Models of Exotic Mesons

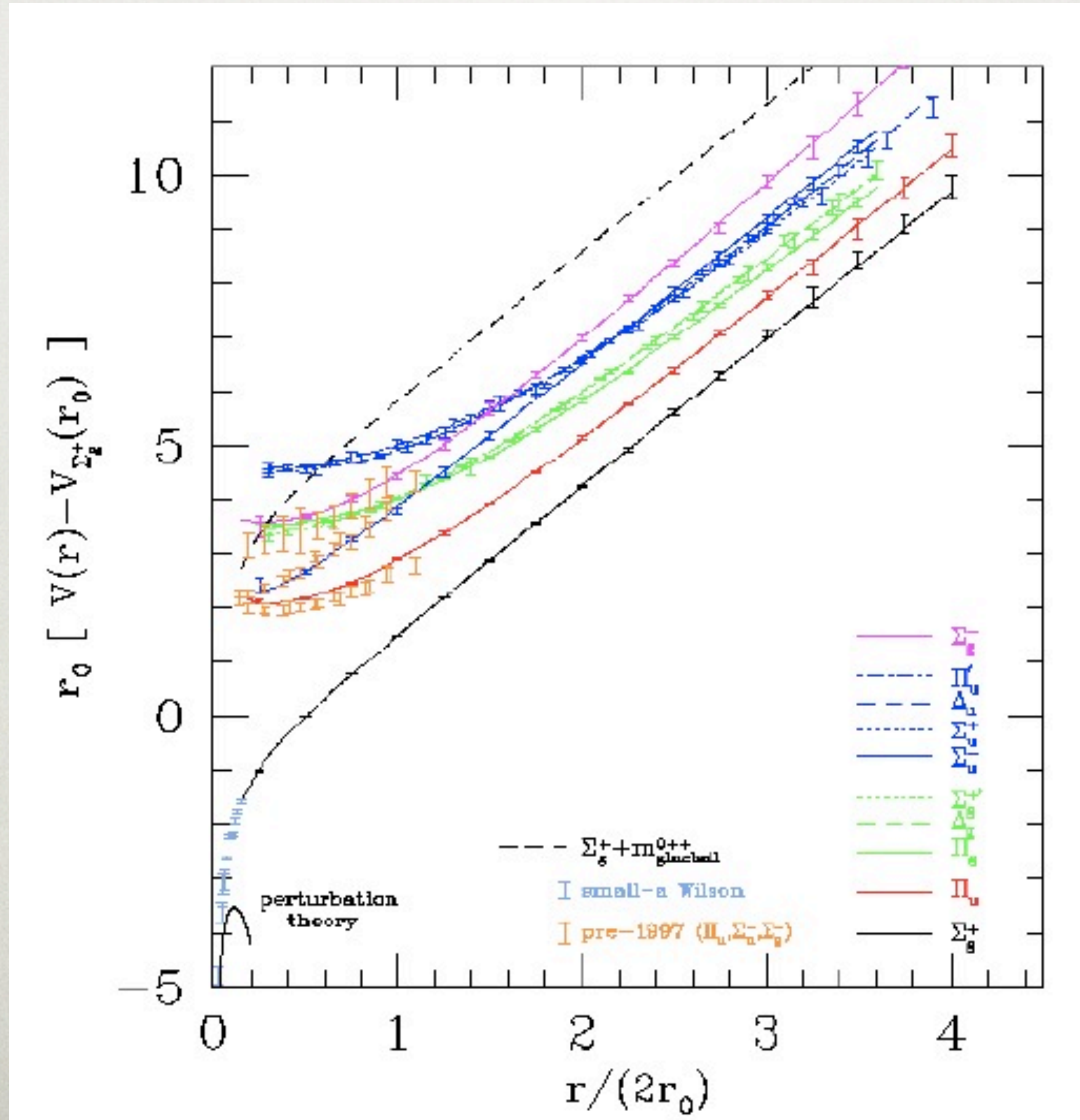
Eric Swanson

MODELLING EXOTIC MESONS

- bound state models
- decay models
- mixing, photocouplings

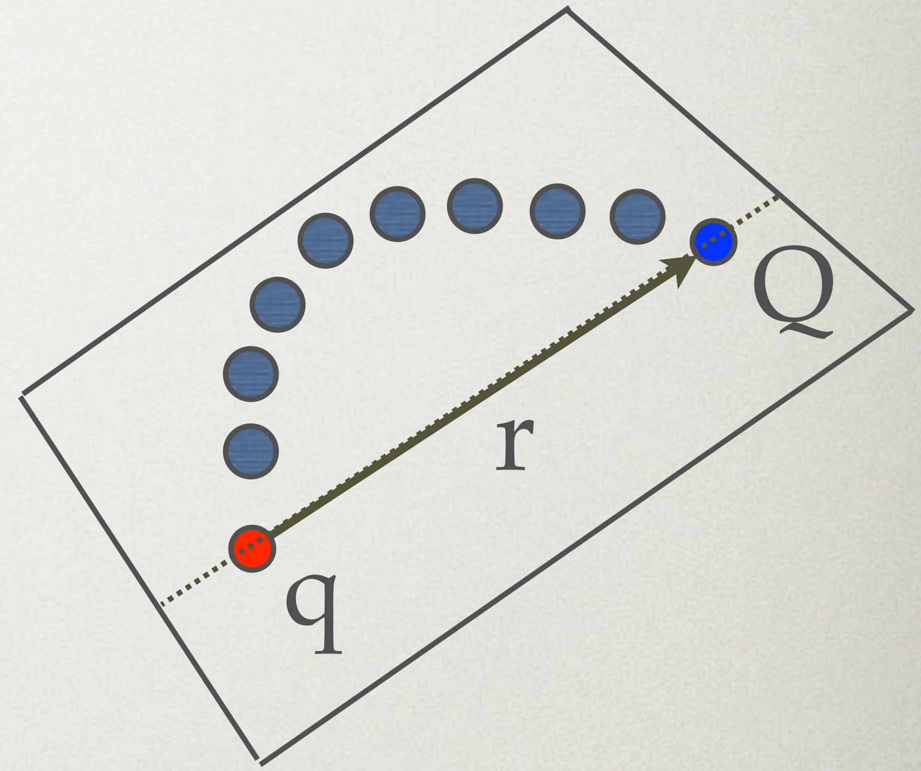
Lattice Results

Juge, Kuti, Morningstar, PRL82,4400,1999



Lattice Results

diatomic molecule quantum numbers



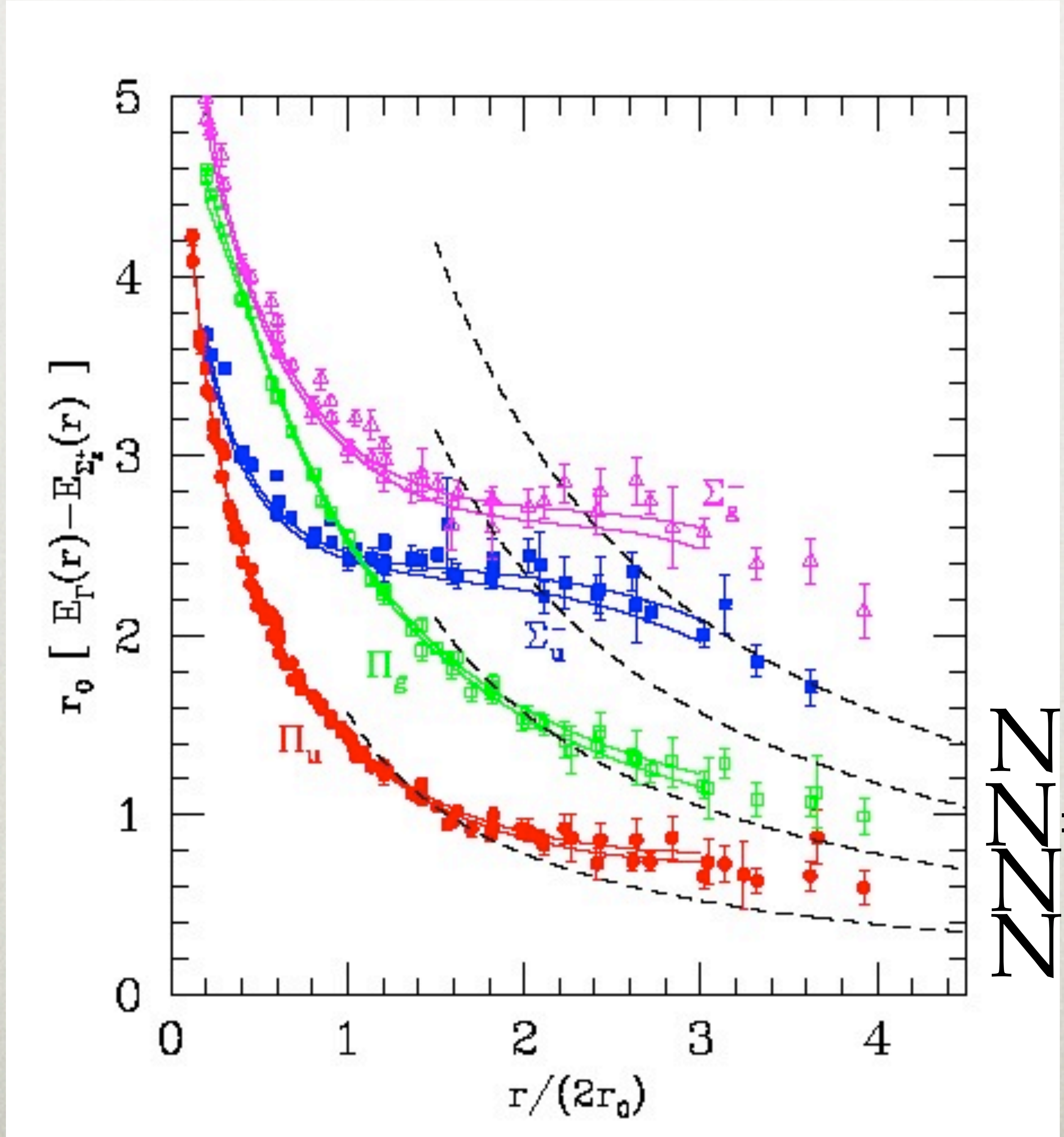
ex: $\Lambda^{Y}_{\eta} = \Sigma^{+}_{g}$ is the ground state

