Interpretation of N* parameters in a dynamical coupled-channel approach

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The Terrain

I. Reaction Theory
  – Dynamical reaction theory
    • Scattering equations
  – Dressed quantities
    • Propagators & vertex functions

II. Model calculations
  – Quantum Monte Carlo
    • Model Hamiltonian
    • QMC yields accurate many-body wave functions
    • Bare vertex functions → Dressed vertex functions

Scattering equations:

In/Out states

$$(E - H_0)\Psi^{\pm} = V\Psi^{\pm} \Rightarrow \Psi^{\pm} = \Phi + G_0 V \Psi^{\pm}$$

$$V\Psi^{\pm} = T\Phi \qquad G_0 = [E - H_0 \pm i\epsilon]^{-1}$$

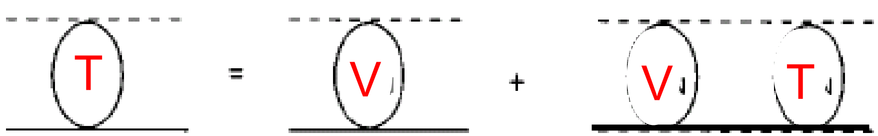
$$H_0 \Phi = E_0 \Phi$$

$$\Phi = a_i^{\dagger} \cdots b_j^{\dagger} \cdots d_k^{\dagger} \cdots |0\rangle$$

H_0 Eigenstates

$$T = V + VG_0T = V[1 - G_0V]^{-1}$$

$$S = 1 - 2\pi i T$$



- In/Out states time independent
- Lorentz covariant
- Particle/anti-particle states
- H_0 spectrum stable states
- H_0 has physical masses
  (cf. bare mass of H)

Hamiltonian Formalism

I. Hamiltonian formalism

$$\mathcal{L}_I \to \mathcal{H}_I$$
$$H_I = \int d^3x \, \mathcal{H}_I \sim V$$

– Main advantage: *readily interpret model calculations*
  - Constituent quark models: test model predictions using many body wave functions
    – SU(6) predictions ~ "missing resonance" problem
    – Di-quark models – suggested by studies of proton form factor
    – Collective string-like model [Bijker, Iachello, & Leviatan]
    – Etc…

Non-resonant and resonant contributions

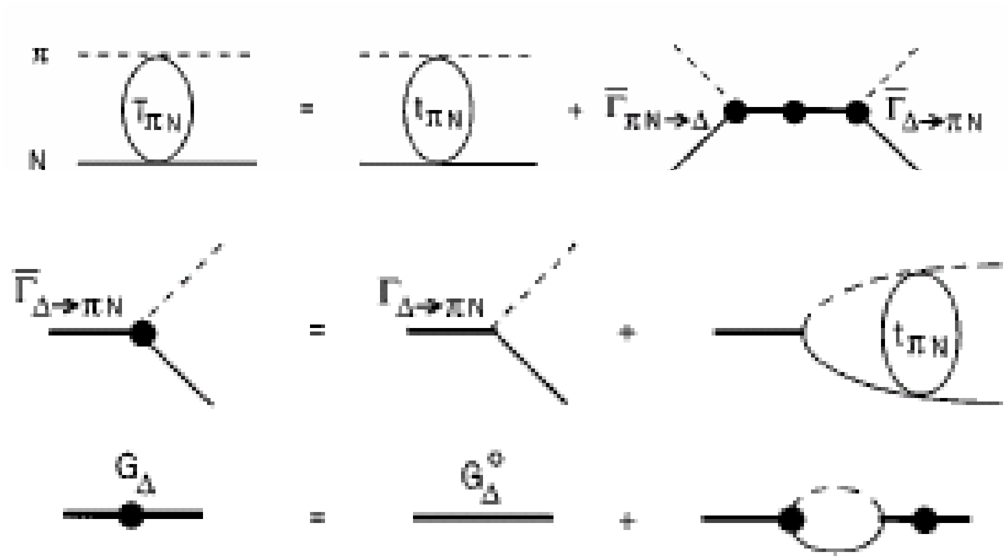
$$V = v_{NR} + v_R$$

$$T = t_{NR} + t_R$$

$$= (1 - v_{NR} G_0)^{-1} v_{NR} + \overline{\Gamma}^\dagger \frac{1}{E - H_0 - \Sigma} \overline{\Gamma}$$

$$\overline{\Gamma}_{i,c} = \sum_{c'} \Gamma_{i,c'} (1 + G_0 t_{NR})_{c',c} \quad c, c' \leftrightarrow \text{channels}$$

$$\Sigma_{i,j} = \overline{\Gamma}_i G_0 \Gamma_j^\dagger \quad i, j \leftrightarrow \text{resonance number}$$



Simple model – for exploration

I. Channel subspace $c = \{\pi N \oplus \eta N \oplus \sigma N\}$

II. Hamiltonian

$$H_{Eff} = H_0 + \sum_c v_{NR}^c + \sum_{i,c} [\Gamma_{N_i^* \to c} + H.c.]$$

III. T-matrix:

$$T(E) = t_{NR}(E) + \sum_{i,j} \overline{\Gamma}_i^\dagger(k_0) \left[\frac{1}{E - H_0 - \Sigma} \right]_{i,j} \overline{\Gamma}_j(k_0)$$

IV. Warm-up project: Fit S_{11} T-matrix

– Non-resonant:
  - Allow variation in $g_{\pi\rho N} = g_{\rho NN} g_{\rho\pi\pi}, g_{\eta NN}$

– Resonant S_{11}(1535,1650) (****)
  - No mixing: $\Sigma_{i,j} \propto \delta_{i,j}$
  - No vertex dressing: $\overline{\Gamma} \to \Gamma$

No mixing, no dressing: `PDG' fit

$$t_{NR}(E) = \sum_{i,j} \bar{\Gamma}_i^\dagger(k_0)\left[\frac{1}{E - H_0 - \Sigma}\right]_{i,j} \bar{\Gamma}_j(k_0) = \sum_i \frac{|\tilde{\Gamma}_i(k_0)|^2}{E - M_i(E) + \frac{i}{2}\Gamma_i(E)}$$

