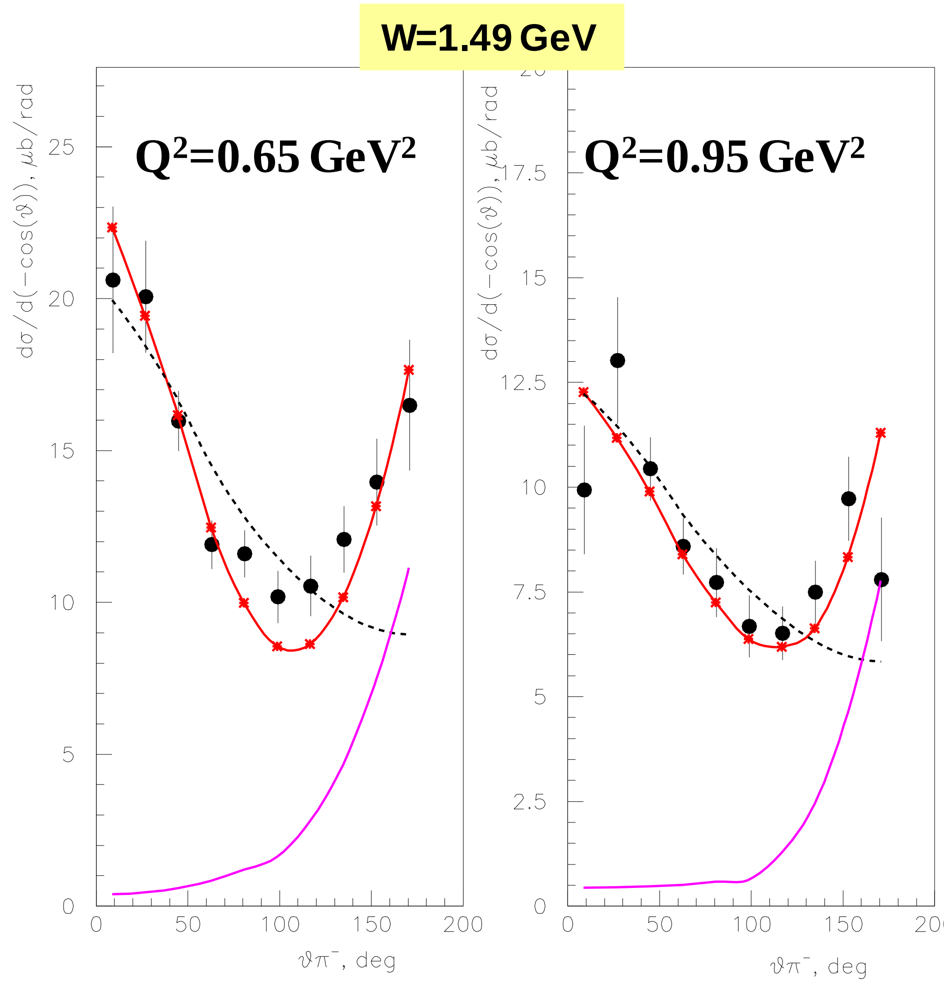
Manifestation of direct 2π production in angular distributions

full JM with 2π dir.shown by diagrams below

phase space for 2π direct

contr. from2π direct production parametrized as shown below





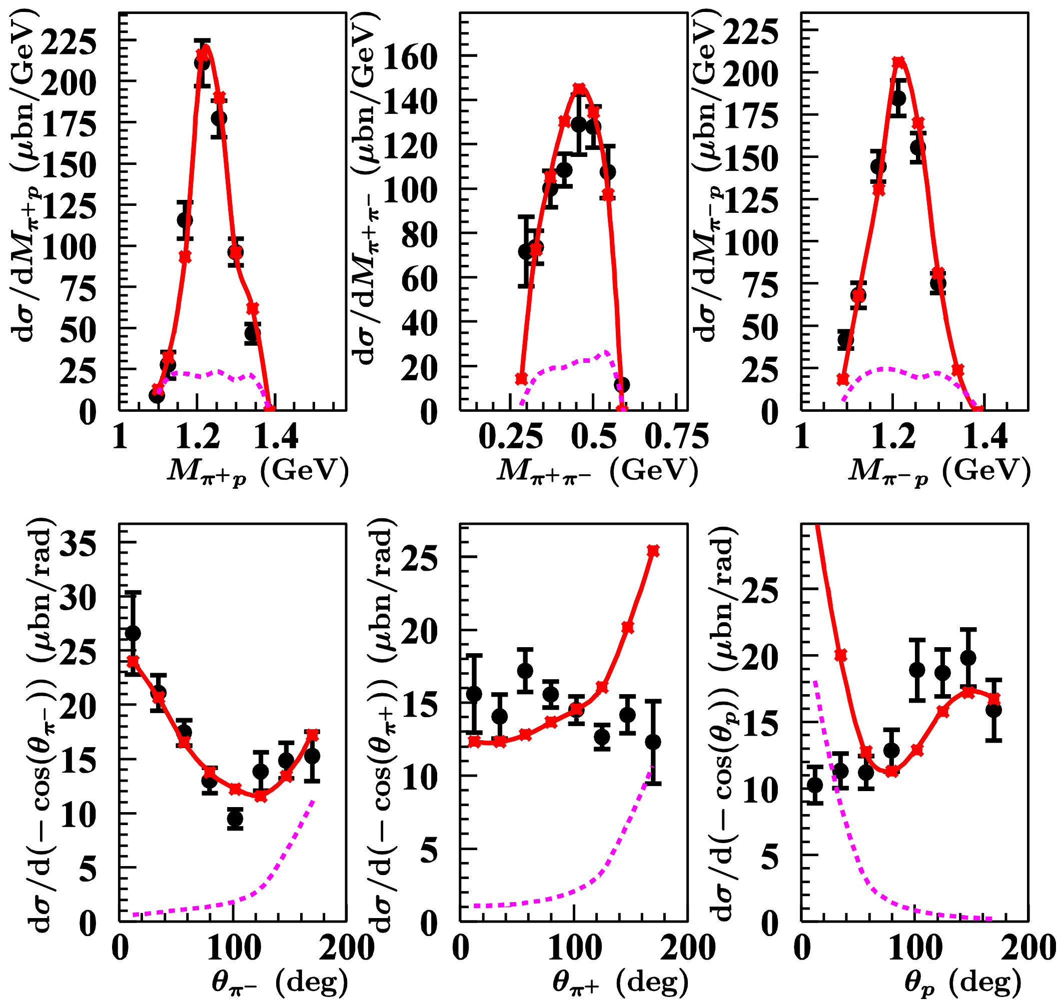
$$M_d = A(W,Q^2)\varepsilon_\mu^\gamma \overline{U}_{p'}\gamma_\mu U_p \frac{1}{W^4} e^{b(P_\mu^2 - P_{\mu\min}^2)}(P_1P_2)$$