Y is PC acting on the glue

$\eta = u/g$ is reflection in a plane containing the q$\bar{\text{q}}$ axis

$\Lambda = \Sigma, \Pi, \Delta, \ldots$ is the projection of the string angular momentum onto the q$\bar{\text{q}}$ axis

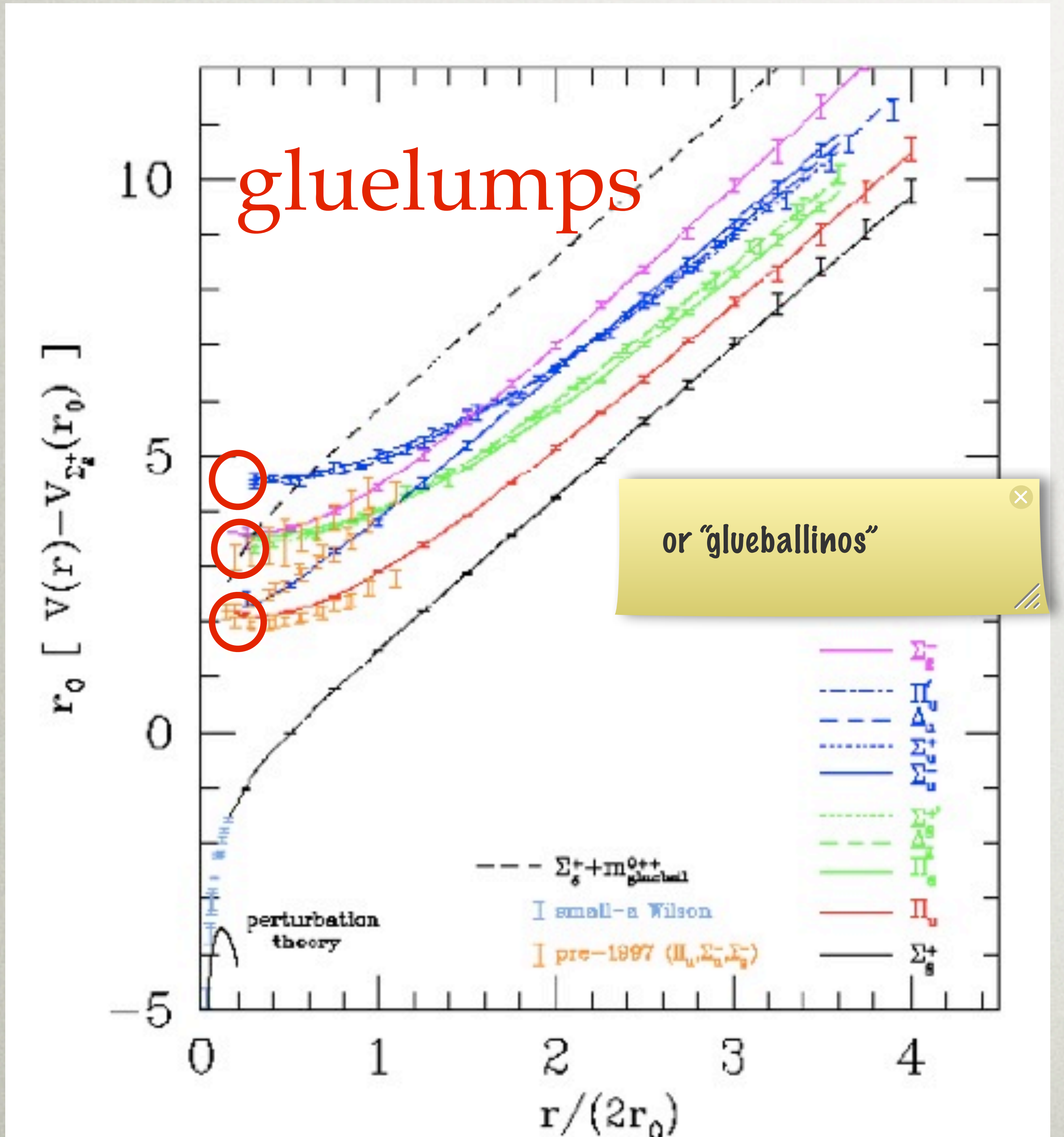
Lattice Results



string splitting not evident until R ~ 4 fm

Lattice Results

Foster and Michaels, PRD59, 094509 (99)



J^{PC}	potentials	ΔE (MeV)
1^{+-}	$\Pi_u,\ \Sigma_u^-$	—
1^{--}	$\Pi_g,\ \Sigma_g^+$	368(6)
2^{--}	$\Sigma_g^-,\ \Delta_g$	584(10)
3^{+-}	–	972(24)
2^{+-}	–	973(36)

MODELS

BOUND STATE MODELS

Ingredients

- model quarks -> (constituent quarks, bag modes)
- model glue -> (constituent gluons, bag modes, flux tubes)
- model dynamics

1. Bag Models

T. Barnes and F. E. Close, Phys. Lett. B 116, 365 (1982)
T. Barnes, F. E. Close and F. de Viron, Nucl. Phys. B 224, 241 (1983)
M. Chanowitz and S. Sharpe, Nucl. Phys. B 222, 211 (1983)

lowest mode is a 1+ TE gluon

the lightest multiplet is $0^{-+}, 1^{--}, 1^{-+}, 2^{-+}$

with mass of roughly 1.5 GeV

1. Bag Models

apply to gluelumps

Karl and Paton, PRD60, 034015 (99)

free gluon spectrum

$1^+, 2^-, 1^-, 3^+, 2^+, ...$

include Coulomb energy

$$
\begin{aligned}
1^+ &= 1.43 \text{ GeV} \\
2^- &= 1.97 \text{ GeV} \\
1^- &= 1.98 \text{ GeV} \\
3^+ &= 2.44 \text{ GeV} \\
2^+ &= 2.64 \text{ GeV}
\end{aligned}
$$

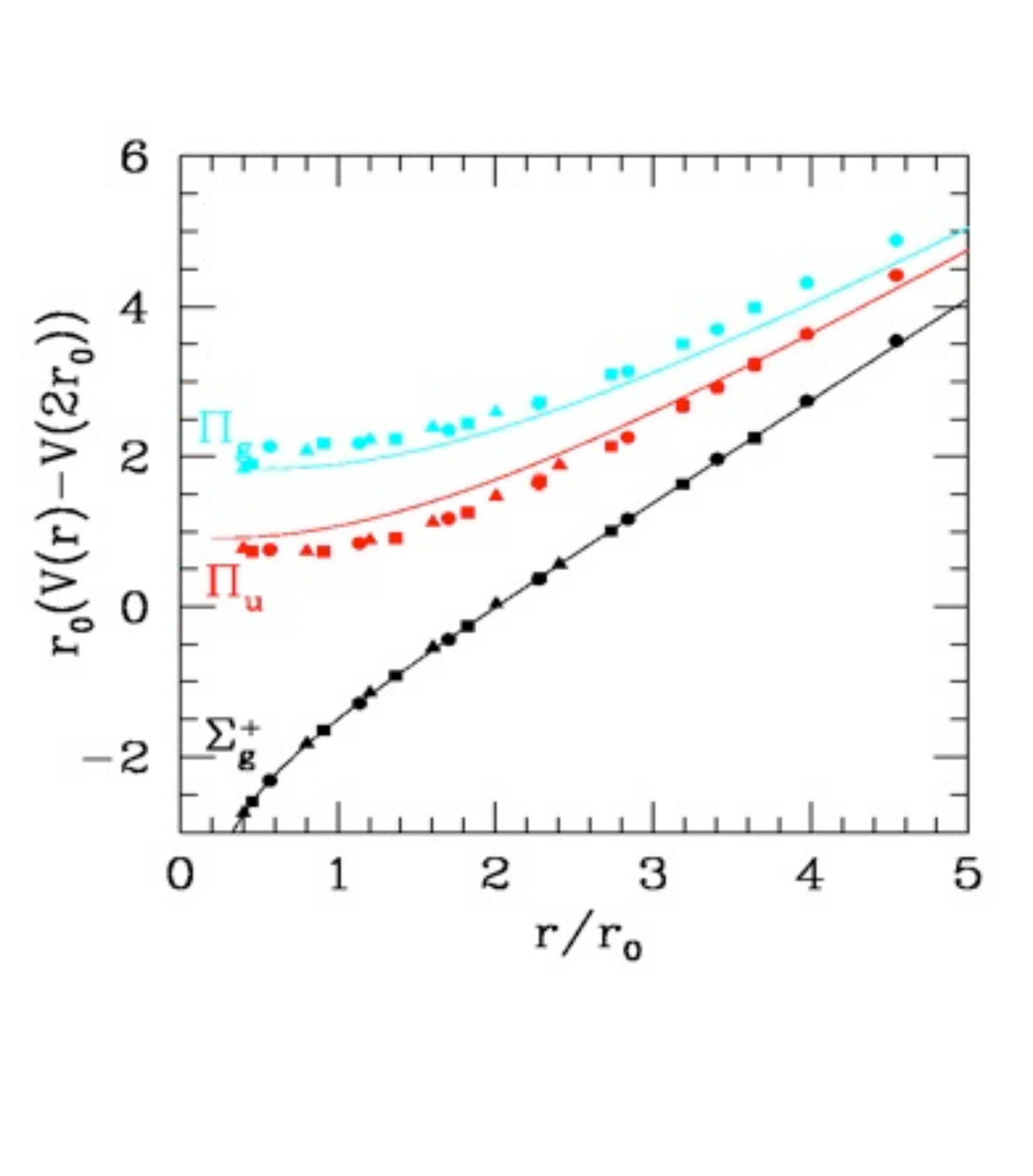
At small r, energies are shifted because a colour dipole develops:

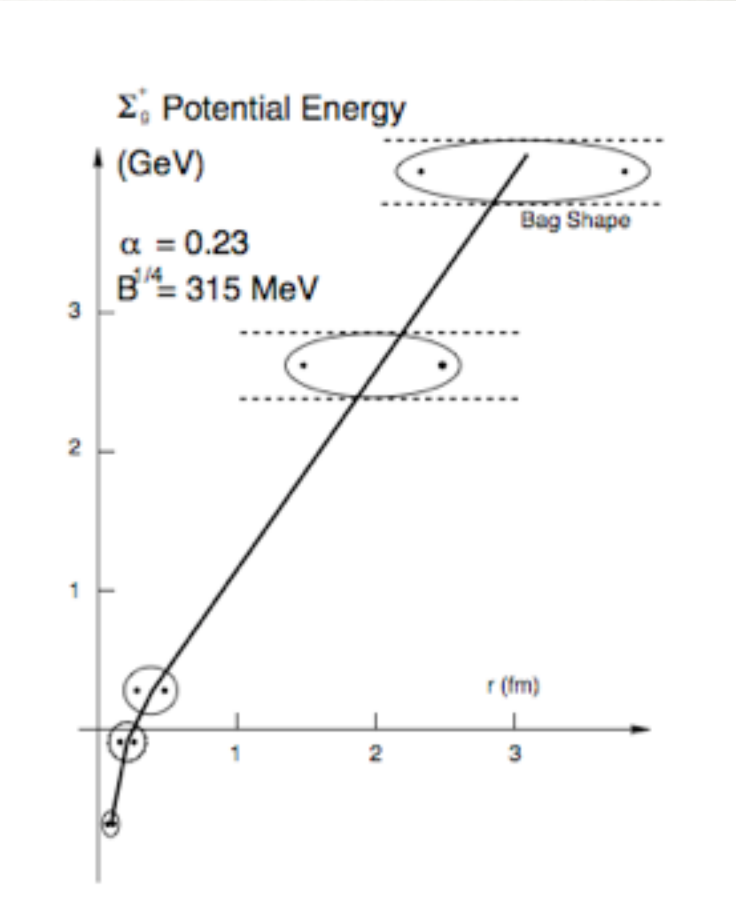
$$\delta E(r) = E(0) + \frac{9}{4}\alpha_s^2 \langle \frac{\cos^2\theta}{r^4} \rangle \frac{r2}{\bar{E}}$$

1. Bag Models

HHKR bag model computation

Juge, Kuti, and Morningstar, (98)





2. Sum Rules

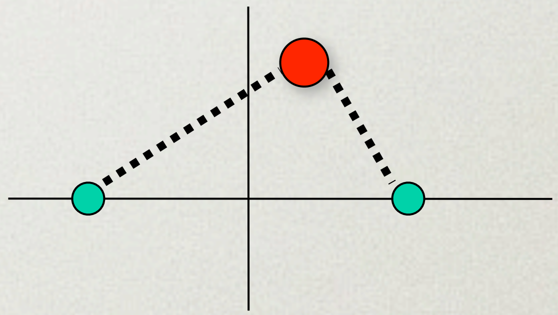
1^{-+} mass (GeV)	ref
1.3	J. Govaerts et al., PLB 128, 262 (1983); 136, 445E (1983); NPB 248, 1 (1984).
~1	I. I. Balitsky, D. I. Dyakonov and A. V. Yung, Z. Phys. C 33, 265 (1986)
1.7(1)	J. I. Latorre, P. Pascual and S. Narison, Z. Phys. C 34, 347 (1987)
1.65(5)	K.G. Chetyrkin and S. Narison, PLB 485, 145 (00)
1.81(6)	S. Narison, PLB 675, 319 (09)

3. Constituent Glue Models

Constituent quarks plus pointlike, massless, spinless, colorless 'glue'.

Barnes, PhD thesis, Caltech, (77)

D. Horn and J. Mandula, PRD 17, 898 (78)



Neglect the repulsive short range colour octet interaction, $V_{\underline{8}} = +\frac{\alpha_s}{6r}$

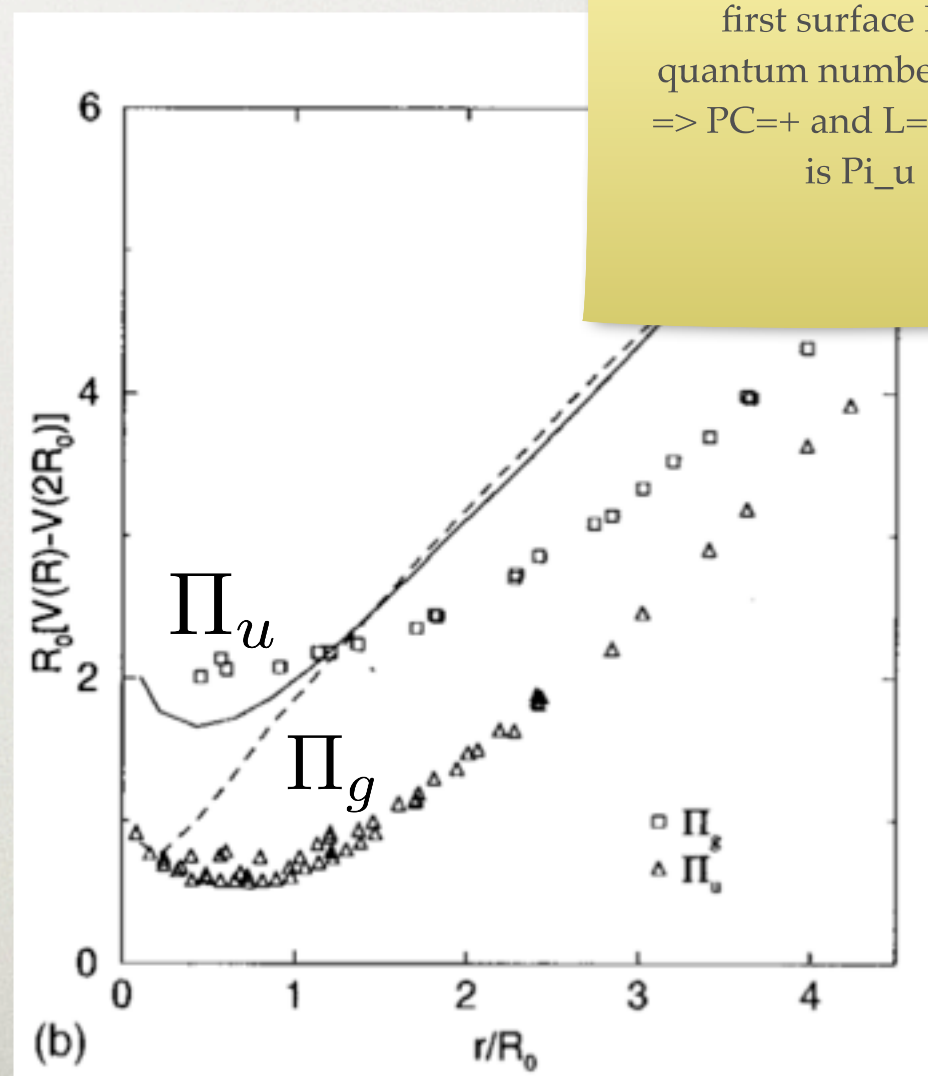
3. Constituent Glue Models

Swanson and Szczepaniak, PRD59, 014035 (98)

Coulomb gauge QCD, transverse gluons with colour, spin, and dynamically generated mass.