Diagonalize for each E

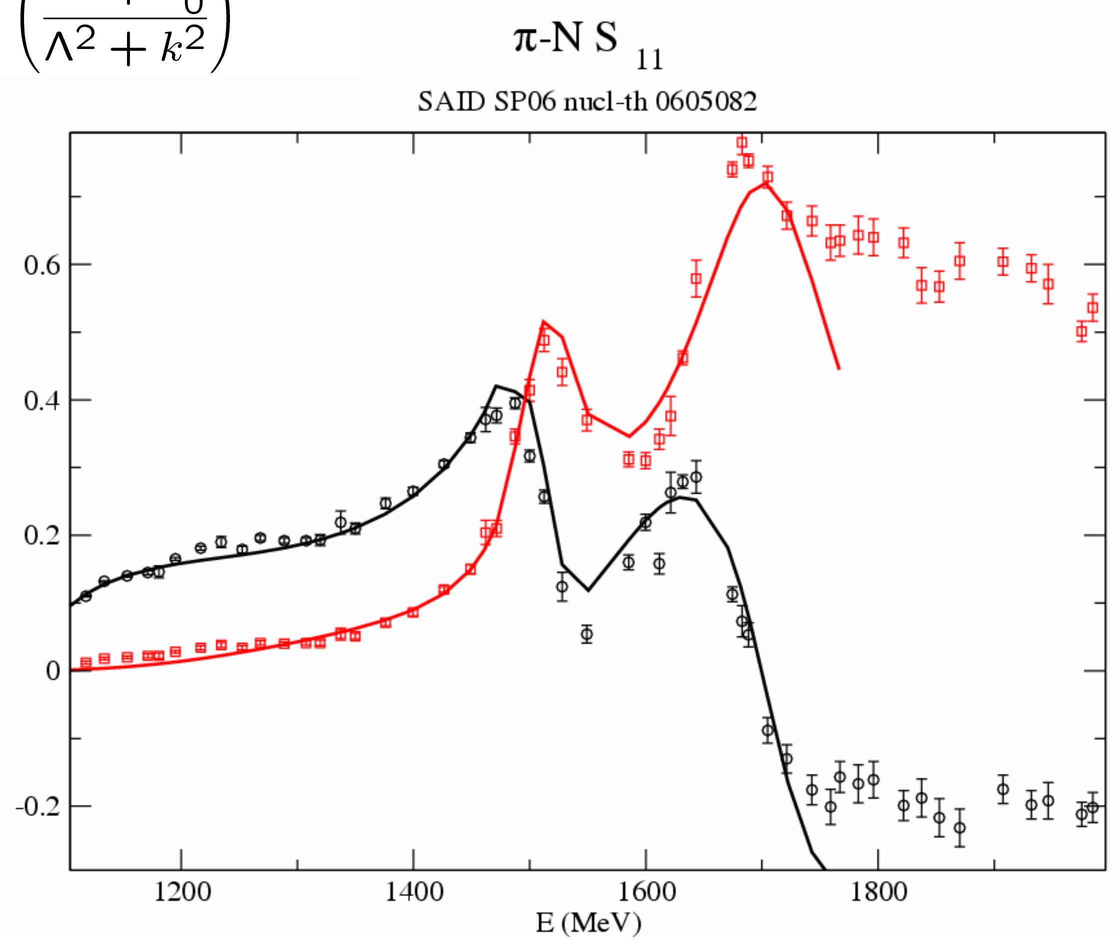
$$M_i(E) \to M_i$$

$$\Gamma_i(E) = \Gamma_i^0 \frac{\rho(k)}{\rho(k_0)}\left(\frac{k}{k_0}\right)^{2L}\left(\frac{\Lambda^2 + k_0^2}{\Lambda^2 + k^2}\right)^{L/2+2}$$

- Includes background
- χ^2 min.: 10 params
- Parallelized code: fast
- Background params:

$$g_{\pi\rho N} = 20.0$$

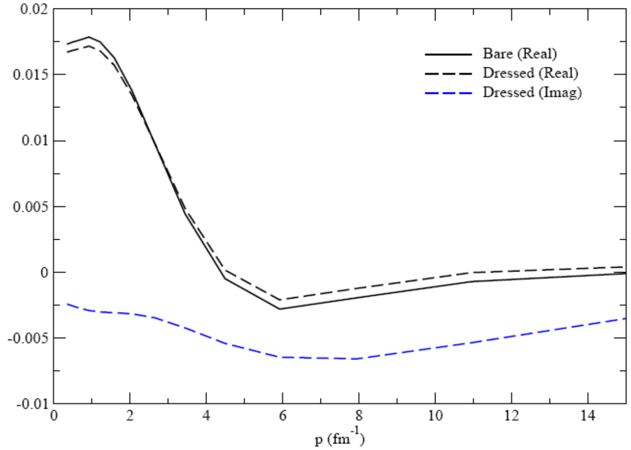
$$g_{\eta NN} = 3.3$$



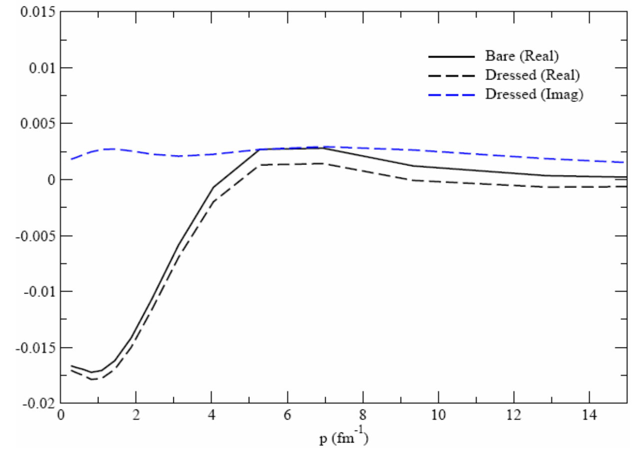
VMC transition form factors

$$\overline{\Gamma}(k) = \Gamma(k) + \int_0^\infty dk'\, k'^2\, \Gamma(k') G_0(k') t_{NR}(k', k; M_{N^*})$$