Need for improvements in description of direct 2π production

full JM

2π direct

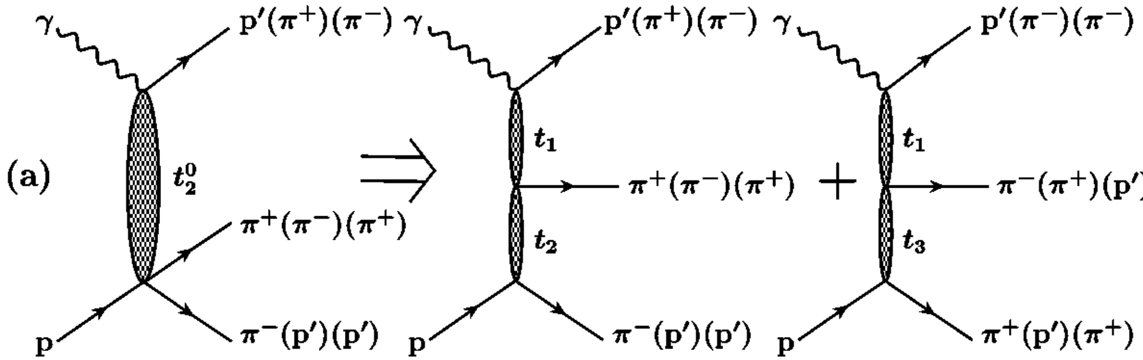
W=1.51 GeV Q^2=0.43 GeV^2



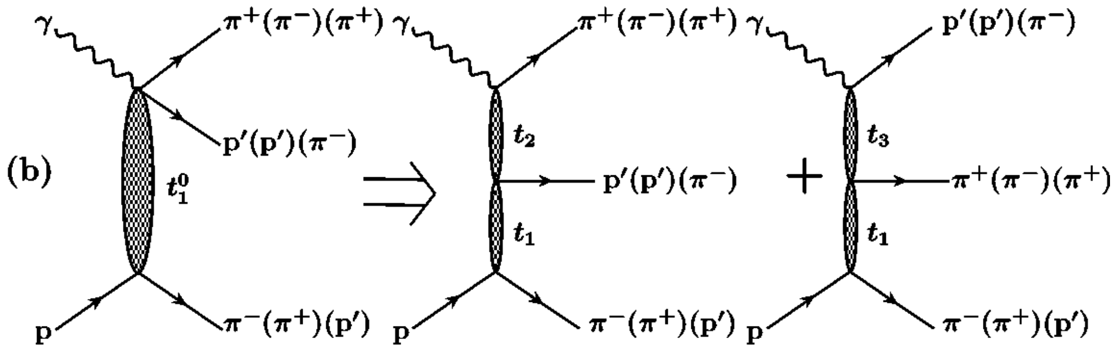
failure in description of angular distributions is related to the shortcomings in parameterization of direct 2π production

Final parametrization of direct 2π production amplitudes

$$A(W,Q^2)\varepsilon_\mu(q_\gamma)\bar{U}_{p'}(P_{p'})\gamma^\mu U_p(P_p)*$$
$$e^{b(t_1-t_{1max})}(P_2P_3)\{\alpha_1(W)e^{b(t_2-t_{2max})}+$$
$$\alpha_2(W)e^{b(t_3-t_{3max})}\}$$
$$t_1=(q_\gamma-P_1)^2$$
$$t_2=(P_p-P_3)^2$$
$$t_3=(P_p-P_2)^2$$