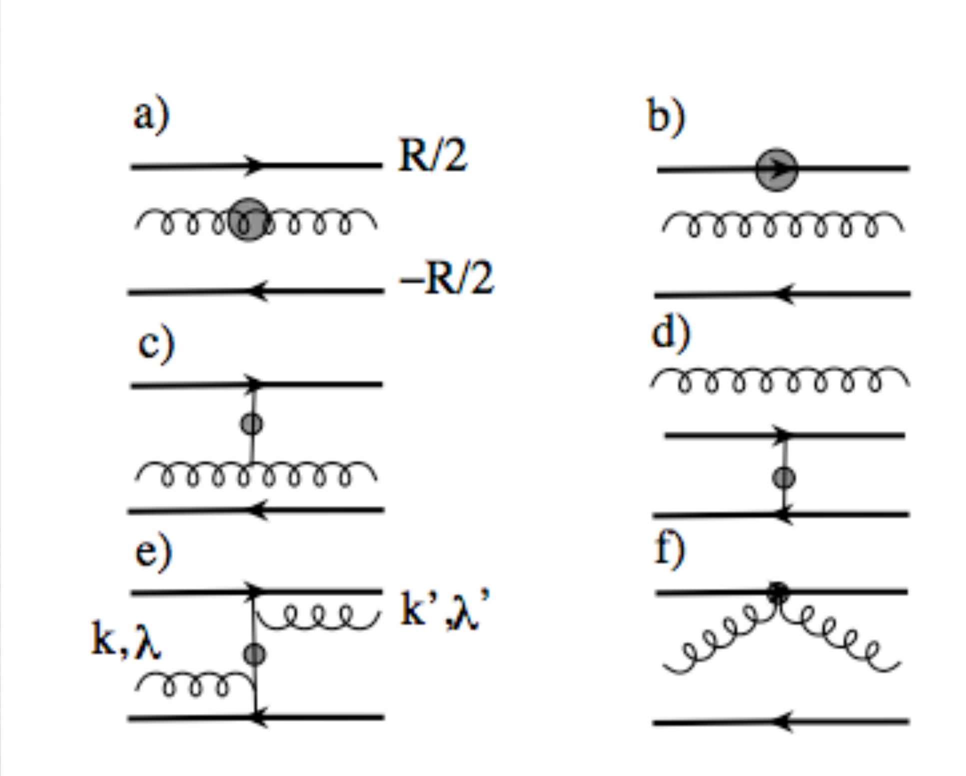
-- level ordering problem

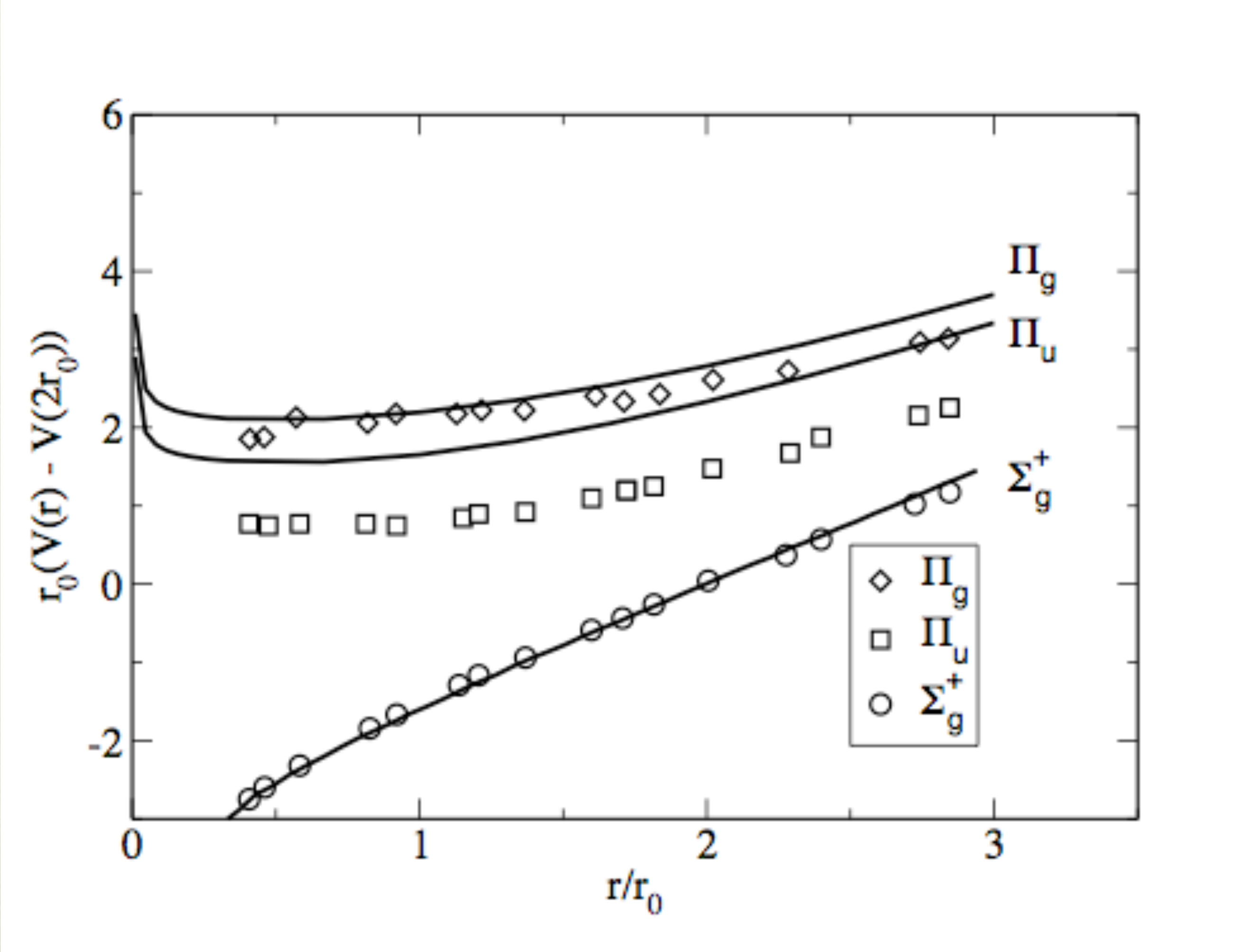
throw a gluon into a QQ: one expects this gives the first surface Pi_g quantum number (eg 1-- => PC=+ and L=1). But it is Pi_u



3. Constituent Glue Models

Szczepaniak and Krupinski, hep-ph/0604098





4. Flux Tube (& String) Models

Giles and Tye, PRL37, 1175 (76). ①

Coupled quarks to a relativistic 2d sheet... the "Quark Confining String Model".

"The presence of vibrational levels gives ... extra states in quantum mechanics. ... that are absent in the charmonium model."

$$V_N = \sigma r \left(1 + \frac{2N\pi}{\sigma r^2}\right)^{1/2}$$

GT

$$V_{NG} = \sigma r \left(1 - \frac{D-2}{12\sigma r^2} + \frac{2N\pi}{\sigma r^2}\right)^{1/2}$$