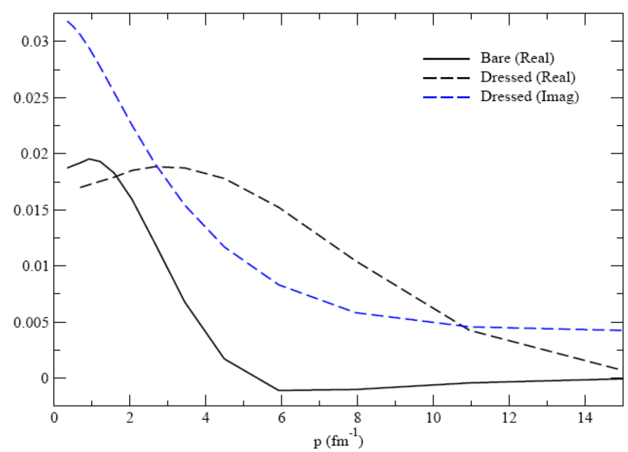
$\Gamma_{N^*(1535)\to\pi N}(S_{11})$



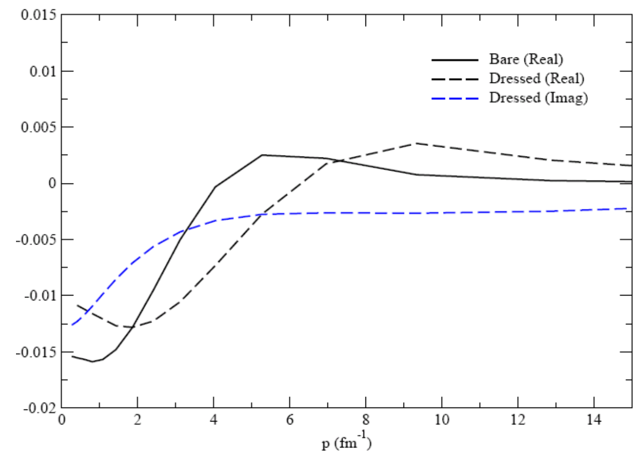
$\Gamma_{N^*(1650)\to\pi N}(S_{11})$



$\Gamma_{N^*(1535)\to\eta N}(S_{11})$



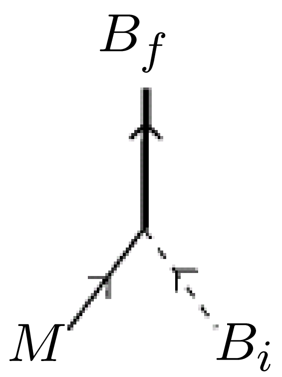
$\Gamma_{N^*(1650)\to\eta N}(S_{11})$



Quantum Monte Carlo

- Bare transition form factors

$$
\begin{aligned}
&\Gamma_{MB_i\to B_f}(\mathbf{k}) \\
&= \langle B_f(\mathbf{p}_f JMTT_z)|H_{\pi qq}|B_i(\mathbf{p}_i S_B M_B T_B T_{z,B}); M(\mathbf{k} S_M M_M T_M T_{z,M})\rangle \\
&= \langle TT_z|T_B T_M T_{z,B} T_{M,z}\rangle \\
&\times \sum_{LM_L}\sum_{SM_S} \langle JM|SLM_SM_L\rangle\langle SM_S|S_BS_MM_BM_M\rangle i^L Y_L^{M_L}(\hat{\mathbf{k}}) \\
&\times \Gamma^{JT}_{LS}(k)\,\delta^{(3)}(\mathbf{k}+\mathbf{p}_f-\mathbf{p}_i)
\end{aligned}
$$



- Transition operators
  - One body

$$
H^{(a)}_{\pi qq}(\mathbf{k}) = \frac{f_{\pi qq}}{m_\pi}\frac{i}{\sqrt{(2\pi)^3 2\omega_k}}\sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i}\tau_i^z\boldsymbol{\sigma}_i\cdot\left[\mathbf{k}-\frac{\omega_k}{2m_q}(\mathbf{p}_i+\mathbf{p}_i')\right]F_\pi(|\mathbf{k}|^2)
$$

  - Two-body and higher: to be considered

Constituent quark model

- Dynamical Hamiltonian model
  - non-relativistic kinetic energy
  - two-body potential interactions
    - strong state dependence T=0,1 isospin
      S=0,1 spin
      C=**3***,**6** color
    - just about any configuration space potential can be handled
    - work with one-gluon exchange + one-pion exchange

$$v_{ij}^{\pi} = v_{\sigma\tau}(r)\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j + v_{t\tau}(r)S_{ij}\tau_i\cdot\tau_j$$
$$v_{ij}^{g} = [v_{cg}(r) + v_{\sigma g}(r)\sigma_i\cdot\sigma_j + v_{tg}(r)S_{ij} + v_{\ell s}(r)(\mathbf{L}\cdot\mathbf{S})_{ij}]\mathsf{T}_i\cdot\mathsf{T}_j$$

    - linear confining interaction acting between all quark pairs

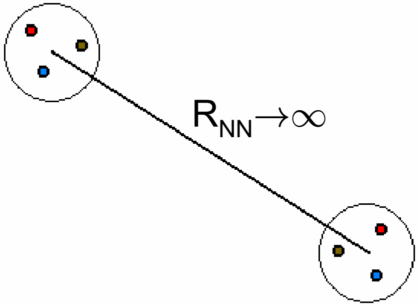
$$v^c(r) = \frac{1}{N_q-1}\sqrt{\sigma}r_{ij}$$

  - many-body confining term
    - flux tube model
  - constant term
    - scales with number of quarks
      - flux tube ends
      - constant term $\sum_{i<j}\mathsf{T}_i\cdot\mathsf{T}_j\Phi = -\frac{1}{2}\left(\sum_i \mathsf{T}_i^2\right)\Phi = -\frac{8}{3}N_q\Phi$

$$H_{CQM} = \sum_{i=1}^{N_q}\frac{\hbar^2}{2m_i}\nabla_i^2$$
$$+\sum_{i<j=1}^{N_q} v_{ij}(\mathbf{r}_{ij})$$
$$+V_{MB}(\mathbf{r}_1,\ldots,\mathbf{r}_{N_q})$$
$$+N_q v_0$$