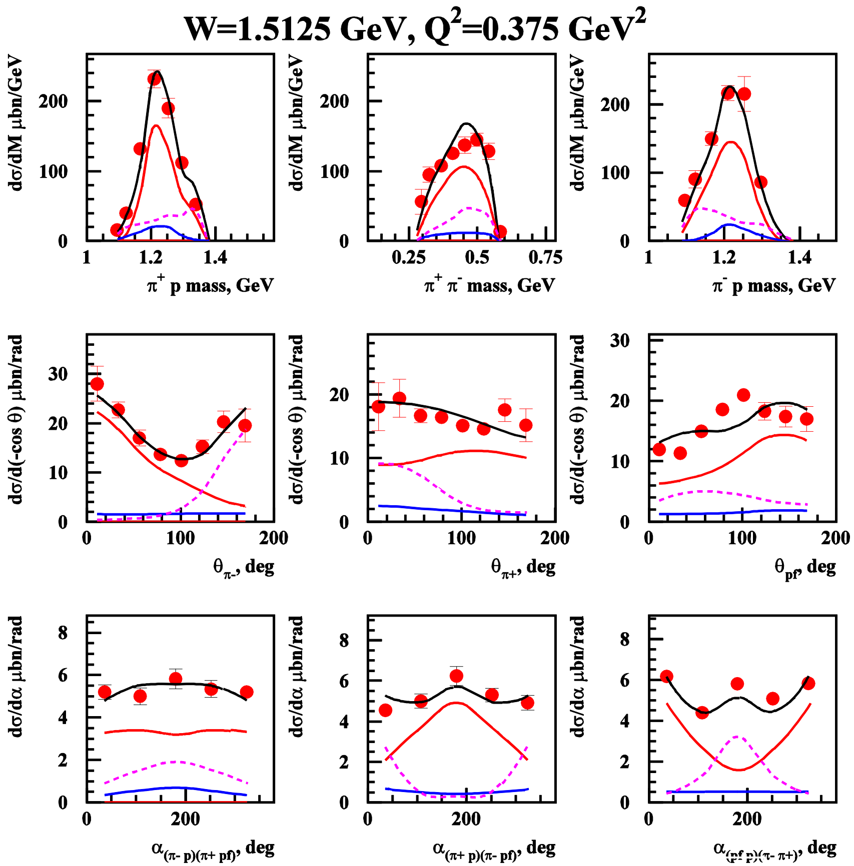


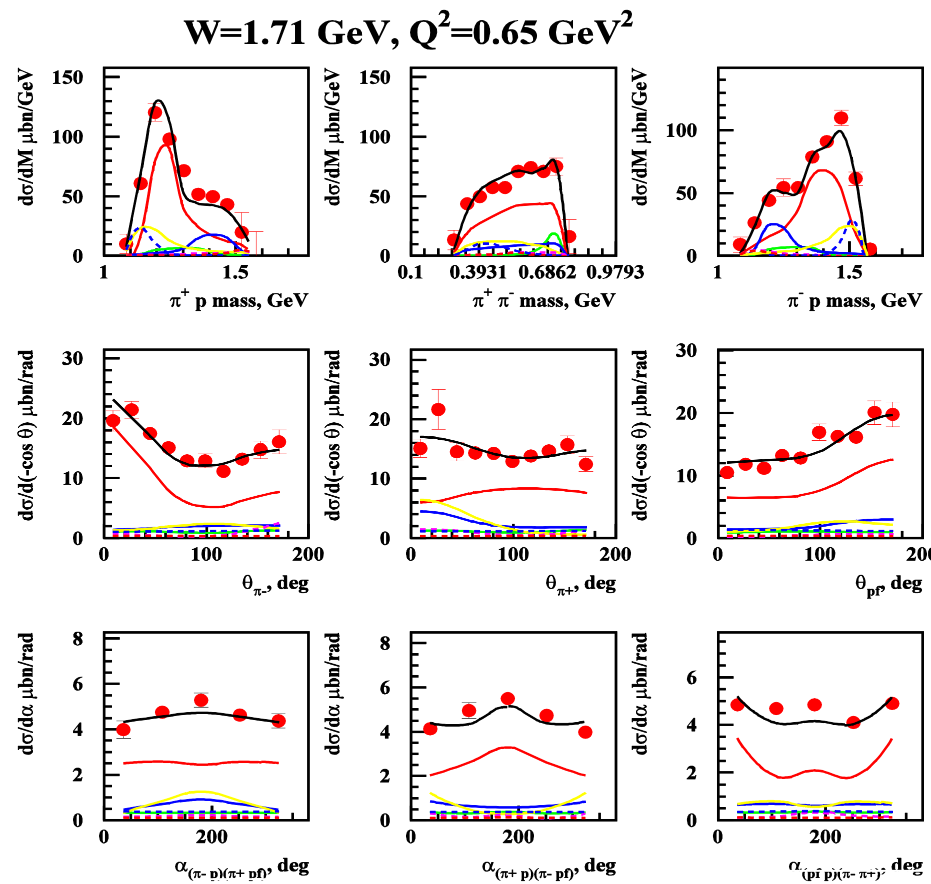
$$A(W,Q^2)\varepsilon_\mu(q_\gamma)\bar{U}_{p'}(P_{p'})\gamma^\mu U_p(P_p)*$$
$$e^{b(t_1-t_{1max})}(P_1P_2)\{\alpha_1(W)e^{b(t_2-t_{2max})}(t_2-t_{2max})+$$
$$\alpha_2(W)e^{b(t_3-t_{3max})}\}$$
$$t_1=(p_p-P_3)^2$$
$$t_2=(q_\gamma-P_1)^2$$
$$t_3=(q_\gamma-P_2)^2$$



The CLAS data on $\pi^+\pi^-p$ differential cross sections and the description within the JM model

G.V.Fedotov et al, PRC 79 (2009), 015204 | **M.Ripani et al, PRL 91 (2003), 022002**





full JM calc. — $\pi^+\Delta^0$ — ρp — $\pi^+F^0_{15}(1685)$

$\pi^-\Delta^{++}$ — 2π direct — $\pi^+D^0_{13}(1520)$

Extraction of resonance parameters and non-resonant contributions from the CLAS $\pi^+\pi^-p$ data

Fitting Procedures

Simultaneous variation of the following resonant/non-resonant JM model parameters according to the normal distribution:

-γ_vNN* electrocouplings with σ–parameter equal to 30% from their initial values, taken from interpolation of the CLAS/world data;

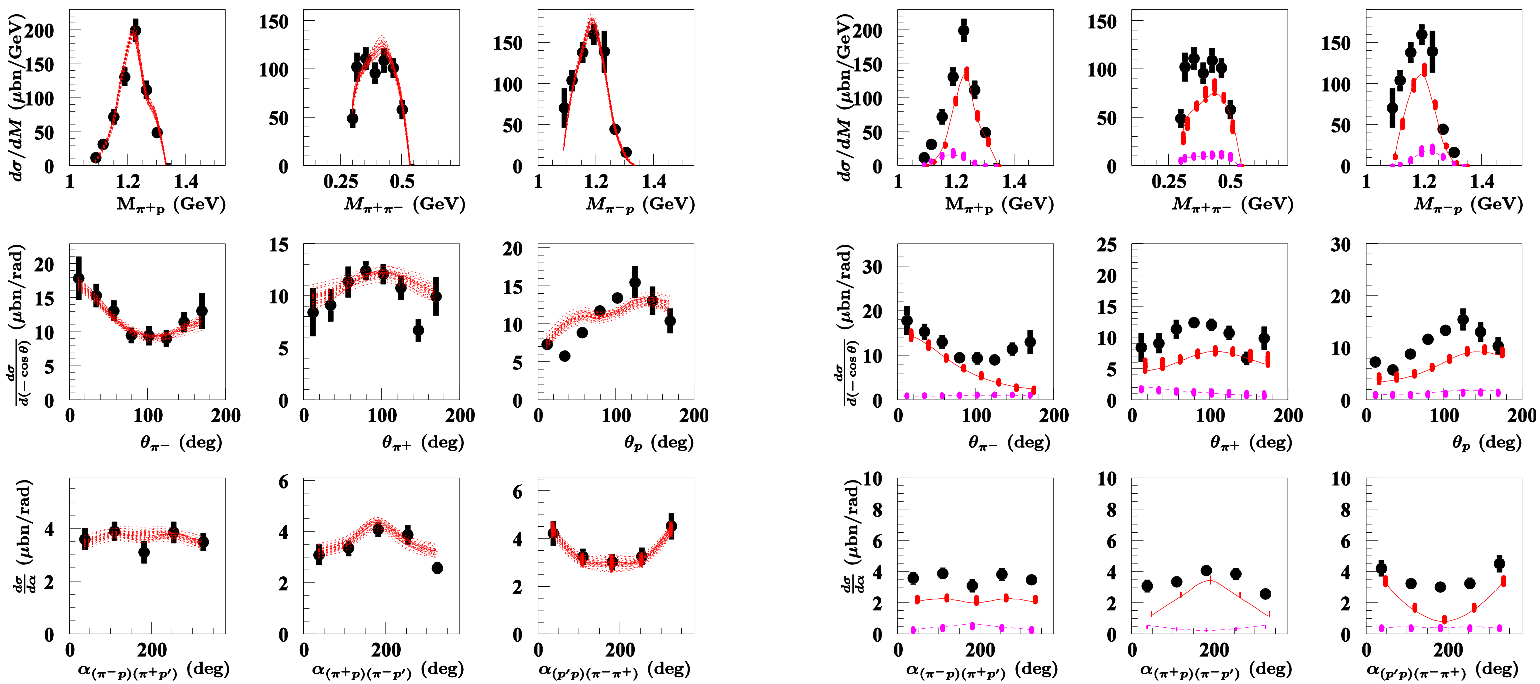
-$\pi\Delta$ and ρp hadronic decays decomposed over LS p.w for part of N*'s with σ–parameters, that cause total N* width float from 40 to 600 MeV

-magnitudes of complementary contact terms in $\pi\Delta$ isobar channels; magnitudes of the $\pi^+D^0_{13}(1520)$, $\pi^+F^0_{15}(1685)$, $\pi^-P^{++}_{33}(1620)$ isobar channel amplitudes; magnitudes of direct 2π production mechanisms ; the respective σ-parameters were chosen 10 -30 % from the starting magnitude values

- χ^2/d.p. fit of nine 1-fold diff. cross sections in each bin of W and Q^2 were carried out . Closest to the data calculated section were selected with
- $\chi^2/d.p. < \chi^2/d.p._{thr}$. The $\chi^2/d.p._{thr}$. were defined so that selected in the fit calc. cross sections are inside the uncertainties of measured cross sections for a major part of the data points.