J.F. Arvis, PLB127, 106 (83); Luescher

16 APPLICATION OF THE QUARK-C

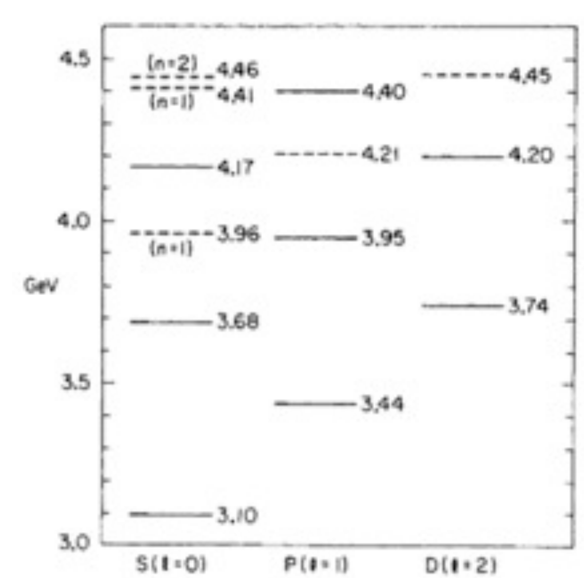
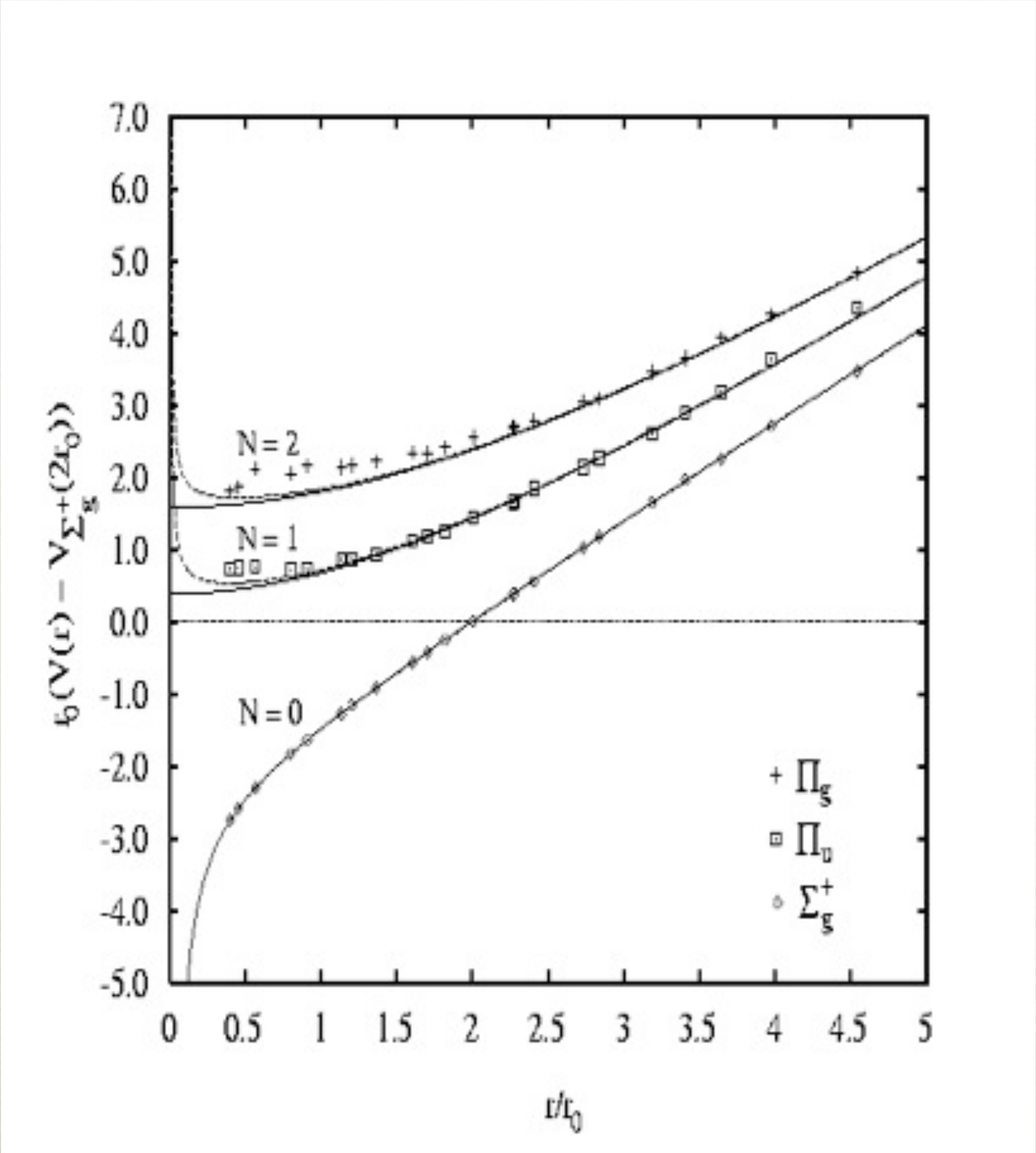


FIG. 4. The nonrelativistic spectroscopy of the charm string. $\psi(3.10)$ and $\psi(3.68)$ are fitted to obtain $M = 1.154$ GeV and $k = 0.21$ GeV^2. The dashed lines are the vibrational levels absent in the charmonium model. Levels with $E > 4.5$ GeV or $l > 2$ are not shown.

4. Flux Tube (& String) Models

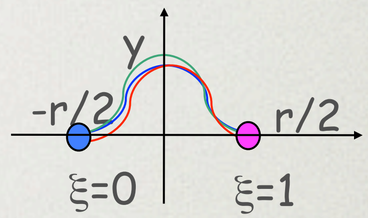
T. Allen and M.G. Olsson, PLB434, 110 (98)



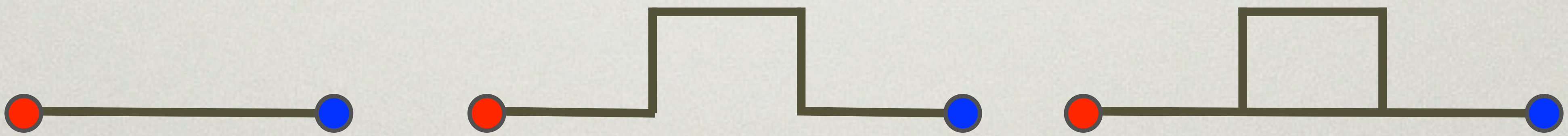
4. Flux Tube (& String) Models

Isgur and Paton, PRD31, 2910 (85).

strong coupling Hamiltonian lattice gauge theory



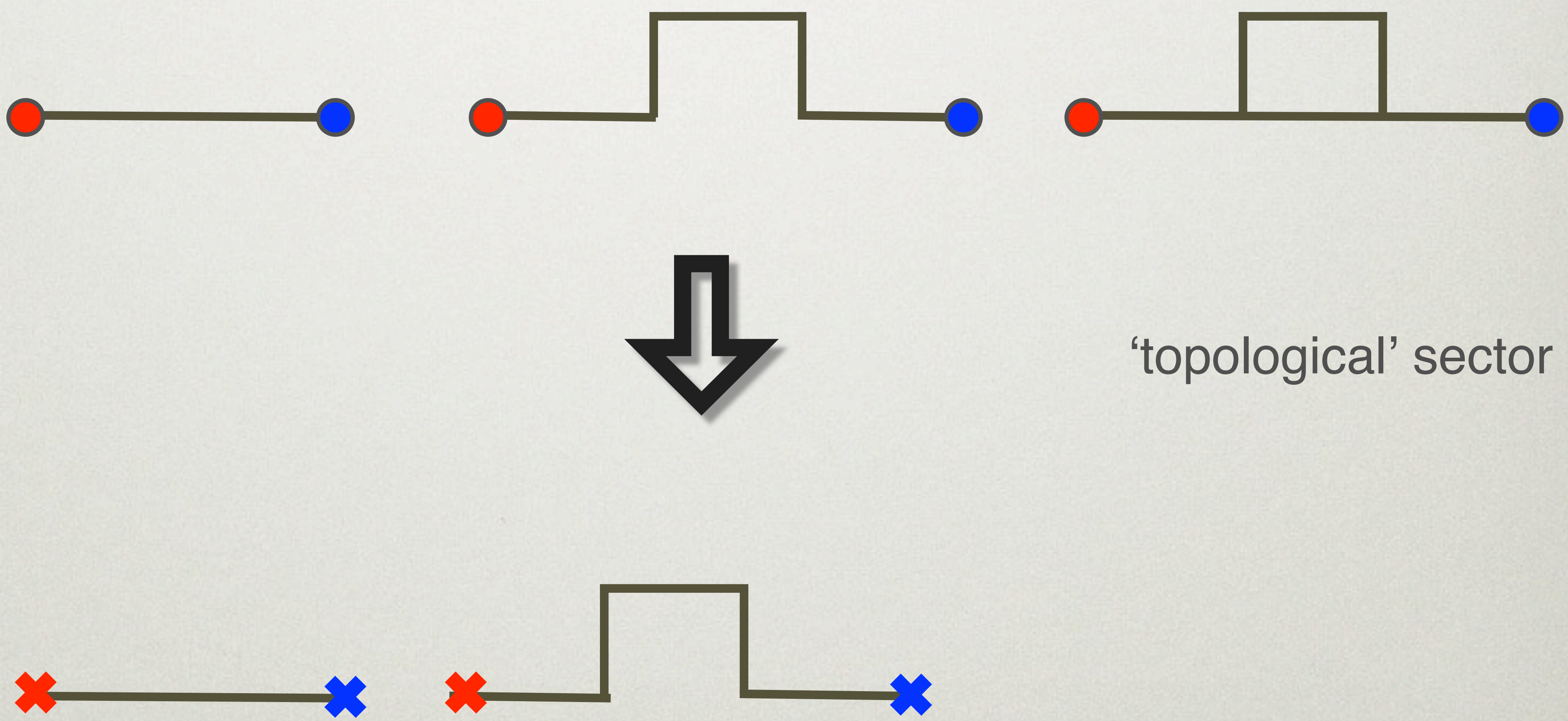
$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi^\dagger_n\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$



4. Flux Tube (& String) Models

Isgur and Paton, PRD31, 2910 (85).