Jefferson Lab

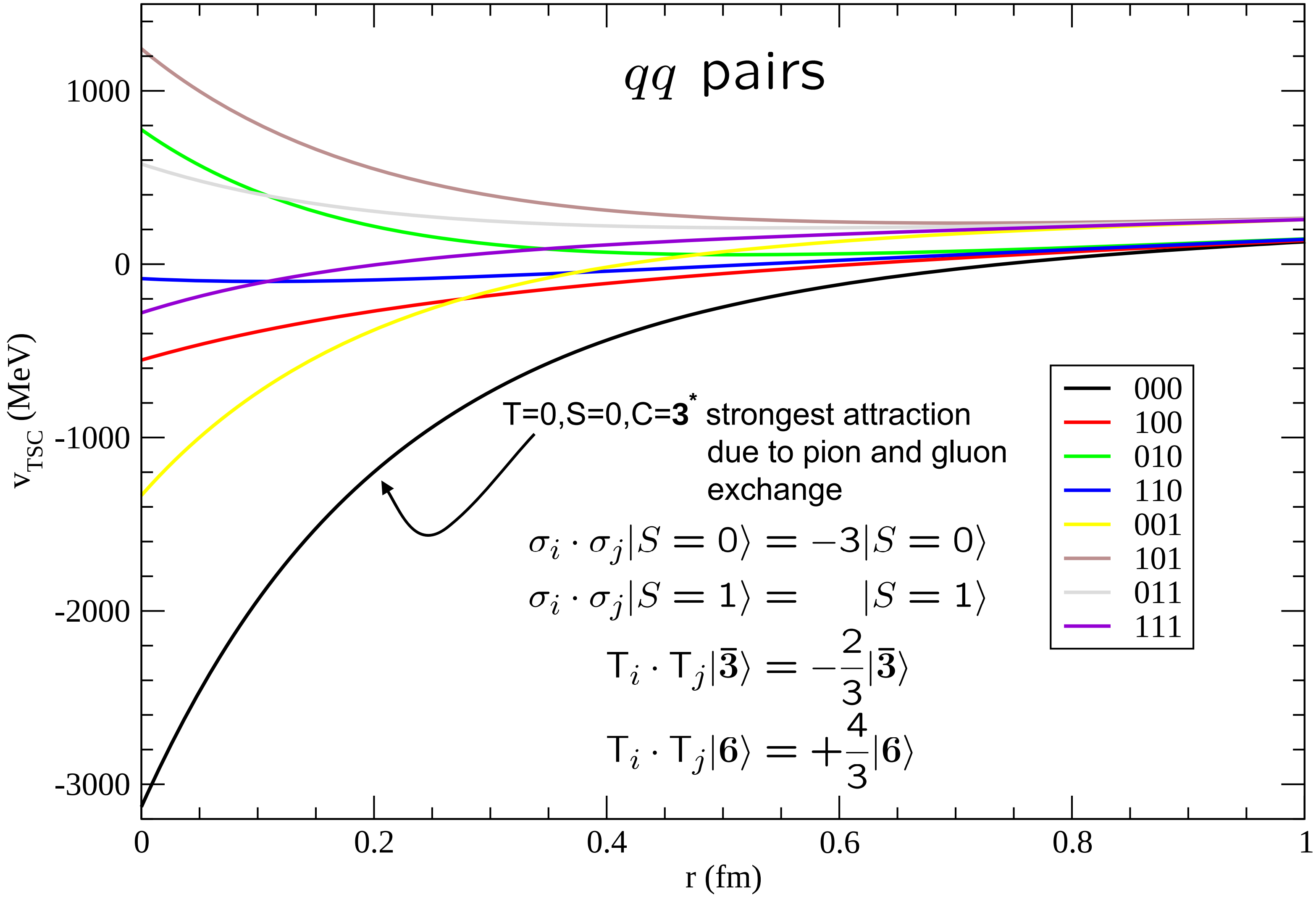
Model parameters



- Constituent quark mass
  - light quark mass $m_q = m_N/3$
    - fixed by non-relativistic form
  - strange quark mass m_s=510 MeV
    - parameter
- string tension
  - N and Δ trajectories
  - E^2 vs. J for "stringlike" configurations $E^2=2\pi\sigma^{1/2}J$
  - $\sigma^{1/2}$=0.88 GeV/fm
- perturbative gluon coupling constant
  - $m_\Delta - m_N \Rightarrow \alpha_s=0.61$
- pion quark coupling
  - $R_{NN}\to\infty \Rightarrow f_{\pi\, qq}=3f_{\pi\, NN}/5$
- quark form factor
  - $F(\mathbf{q}^2) = \frac{\Lambda^2-\mu^2}{\Lambda^2-\mathbf{q}^2}$
  - Λ=5 fm^{-1}; parameter
- constant term fitted to $m_N \Rightarrow v_0\approx$130 MeV (per flux tube end)

Central potential

v_{TSC} vs. r



Solution of the Schrödinger equation

- Variational Monte Carlo

$$\langle H_{CQM}\rangle = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle}$$

$$\frac{\delta\langle H_{CQM}\rangle}{\delta\Psi_V} = 0 \text{ S.T. } \langle\Psi_V|\Psi_V\rangle = \text{const.}$$

$$\langle H_{CQM}\rangle \geq E_0$$

- How to write a good many-body wave function
  - Correlation operator $|\Psi_V\rangle = \widehat{\mathcal{G}}|\Phi\rangle$
  - Ansätz $\widehat{\mathcal{G}} \approx \mathcal{S}\prod_{i<j=1}^{N_q}\widehat{F}_{ij}$
  - Two-body correlation