Isobar channel cross sections derived from Nππ CLAS data

W=1.51 GeV Q^2=0.43 GeV^2



band of differential cross sections calculated in JM model that are closest to experimental Nππ data, being determined under requirement : χ^2/d.p.< χ^2/d.p._max

differential cross sections for contributing isobar channels:

$\pi^-\Delta^{++}$

$\pi^+\Delta^0$

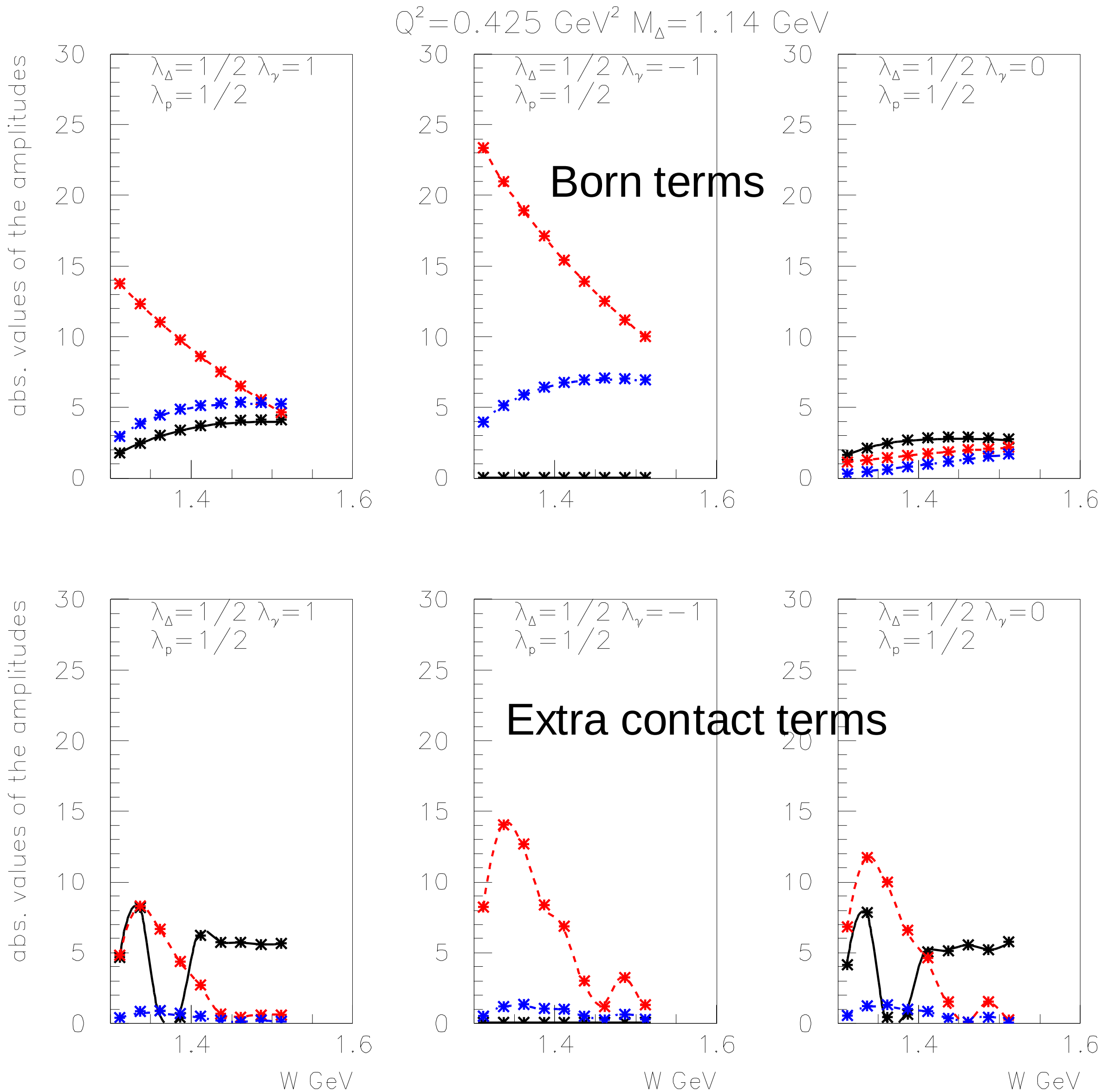
may be used in development of reaction models

Non-resonant amplitudes contributing to $\pi^- \Delta^{++}$ isobar channel from the CLAS data fit

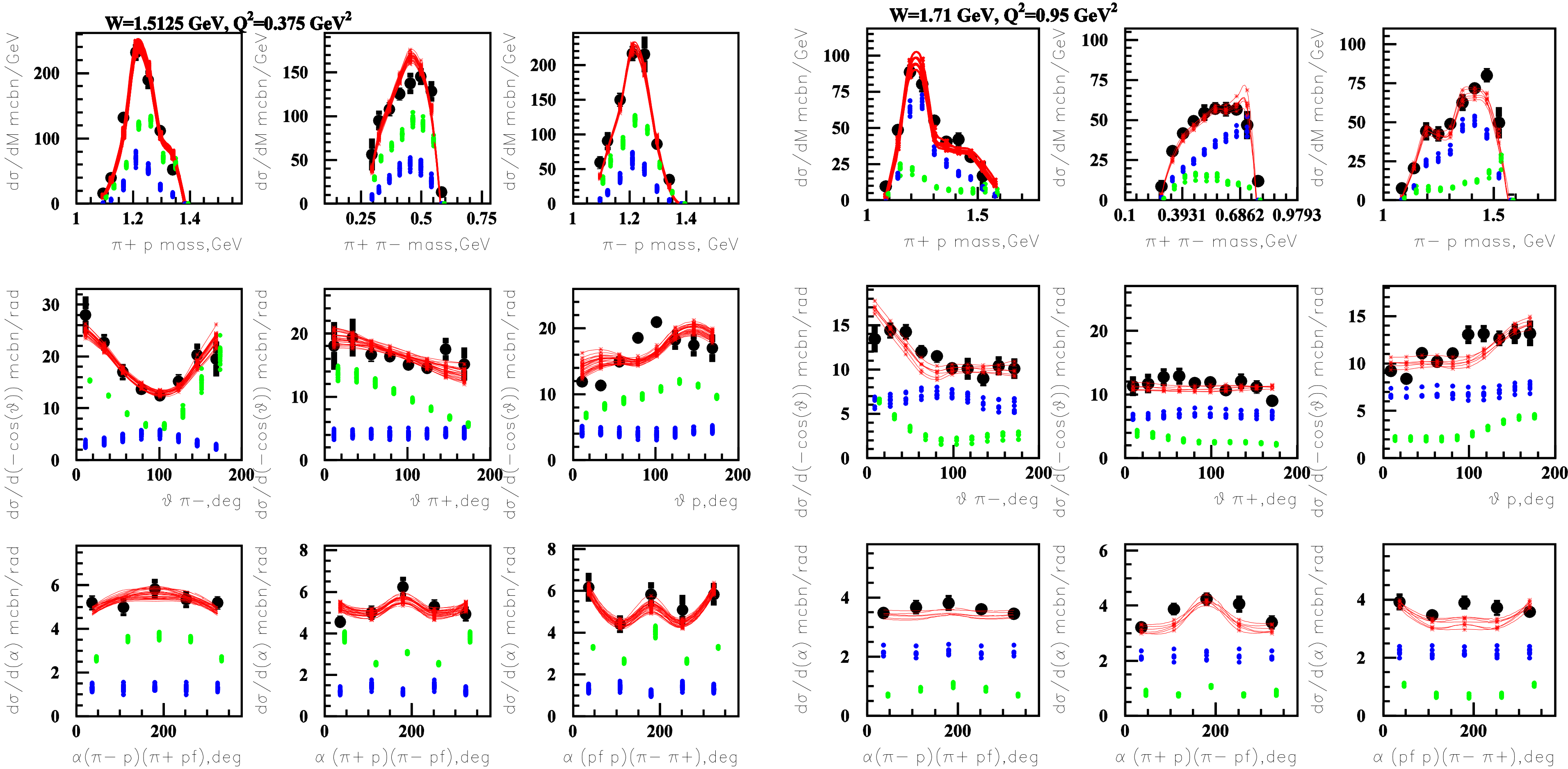
J	
black line	1/2
red line	3/2
blue line	5/2