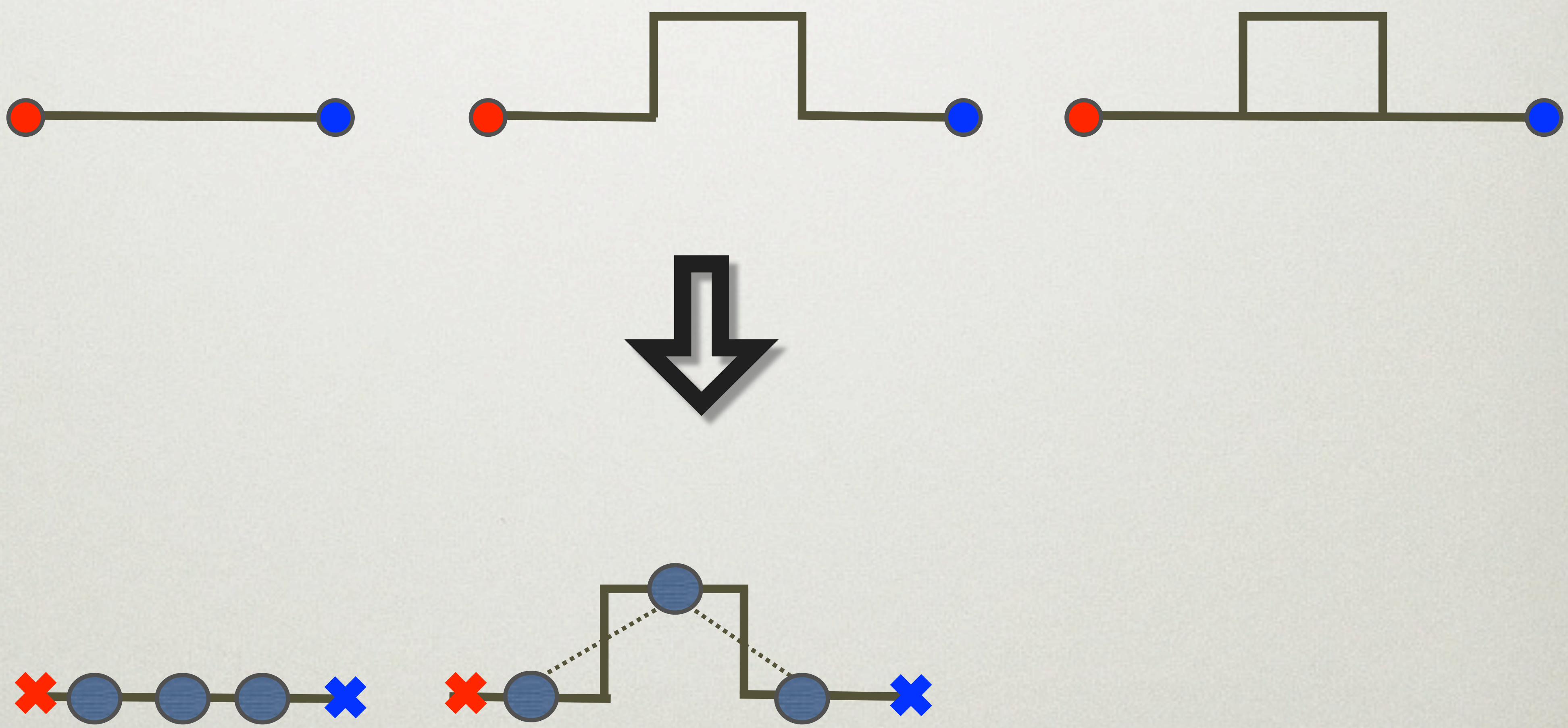
$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n \psi_n + \frac{1}{a}\sum_{n,\mu} \psi^\dagger_n \alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$



4. Flux Tube (& String) Models

Isgur and Paton, PRD31, 2910 (85).

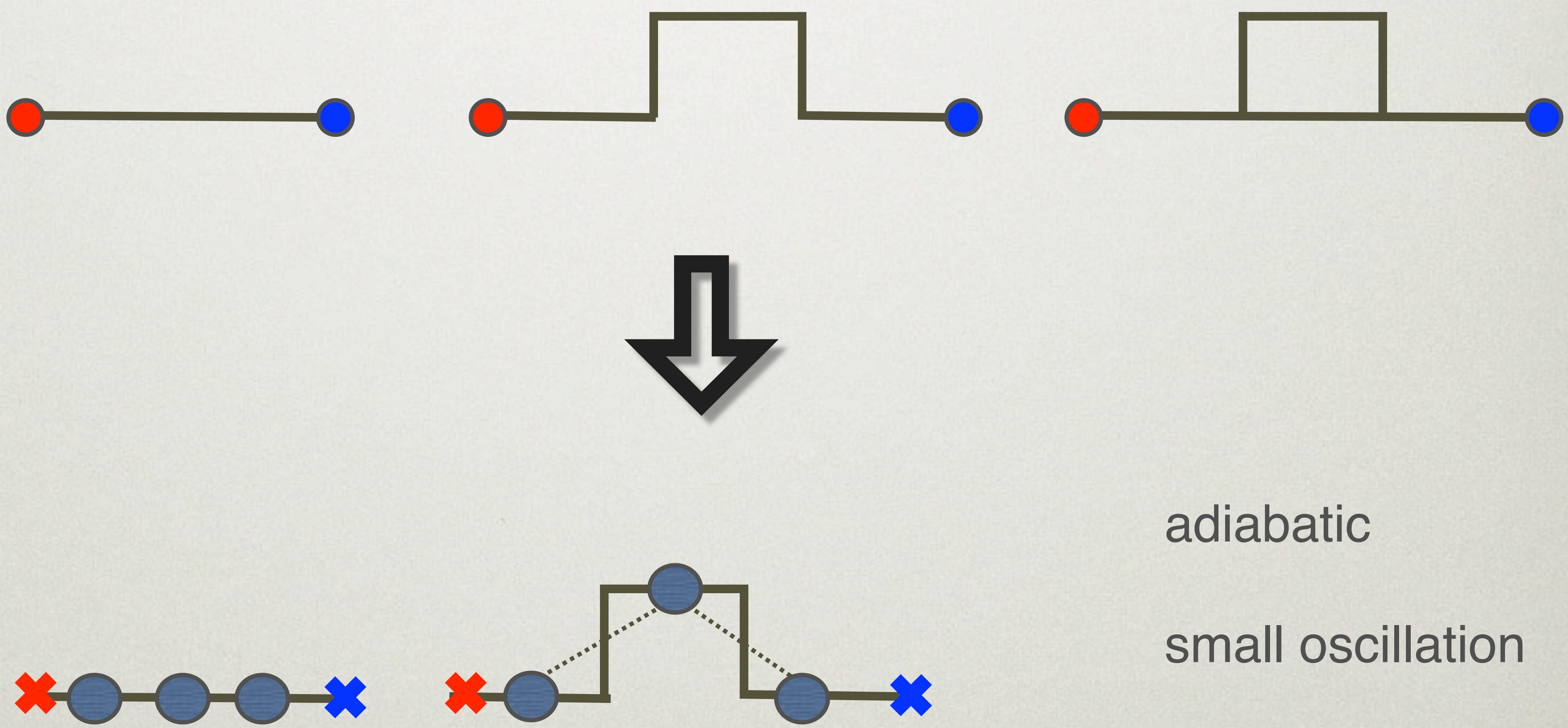
$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi^\dagger_n\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U^\dagger_P)$$



4. Flux Tube (& String) Models

Isgur and Paton, PRD31, 2910 (85).

$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi^\dagger_n\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$



adiabatic

small oscillation

nonrelativistic beads

$m_b = b\,a$

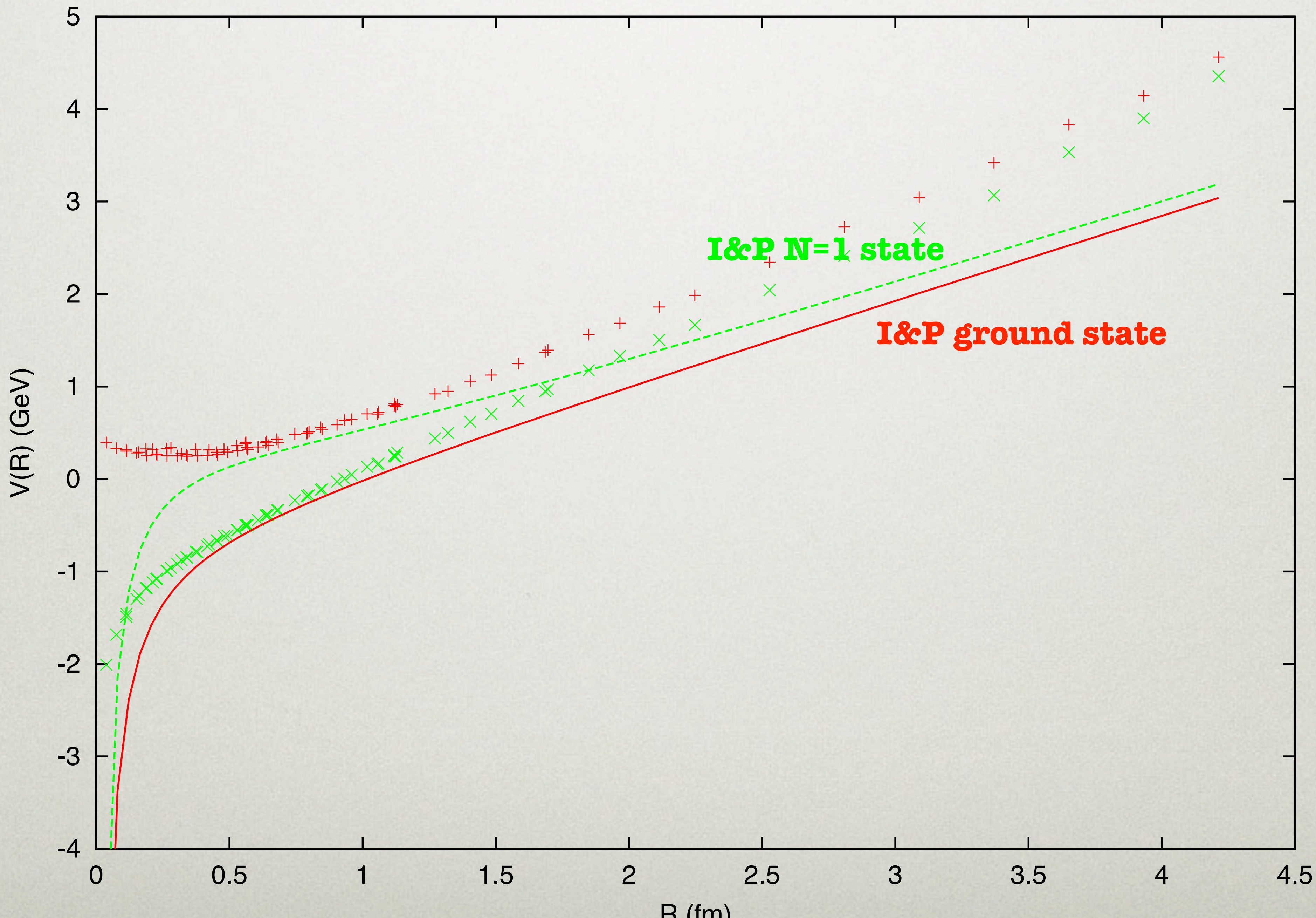
4. Flux Tube (& String) Models

$$H = b_0 R + \left(\frac{4}{\pi a^2} R - \frac{1}{a} - \frac{\pi}{12R} \right) + \sum_{n\lambda} \omega_n \alpha^\dagger_{n\lambda} \alpha_{n\lambda}$$