$$\widehat{F}_{ij} = \sum_p f_p(r_{ij})\mathcal{O}_{ij}^p$$

p	$\mathcal{O}_p$	Symb
1	1	c
2	$\tau_i\cdot\tau_j$	τ
3	$\sigma_i\cdot\sigma_j$	σ
4	$\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j$	$\sigma\tau$
5	$\mathsf{T}_i\cdot\mathsf{T}_j$	g
6	$\tau_i\cdot\tau_j\mathsf{T}_i\cdot\mathsf{T}_j$	τg
7	$\sigma_i\cdot\sigma_j\mathsf{T}_i\cdot\mathsf{T}_j$	σg
8	$\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j\mathsf{T}_i\cdot\mathsf{T}_j$	$\sigma\tau g$
9	S_{ij}	t
10	$S_{ij}\tau_i\cdot\tau_j$	$t\tau$
11	$S_{ij}\mathsf{T}_i\cdot\mathsf{T}_j$	tg
12	$S_{ij}\tau_i\cdot\tau_j\mathsf{T}_i\cdot\mathsf{T}_j$	$t\sigma g$

$$\sum_{p=1}^{8} f_p(r_{ij})\mathcal{O}_{ij}^p|TSC\rangle = f_{TSC}(r_{ij})|TSC\rangle$$

$$\sum_{p=9}^{12} f_p(r_{ij})\mathcal{O}_{ij}^p S_{ij}|TSC\rangle = f_{tTC}(r_{ij})S_{ij}|TSC\rangle$$

  - Solve two-body Schrodinger-like equation, eg.

$$\left\{-\frac{\hbar^2}{2\mu}\nabla_{ij}^2 + \left[v_{TSC}(r_{ij}) - \lambda_{TSC}(r_{ij})\right]\right\} f_{TSC}(r_{ij}) = 0$$

Many—body wave function

- Features
  - Many—body: 2- & 3-body correlations
  - Translationally invariant $\Psi_V(\mathbf{R}+\mathbf{a}) = \Psi_V(\mathbf{R})$
  - Accurate

$$E_V(\mathbf{R}) = \frac{\langle \Psi_V(\mathbf{R}) | \hat{H} | \Psi_V(\mathbf{R}) \rangle}{\langle \Psi_V(\mathbf{R}) | \Psi_V(\mathbf{R}) \rangle}$$

$$\text{If } |\Psi_V\rangle = |\Psi_0\rangle$$

$$\Rightarrow E_V(\mathbf{R}) = \frac{\langle \Psi_0(\mathbf{R}) | \hat{H} | \Psi_0(\mathbf{R}) \rangle}{\langle \Psi_0(\mathbf{R}) | \Psi_0(\mathbf{R}) \rangle} = E_0$$

$$\sigma(E_V) = \sqrt{\frac{\langle \Psi_V | \hat{H}^2 | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} - \frac{\langle \Psi_V | \hat{H} | \Psi_V \rangle^2}{\langle \Psi_V | \Psi_V \rangle^2}}$$

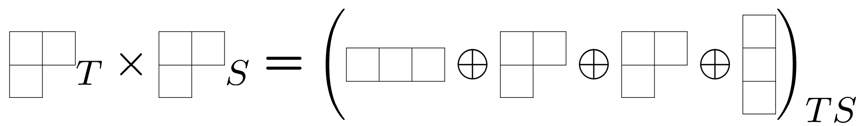
$$\sigma(E_V)/E_V \sim 0.1\%$$

- Limitations: “Cartoon”
  - No $q\bar{q}$ pairs
  - No glue
  - Non-relativistic

Single hadron wave function

- S- and P-wave N & Δ states
  - uncorrelated $|qqq\rangle$ states
    - color singlet $\Rightarrow T_i \cdot T_j = -2/3 \; \forall \; i,j$
    - spin—isospin

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2}^{S} \oplus \frac{1}{2}^{\rho} \oplus \frac{1}{2}^{\lambda}$$



$N^{\frac{1}{2}+}(939)$	$\chi_\lambda(m_S)\chi_\lambda(m_T) + \chi_\rho(m_S)\chi_\rho(m_T)$
$\Delta^{\frac{3}{2}+}(1232)$	$\chi_S(m_S)\chi_S(m_T)$
$N^{\frac{1}{2}-}(1535)$ $N^{\frac{3}{2}-}(1520)$	$\chi_\rho(m_T)\left[\phi_\lambda(m_L)\chi_\rho(m_S) + \phi_\rho(m_L)\chi_\lambda(m_S)\right]$ $+\chi_\lambda(m_T)\left[-\phi_\lambda(m_L)\chi_\lambda(m_S) + \phi_\rho(m_L)\chi_\rho(m_S)\right]$
$\Delta^{\frac{1}{2}-}(1620)$ $\Delta^{\frac{3}{2}-}(1700)$	$\chi_S(m_T)\left[\phi_\lambda(m_L)\chi_\lambda(m_S) + \phi_\rho(m_L)\chi_\rho(m_S)\right]$
$N^{\frac{1}{2}-}(1650)$ $N^{\frac{3}{2}-}(1700)$ $N^{\frac{5}{2}-}(1675)$	$\chi_S(m_S)\left[\phi_\lambda(m_L)\chi_\lambda(m_T) + \phi_\rho(m_L)\chi_\rho(m_T)\right]$