$$\langle \lambda_f | T^J | \lambda_\gamma \lambda_p \rangle = \int \frac{2J+1}{2} \langle \lambda_f | T | \lambda_\gamma \lambda_p \rangle \bullet d^J_{\mu\nu}(\theta_f) \sin\theta_f \, d\theta_f$$

Can be used for N* studies in coupled channel approaches.



Resonant & non-resonant parts of Nππ cross sections as determined from the CLAS data fit within the framework of JM model



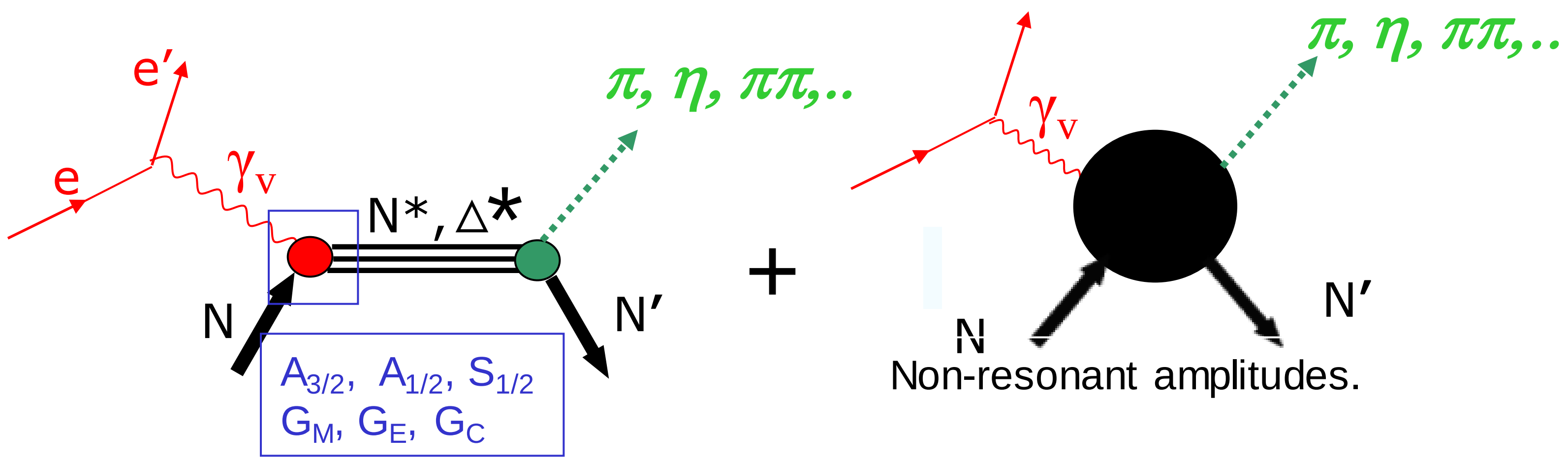
full cross sections

resonant part

non-resonant part

How N* electrocouplings are accessed in JM model

- **Isolate the resonant part of production amplitudes by fitting the measured observables.**
- **N* electrocouplings are determined from resonant amplitudes, parameterized within the framework of unitarized BW ansatz.**



Consistent results on N* electrocouplings obtained in analyses of various meson channels (e.g. *πN, ηp, ππN)* ***with entirely different non-resonant*** **amplitudes will show that they are determined reliably.**

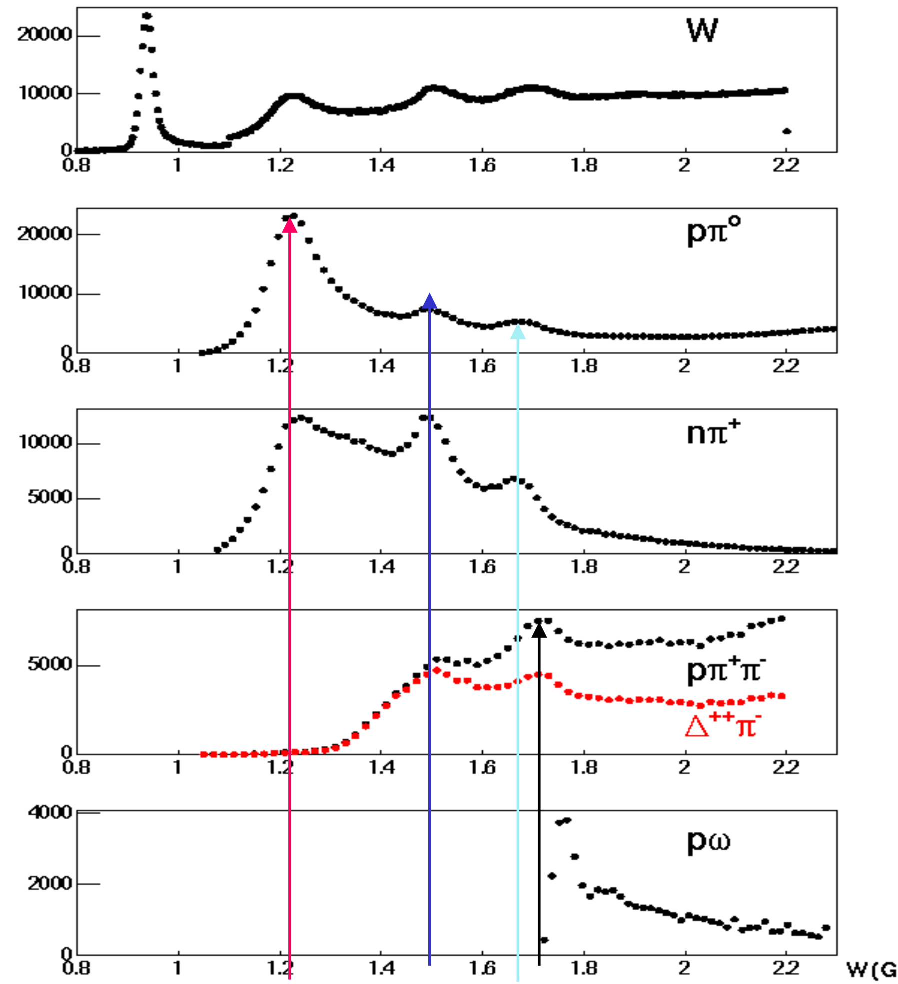
Input from advanced multi channel analyses accounting for FSI and rigorously employing Untiratity is required!