	L	S	J^{PC}
	1	0	1^{++}
	1	1	$(2,1,0)^{+-}$
$\zeta = +$	2	0	2^{--}
	2	1	$(3,2,1)^{-+}$
	1	0	1^{--}
$\zeta = -$	1	1	$(2,1,0)^{-+}$
	2	0	2^{++}
	2	1	$(3,2,1)^{+-}$

4. Flux Tube (& String) Models

Comparison to the lattice



5. Schwinger-Dyson Formalism

C.J. Burden et al, PRC55, 2649 (97).

C.J. Burden & M.A. Pichowsky, Few Body Sys. 32, 119 (02).

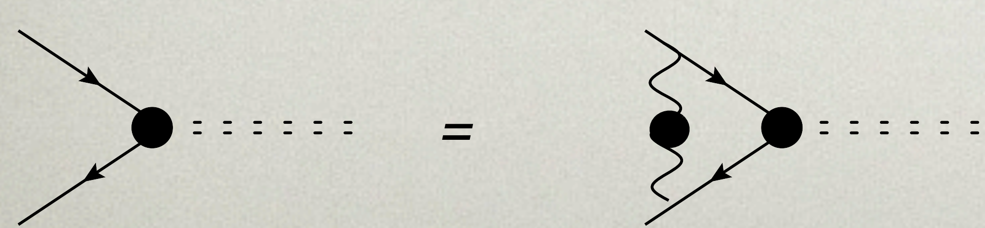
separable Ansatz for the scattering kernel:

$$D_{\mu\nu}(p-q) = \delta_{\mu\nu}[G(p^2)G(q^2) + p\cdot q F(p^2)F(q^2)]$$

$$F \propto A - 1$$

$$G \propto B - m$$

$$S = \frac{i}{A\not{p} + B}$$



J^{PC}	mass (MeV)	state
0^{++}	749	$\sigma(540)$, $a_0(980)$
0^{-+}	1082	ex
0^{--}	1319	ex
1^{--}	730	$\rho(770)$
1^{+-}	1244	$h_1(1170)$
1^{++}	1337	$a_1(1260)$
1^{-+}	1439, 1498	$\pi_1(1400)$, $\pi_1(1600)$

presumably 0+- NOT 0-+

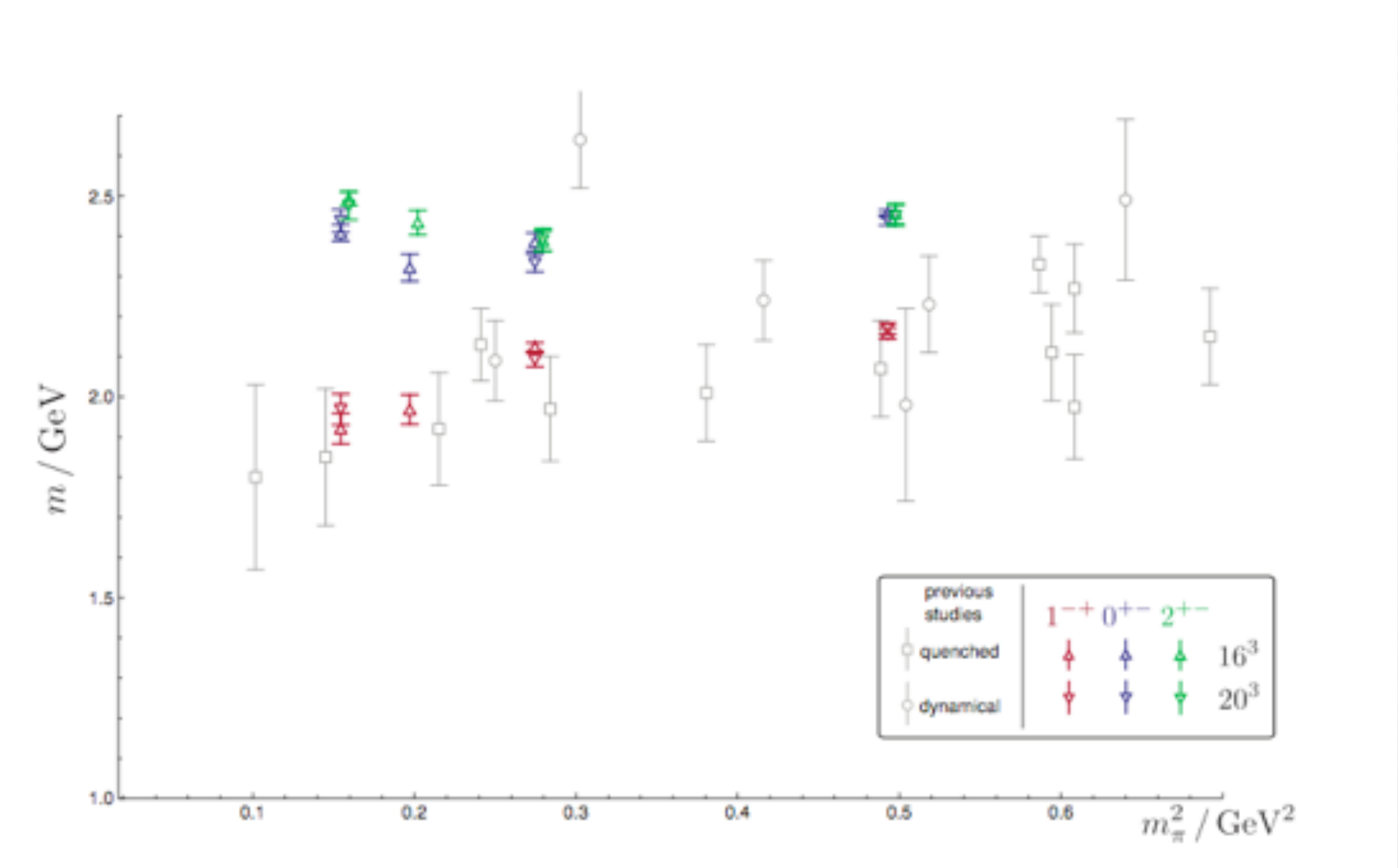
6. Lattice Gauge Theory

JKM, nucl-th/0307116

C. Bernard et al., hep-lat/0301024.

J.J. Dudek et al. PRL103, 262001 (09)