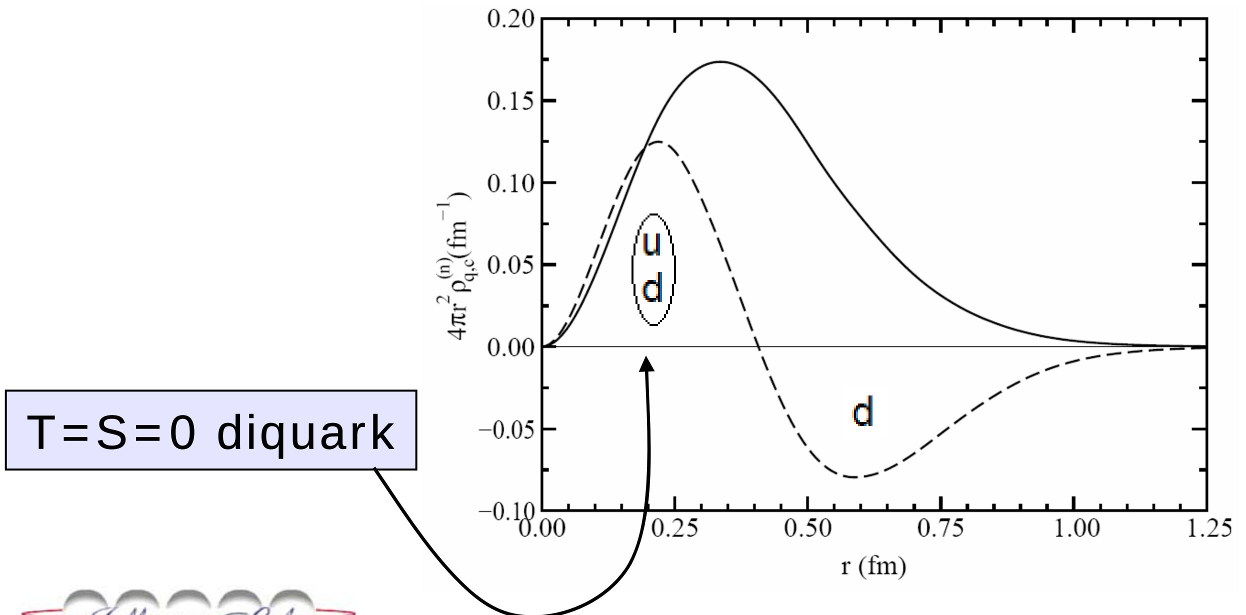
- Apply correlation operator: $|\Psi_V\rangle = \hat{\mathcal{G}}|\Phi\rangle$

Single hadron results

- s- & p-wave spectra

	$N\frac{1}{2}^+$	$\Delta\frac{3}{2}^+$	$N\frac{1}{2}^-$	$N\frac{3}{2}^-$	$\Delta\frac{1}{2}^-$	$\Delta\frac{3}{2}^-$	$N\frac{1}{2}^-$	$N\frac{3}{2}^-$	$N\frac{5}{2}^-$
Exp.	939(0)	1232(2)	1535(13)	1520(8)	1620(30)	1700(50)	1650(20)	1700(50)	1675(8)
H	939(1)	1232(1)	1581(1)	1571(1)	1703(1)	1702(1)	1603(1)	1672(1)	1734(1)
T	1007(5)	767(3)	1063(3)	1024(4)	965(3)	949(3)	1021(3)	940(3)	904(3)
V	308(5)	840(3)	894(3)	923(4)	1114(2)	1129(3)	958(3)	1108(3)	1206(2)

- Neutron quark & charge densities



GFMC

- Green's function Monte Carlo
  - Project true ground state from Ψ_V

$$\Psi(\tau) = e^{-\tau H}\Psi_V$$
$$\lim_{\tau\to\infty}\Psi(\tau) \to c_0 e^{-\tau E_0}\Psi_0$$

  - Short—time Feynman—Carlson propagator

$$e^{-\Delta\tau H} \approx e^{-\Delta\tau V/2} e^{-\Delta\tau \hat{T}} e^{-\Delta\tau V/2} + \mathcal{O}(\Delta\tau^3)$$

  - Operator expectations

$$\langle\mathcal{O}\rangle_M = \frac{\langle\Psi_V|\mathcal{O}|\Psi(\tau)\rangle}{\langle\Psi_V|\Psi(\tau)\rangle}$$
$$= \frac{\int d\mathbf{P}_n [\mathcal{O}^\dagger\Psi_V(\mathbf{R}_n)]^\dagger \prod_{i=1}^n G(\mathbf{R}_i,\mathbf{R}_{i-1})\Psi_V(\mathbf{R}_0)}{\int d\mathbf{P}_n \Psi_V^\dagger(\mathbf{R}_n) \prod_{i=1}^n G(\mathbf{R}_i,\mathbf{R}_{i-1})\Psi_V(\mathbf{R}_0)}$$

Many—body sign problem

- Sampling paths

$$\langle \mathcal{O} \rangle_M = \frac{\sum_{\{\mathbf{P}\}} N_{\mathbf{P}}}{\sum_{\{\mathbf{P}\}} D_{\mathbf{P}}}$$

  – Drawn from probability distribution for paths

$$P(\mathbf{P}_n) = \prod_{i=1}^{n} \frac{I[\Psi_V(\mathbf{R}_i), \Psi_i(\mathbf{R}_i)]}{I[\Psi_V(\mathbf{R}_{i-1}), \Psi_{i-1}(\mathbf{R}_{i-1})]} I[\Psi_V(\mathbf{R}_0), \Psi_V(\mathbf{R}_0)]$$