Why Nπ/Nππ electroproduction channels are important

- **Nπ/Nππ channels are the two major contributors in N* excitation region;**
- **these two channels combined are sensitive to almost all excited proton states;**
- **they are strongly coupled by πN→ππN final state interaction;**
- **may substantially affect exclusive channels having smaller cross sections, such as ηp,KΛ, and KΣ.**

Consistent results from independent analyses of individual exclusive channels and global multi channel analysis offer an ultimate test for reliability of extracted N* parameters

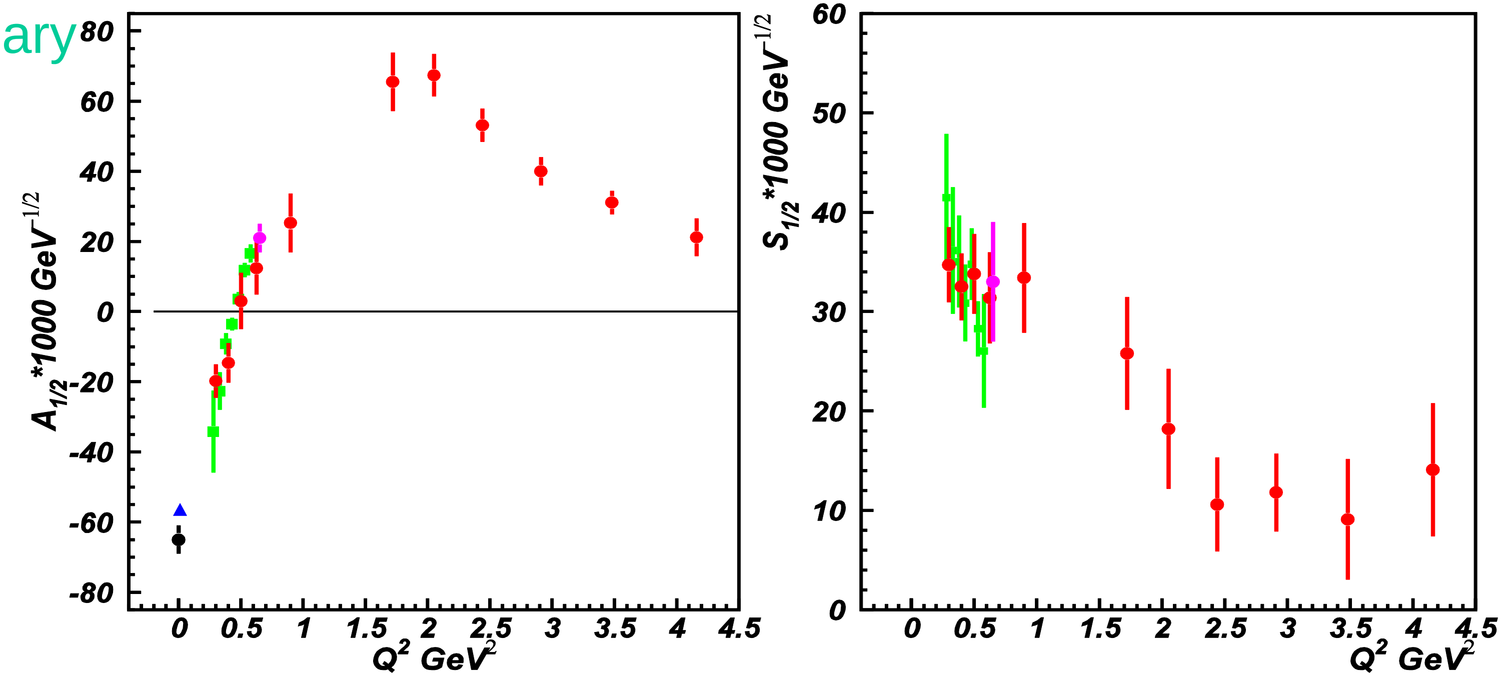
CLAS data on meson electroproduction at $Q^2 < 4.0$ GeV²



Comparison between $P_{11}(1440)$ electrocouplings obtained from the CLAS data on $N\pi/N\pi\pi$ electroproduction