- Many—body wave functions
  – Fermions $P_{ij}\Psi = -\Psi$

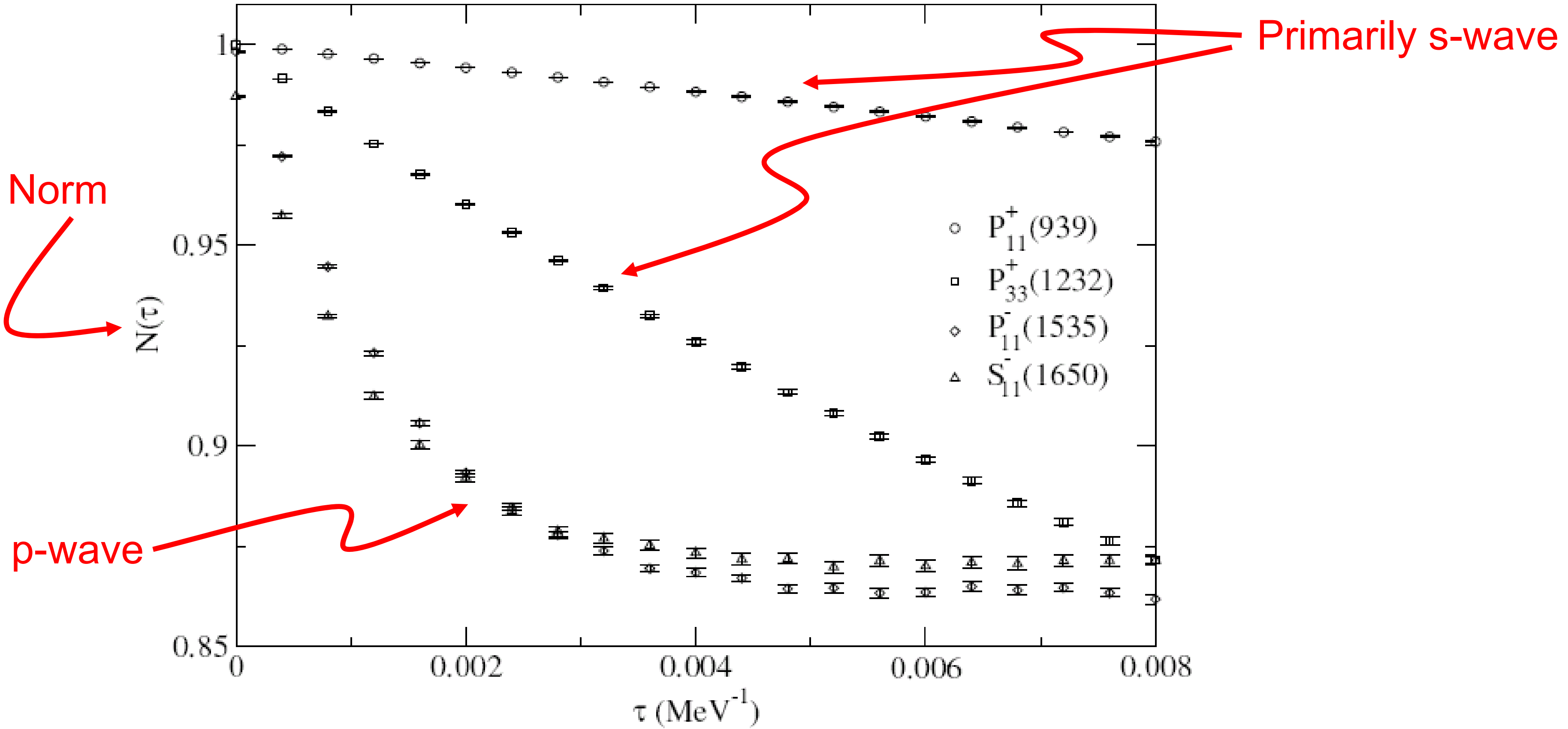
Guarantees nodes at coincidence “planes”

  – Bosonic
    - $P_{ij}\Psi_{\mathrm{Baryon}}(TSL) = +\Psi_{\mathrm{Baryon}}(TSL)$

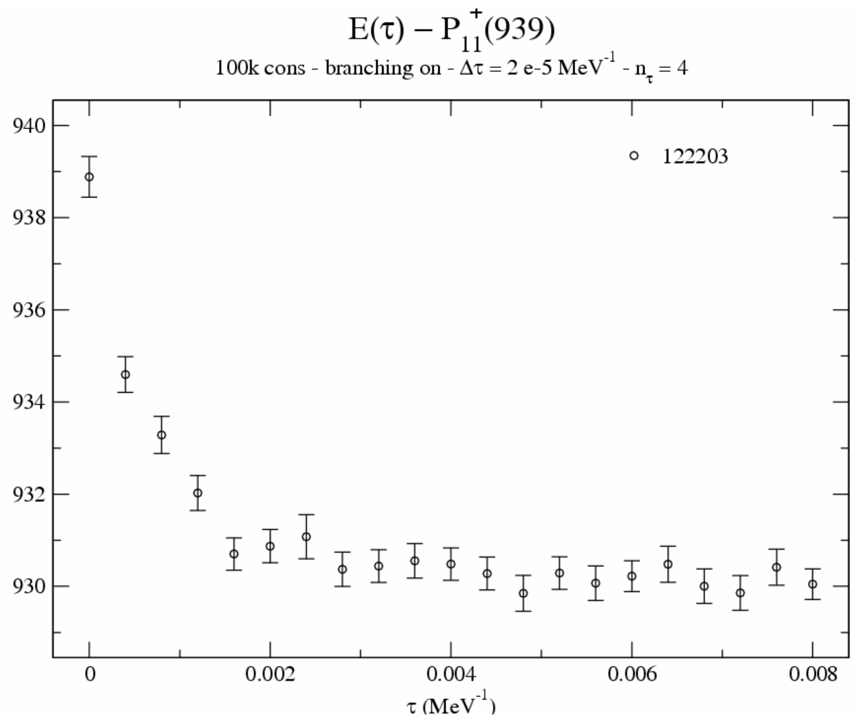
Can still have nodal “surfaces”

Constrained path Monte Carlo

- Discard paths which cross nodal "surfaces" without affecting expectation values



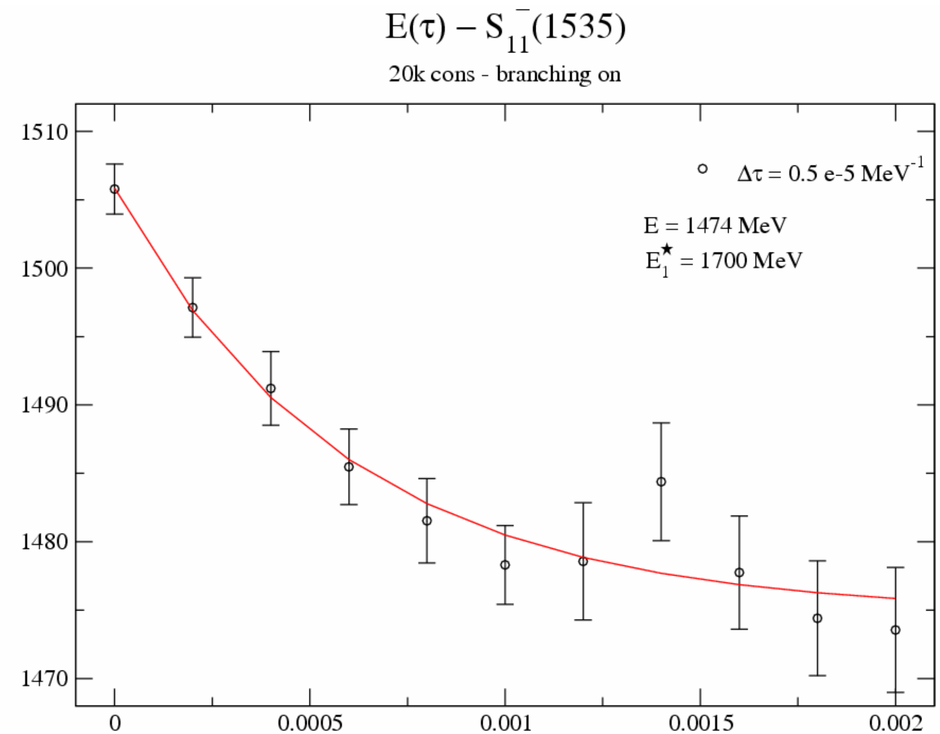
GFMC Results



- Fit parameters

$$E(\tau) = \frac{\langle \Psi(\tau/2)|H|\Psi(\tau/2)\rangle}{\langle \Psi(\tau/2)|\Psi(\tau/2)\rangle} = E_0 + \sum_{n>0} c_n E_n e^{-\tau(E_n - E_0)}$$

- s-wave $S_{11}^{+}(939)$
  - Sign problem negligible
  - Large τ evolution possible
  - Energy reduced $\sim$ 8 MeV



Conclude/Future Objectives

- Features
  - Dynamical coupled channel approach
    - Unitary
    - Off-shell multiple scattering dynamics
  - Model calculations
    - Accurate many—body wave function
    - Many—body current contributions calculable
    - Model interpretation
    - Necessity of pion cloud “dressing”
- Objectives
  - Fit complete set of observables for 1.1 GeV < E_{COM} <1.8 GeV
  - Interpret data in terms of model calculations
  - Make connection to lattice QCD (N-Δ transition)
  - Resolve $SU(6)_{FS}$ violations $\leftrightarrow$ “missing” resonances

EBAC

People

Main
⇒People
Notes
Papers
Links

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Supplementary material

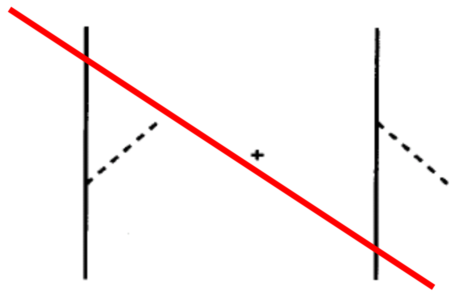
Unitary transformation (KSO) method

KSO: Kobayashi, Sato & Ohtsubo
PTP v98, N.4, 1997

Objective:

- Eliminate virtual={non-vanishing off energy-shell} processes ⇒ *operators diagonal in Fock space*
- Start with Interaction:

$$\mathcal{L}=\left[-\frac{f_{\pi NN}}{m_\pi}\overline{\psi(x)}\gamma_5\gamma^\mu\vec{\tau}\psi(x)\cdot\partial_\mu\vec{\phi}(x)\right.$$
$$\left.+\frac{f_{\pi N\Delta}}{m_\pi}\bar{\vec{\psi}}{}^\mu_\Delta(x)\vec{T}\psi(x)\cdot\partial_\mu\vec{\phi}(x)\right]+[\mathrm{H.c.}]$$



- Separate rea
- Unitary transform

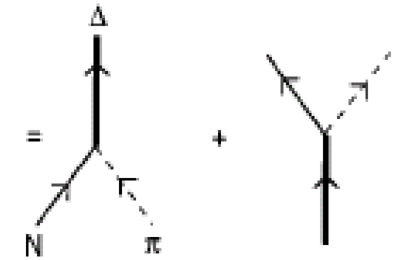
$$H = H_0 + H_I^P + H_I^Q$$

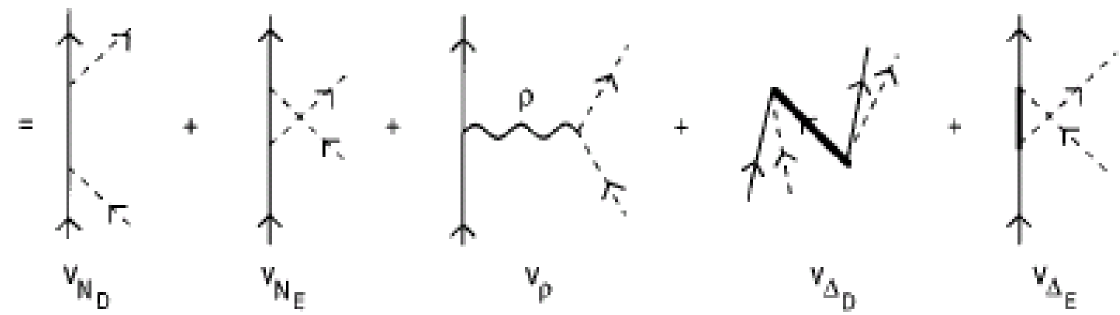
$$H_{Eff} = U^\dagger H U \approx H_0 + H_I^P + \underbrace{(H_I^Q + [H_0, iS])}_{=0} + \cdots$$

$$i\langle f|S|i\rangle = \frac{\langle f|H_I^Q|i\rangle}{E_f - E_i}$$

SKO (cont.)

$$H_{Eff} = H_0 + H_I^P$$
$$+ \left([H_I^P, iS] + \frac{1}{2}[H_I^Q, iS]\right)_{\text{real}}$$
$$+ \left([H_I^P, iS] + \frac{1}{2}[H_I^Q, iS]\right)_{\text{virtual}}$$
$$+ \sum_{n\geq 3} O(g_\alpha^n)$$





The price:

$$\langle f|H_I'^P|i\rangle = \sum_n \Bigg\{ \langle f|H_I^P|n\rangle\langle n|H_I^Q|i\rangle \frac{1}{E_i - E_n}$$
$$- \langle f|H_I^Q|n\rangle\langle n|H_I^P|i\rangle \frac{1}{E_n - E_f}$$
$$+ \langle f|H_I^Q|n\rangle\langle n|H_I^Q|i\rangle \frac{1}{2}\left[\frac{1}{E_i - E_n} - \frac{1}{E_n - E_f}\right]\Bigg\}$$