Nππ preliminary

Nπ

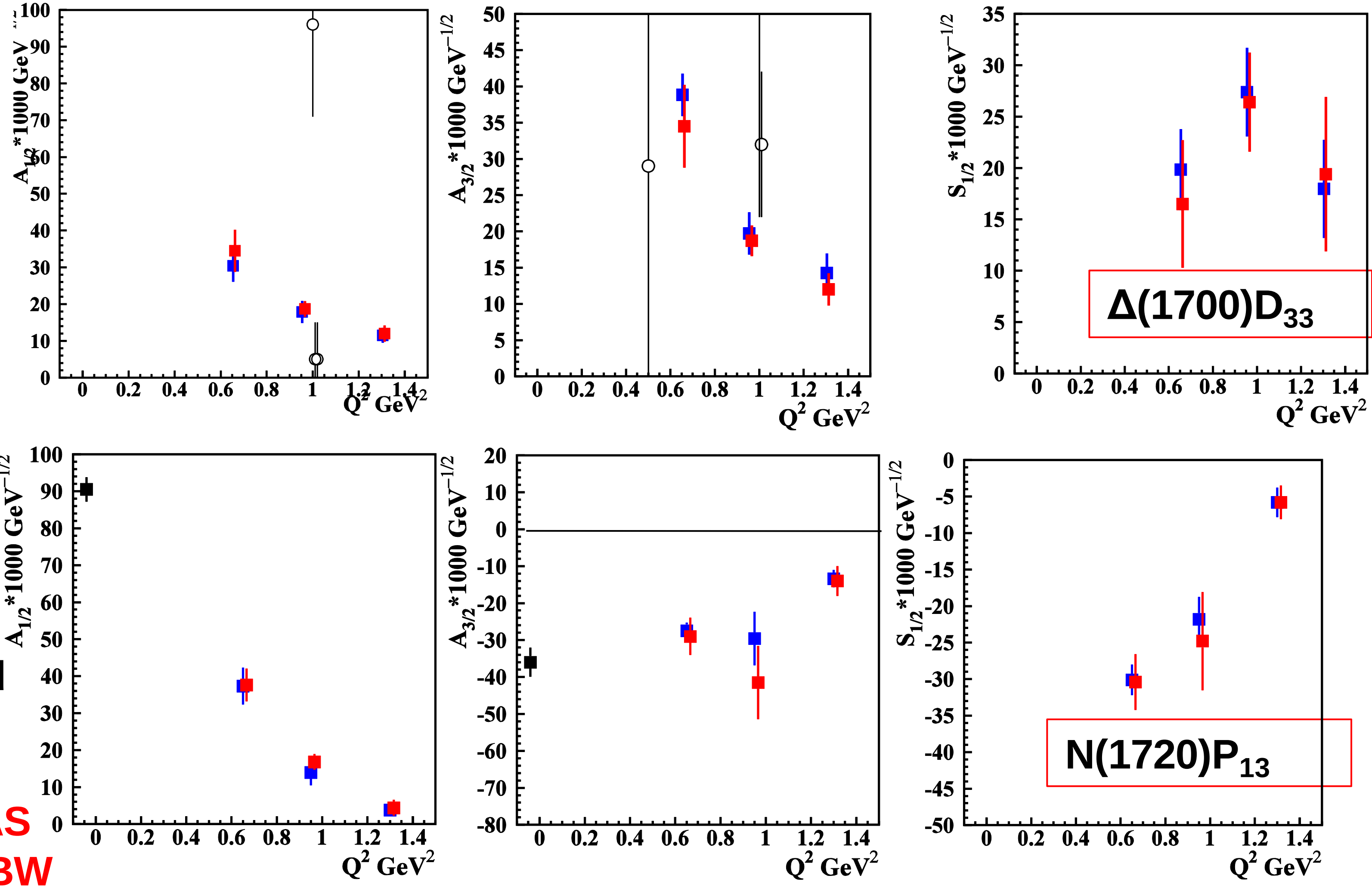


Good *agreement* between the electrocouplings obtained from the *$N\pi$ and $N\pi\pi$ channels*: Reliable measure of the electrocouplings with N* pasrtameters defined within the framework of BW ansatz

Open question: how to compare these electrocouplings with obtained in cc approaches? EBAC-DCC results showed substantial Im parts

High lying resonance electrocouplings & the impact of BW ansatz unitarization

Nππ CLAS unitarized BW ansatz



Nπ world

Nππ CLAS regular BW ansatz

The studies of $\pi^+\pi^-p$ electroproduction are vital for reliable extraction of transition gvNN* electrocouplings for high lying states

N* hadronic parameters derived from the CLAS $\pi^+\pi^-p$ data fit

$P_{13}(1720)$

	Γ_{tot}, MeV	$\Gamma_{\pi\Delta}$, MeV	$\Gamma_{\rho p}$, MeV	M,GeV
Regular BW ansatz	135±12	1.53±1.05	114±12	1.743±0.006
Unitarized BW anstaz	113±3.4	10.9±1.40	82.9±3.26	1.744±0.007

$3/2^+(1720)$ candidate state

	Γ_{tot}, MeV	$\Gamma_{\pi\Delta}$, MeV	$\Gamma_{\rho p}$, MeV	M,GeV
Regular BW ansatz	86±5	44±5.5	6.25±1.62	1.727±0.003
Unitarized BW anstaz after improvements	107±12	61±12	0.63±0.21	1.725±0.006

Unitarization of BW ansatz affects substantially N* hadronic decay parameters, while the impact on $\gamma_v NN^*$ electrocouplings is almost negligible

Back-up

JM amplitudes and cross sections

S-matrix, inv. flux F and single particle phase space element in JM model.

$$S = I + (2\pi)^4 \delta^4 (P_f - P_i) iT,$$

where P_f and P_i are total four momenta of the final and the initial particles respectively. The Dirac spinors were normalized as:

$$\overline{U_p} U_p = 2M_N,$$

where U_p, $(\overline{U_p})$ are Dirac (conjugated Dirac) spinors, M_N is the nucleon mass.

Invariant flux:

$$F = q_\gamma P_N,$$

where q_γ and P_N are four momenta of the initial photon and proton.

The phase space element for the final particle i with three-momentum vector $\vec{p_i}$ and energy E_i:

$$d^3\vec{p_i}/(2E_i(2\pi)^3)$$

Phase space element d⁵Φ for the final π⁺π⁻p state:

$$d^5\phi = \frac{1}{32W^2(2\pi)^5} ds_{\pi^+\pi^-} ds_{\pi^+p} d\theta_{\pi^-} d\alpha_{[p'\pi^+][p\pi^-]} d\varphi_{\pi^-},$$

$$ds_{\pi^+\pi^-} = dM^2_{\pi^+\pi^-},$$

$$ds_{\pi^+p} = dM^2_{\pi^+p},$$

where θ_{π^-}, φ_{π^-} are the final π^- spherical angles with respect to the direction of virtual photon, and $\alpha_{[p'\pi^+][p\pi^-]}$ is the angle between the plane B defined by the momenta of the final $p'\pi^+$ pair and the plane A defined by the initial proton and the final π^-

Con’d

4 –fold differential $\pi^+\pi^-$p cross section after integration over $\varphi_{\pi-}$:

$$d^4\sigma = \frac{4\pi\alpha}{4K_L M_N}\left\{\frac{J_x^* J_x + J_y^* J_y}{2} + \epsilon_L J_z^* J_z\right\} d^4\phi,$$

$d^4\phi$ stands for the 3-body phase space differential after integration over ϕ_{π^-}.

where $\alpha = \frac{1}{137}$, ϵ_L stands for degree of longitudinal polarization of virtual photons. The K_L is the equivalent photon energy:

$$K_L = \frac{W^2 - M_N^2}{2M_N},$$

The hadronic current J_ν and the virtual photon vectors $\epsilon(\lambda_\gamma)$ (λ_γ=-1,0,+1) are related to reaction helicity amplitudes $\langle\lambda_f|T|\lambda_p\lambda_\gamma\rangle$ as:

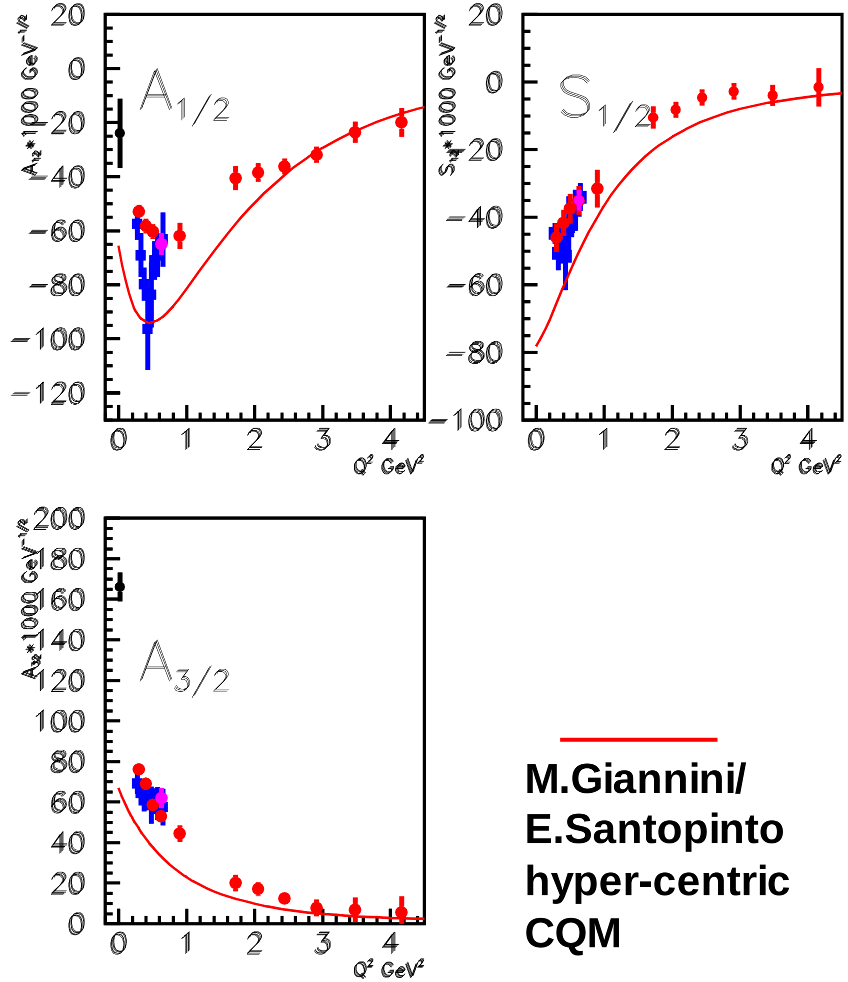
where λ_i (i=γ,p) stand for the initial state photon and proton helicities. The λ_f is generic symbol for the helicities in the final state

$$\epsilon_\nu(\lambda_\gamma = -1)J^\nu(\lambda_p, \lambda_f) = \langle\lambda_f|T|\lambda_p\lambda_\gamma = -1\rangle,$$
$$\epsilon_\nu(\lambda_\gamma = 1)J^\nu(\lambda_p, \lambda_f) = \langle\lambda_f|T|\lambda_p\lambda_\gamma = 1\rangle,$$
$$\epsilon_\nu(\lambda_\gamma = 0)J^\nu(\lambda_p, \lambda_f) = \langle\lambda_f|T|\lambda_p\lambda_\gamma = 0\rangle,$$

The expressions on the slides #10,11 allow us to relate $\pi^+\pi^-$p electroproduction amplitudes of JM model and any other approach.

D_{13}(1520) electrocouplings from the CLAS data on Nπ/Nππ electroproduction

- **electrocouplings as determined from the Nπ & Nππ channels are in good agreement overall**
- ***but* the apparent discrepancies for the $A_{1/2}$ amplitude at Q^2 ~ 0.4 GeV^2 will be further investigated in a combined Nπ/Nππ analysis**
- **hypercentric Consituent Quark Model calculations reasonably describe electrocouplings at Q^2>2.5 GeV^2, suggesting that the 3-quark component is the primary contribution to the structure of this state at high Q^2.**



M.Giannini/ E.Santopinto hyper-centric CQM

$N\pi$ CLAS data at low & high Q^2

Number of data points > 119,000, W < 1.7 GeV

Observable	Q^2 [GeV²]	Number of Data points
$d\sigma/d\Omega(\pi^0)$	0.35-1.6	31 018
$d\sigma/d\Omega(\pi^+)$	0.25-0.65 1.7-4.3	13 264 33 000
$A_e(\pi^0)$	0.40 0.65	956 805
$A_e(\pi^+)$	0.40 0.65 1.7 - 4.3	918 812 3 300
$d\sigma/d\Omega(\eta)$	0.375 0.750	172 412

Low Q^2 results:
I. Aznauryan *et al.*, PRC 71, 015201 (2005); PRC 72, 045201 (2005);

High Q^2 results on Roper:
I. Aznauryan *et al.*, PRC 78, 045209 (2008).

Prelim. high Q^2 results on $D_{13}(1520)$, $S_{11}(1535)$:
V.Burkert, AIP Conf.Proc. 1056, 248 (2008).

Summary paper:
I.G.Aznauryan, V.D.Burkert et al. (CLAS Collaboration) arXiv:0909.2349[nucl-ex]

full data set in:
http://clasweb.jlab.org/physicsdb/

Definition of Resonance Parameters in JM model (including relative phases between resonant/non-resonant amplitues)

Application to the $\pi^+\pi^-$p CLAS data analysis

- 1.66<W<1.76 GeV, 0.5<Q^2<1.5 GeV^2 ; transitions between only two N* states are allowed by conservation laws 3/2^+(1720) ↔P_{13}(1720) . Unitarized BW ansatz was applied to this pair of states, while the other resonant contributions were treated in regular BW ansatz

- Resonant parts of γp→MB amplitudes, derived from the expression (1),(2) is :

$$T_{\gamma p\to MB}=\frac{1}{\det[S^{-1}]}\{f_{1MB}(M_2^2-i\Gamma_{2t}M_2-W^2)f_{1\gamma p}+i\sqrt{M_1M_2}\sum_i\sqrt{\Gamma_{1i}\Gamma_{2i}}\,f_{2MB}f_{1\gamma p}$$
$$+i\sqrt{M_1M_2}\sum_i\sqrt{\Gamma_{1i}\Gamma_{2i}}\,f_{1MB}f_{2\gamma p}+f_{2MB}(M_1^2-i\Gamma_{1t}M_1-W^2)f_{2\gamma p}\}\quad(3)$$

while the denominator det[S^{-1}] for two interacting resonances is:

$$\det[S^{-1}]=(M_1^2-i\Gamma_{1t}M_1-W^2)(M_2^2-i\Gamma_{2t}M_2-W^2)+\Big(\sum_i\sqrt{\Gamma_{1i}\Gamma_{2i}}\Big)^2M_1M_2\quad(4)$$

where Γαt are full decay widths of interacting N* states mentioned above.

Quality of the CLAS $\pi^+\pi^-p$ data description

	Range of χ^2/d.p. for selected in the data fit calculated cross secions
Regular BW ansatz	2.74-2.98
Unitarized BW ansatz before improvements	2.85-3.21
Unitarized BW ansatz after improvements	2.83-3.04

Comparable quality of the CLAS $\pi^+\pi^-p$ data description was achieved within the framework of JM model ,utilizing both unitarized and regular BW ansatzs .

How to compare N* parameters from JM and the approaches, employing analytical continuation of production amplitudes in complex energy plain?

N* parameters from JM model for comparison:

•fully integrated resonant cross sections for any given resonance at $W=M_{res}$ in the channels:

$\gamma p \rightarrow \pi^- \Delta^{++}$

$\pi^+ \Delta^0$

ρp

at the central or maximal (sub-threshold regime) mass of the unstable hadron

•resonant amplitudes for these processes. Phase difference between resonant amplitudes from JM and cc approaches may be incorporated as extra phases in JM resonant ansatz and data fit with "improved " ansatz can be repeated