J.J. Dudek et al. 1004.4930

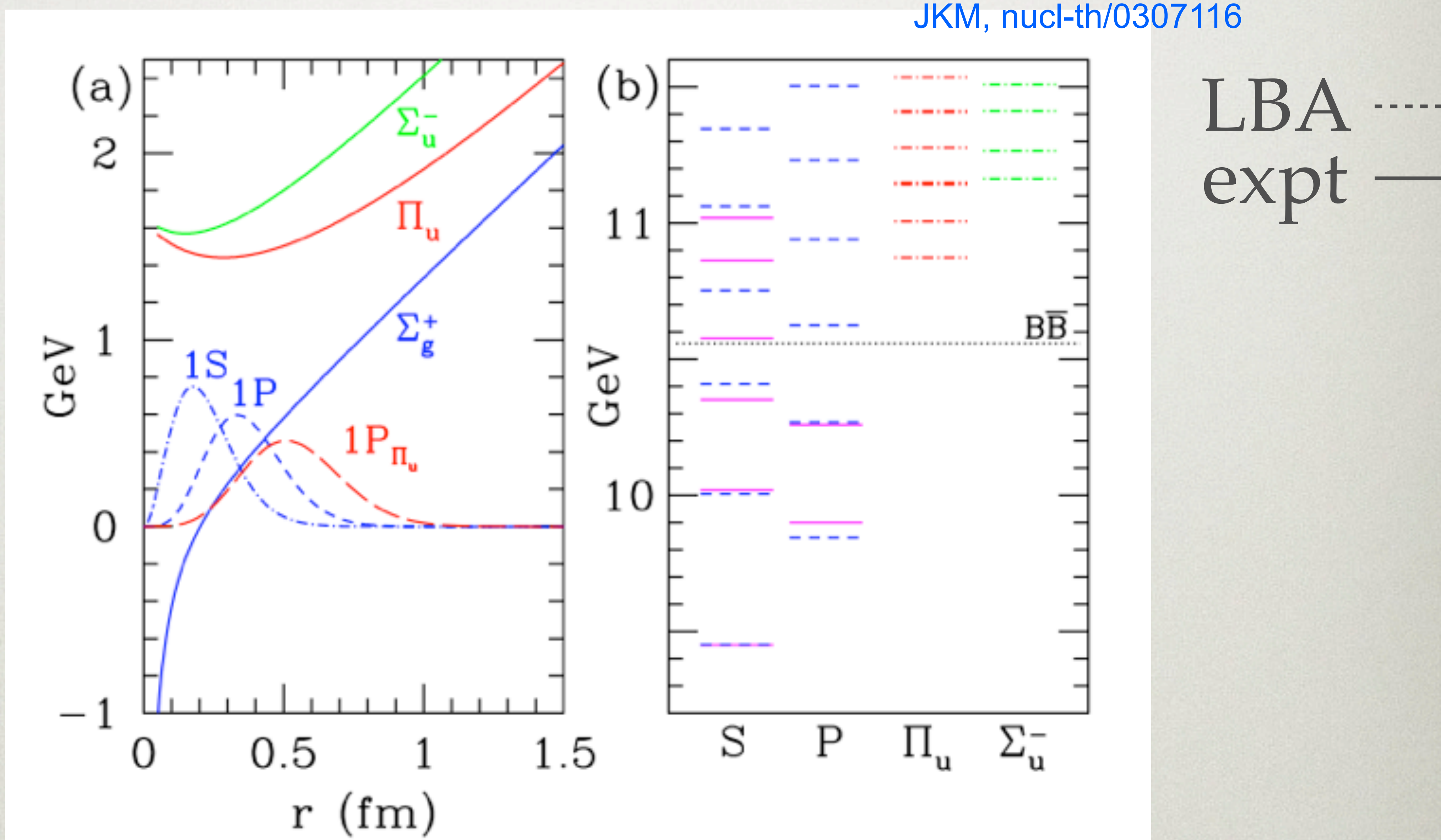


isovector, dynamical lattice, m_u=m_s, m_π=700 MeV

$$M(1^{-+}) \approx 2.17$$

$$M(0^{+-}) \approx M(2^{+-}) \approx 2.51$$

6. Lattice Gauge Theory



$$\Sigma_g^+(S) : 0^{-+}, 1^{--}$$

$$\Sigma_G^+(P) : 0^{++}, 1^{++}, 2^{++}, 1^{+-}$$

$$\Pi_u(P) : 0^{-+}, 0^{+-}, 1^{++}, 1^{--}, 1^{+-}, 1^{-+}, 2^{+-}, 2^{-+}$$

DECAYS

1. IKP Flux Tube Decay Model

quark creation operator

Kokoski & Isgur, PRD35, 907 (87)

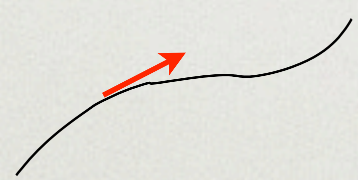
Isgur, Kokoski, & Paton, PRL54, 869 (85)

$$H = \frac{g^2}{2a}\sum_\ell E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi_n^\dagger\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$

$$\Downarrow$$

$$H_{int} \sim \psi_n^\dagger\alpha\cdot\mu\psi_{n+\mu}$$

$$\sim \psi_n^\dagger\alpha\cdot\mu\psi_n + a\psi_n^\dagger\alpha\cdot\mu\,\mu\cdot\nabla\psi_n$$



$\psi_n^\dagger\alpha\cdot\mu\psi_n$

$\psi_n^\dagger\alpha\cdot\nabla\psi_n$

1. IKP Flux Tube Decay Model

Kokoski & Isgur, PRD35, 907 (87)

Isgur, Kokoski, & Paton, PRL54, 869 (85)

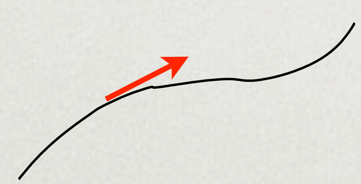
quark creation operator

$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi_n^\dagger \alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$

$$\Downarrow$$

$$H_{int} \sim \psi_n^\dagger \alpha\cdot\mu\psi_{n+\mu}$$

$$\sim \psi_n^\dagger\alpha\cdot\mu\psi_n + a\psi_n^\dagger\alpha\cdot\mu\,\mu\cdot\nabla\psi_n$$



$\psi_n^\dagger\alpha\cdot\mu\psi_n$ $\qquad$ $\psi_n^\dagger\alpha\cdot\nabla\psi_n$

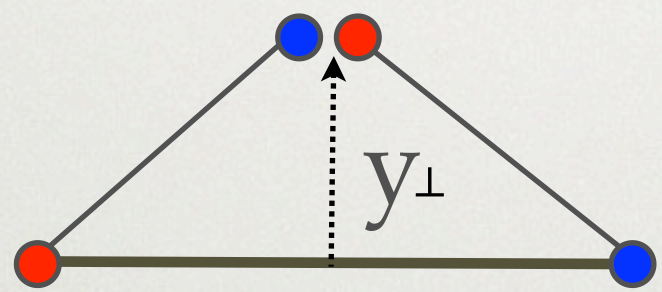
$^3S_1$ $\qquad$ $^3P_0$

1. IKP Flux Tube Decay Model

meson decay

$$\langle \{0\ldots0\}bd; \{0\ldots0\}bd \,|\, O \,|\, \{0\ldots0\}b^\dagger d^\dagger \rangle \sim \langle bd; bd \,|\, {}^3P_0 \,|\, b^\dagger d^\dagger \rangle \cdot$$

$$\langle \{0\ldots0\}; \{0\ldots0\} \,|\, \{0\ldots0\} \rangle$$



$$\downarrow$$

$$e^{-fby_\perp^2}$$

hybrid decay

$$\langle \{0\ldots0\}bd; \{0\ldots0\}bd \,|\, O \,|\, \{1,0\ldots0\}b^\dagger d^\dagger \rangle \sim \langle bd; bd \,|\, {}^3P_0 \,|\, b^\dagger d^\dagger \rangle \cdot$$

$$y_\perp \, e^{-fby_\perp^2} \swarrow \langle \{0\ldots0\}; \{0\ldots0\} \,|\, \{1,0\ldots0\} \rangle$$

2. VECTOR DECAY MODEL

Szczepaniak & Swanson, PRD55, 3987 (97)

Page, Szczepaniak & Swanson, PRD59, 014035 (99)

map chromofields to phonon degrees of freedom

$$E^a_\lambda(n) = \tfrac{\kappa}{a^3}\left(y^a_\lambda(n+1) - y^a_\lambda(n)\right)$$

$$B^a_\lambda(n) = \tfrac{-i}{\kappa a}\frac{\partial}{\partial y^a_\lambda(n)} \qquad\qquad \kappa = a\sqrt{b_0}$$

$$B^a_\lambda(n) = \tfrac{-i}{\kappa}\sqrt{\tfrac{b_0}{r}}\textstyle\sum_m \sin\tfrac{m\pi}{N+1} n\sqrt{\omega_m}\left(\alpha^a_{m\lambda}\mathrm{e}^{-i\omega_m t} - \alpha^{a\dagger}_{m\lambda}\mathrm{e}^{i\omega_m t}\right)$$

2. VECTOR DECAY MODEL

use the same mapping to obtain $\bar{\psi}\alpha \cdot A\psi$

$$H_{int} = \frac{iga^2}{\sqrt{\pi}} \sum_{m,\lambda} \int_0^1 d\xi \cos(\pi\xi) T^a_{ij}\, h^\dagger_i(\xi \mathbf{r}_{Q\bar{Q}}) \sigma \cdot \hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}}_{Q\bar{Q}}) \left(\alpha^a_{m\lambda} - \alpha^{a\dagger}_{m\lambda}\right) \chi_j(\xi \mathbf{r}_{Q\bar{Q}})$$

$$\langle H|H_{int}|AB\rangle = i\frac{ga^2}{\sqrt{\pi}} \frac{2}{3} \int_0^1 d\xi \int d\mathbf{r} \cos(\pi\xi) \sqrt{\frac{2L_H+1}{4\pi}} \mathrm{e}^{\frac{i\mathbf{p}\cdot\mathbf{r}}{2}} \varphi_H(r) \varphi^*_A(\xi\mathbf{r}) \varphi^*_B((1-\xi)\mathbf{r}) \cdot \left[\mathcal{D}^{L_H *}_{M_L \Lambda}(\phi, \theta, -\phi) \chi^{PC}_{\Lambda,\lambda} \hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}}) \cdot \langle\sigma\rangle\right]$$

both models obtain the "S+P" decay selection rule: hybrids cannot decay to two S-wave states with identical spatial wavefunctions.

P.R. Page, PLB401, 313 (97)

3. Lattice Decay

C. McNeile, C. Michael, and P. Pennanen [UKQCD], PRD 65, 094505 (02).

$$1^{-+}(b\bar{b}) \to \eta_b \eta(s\bar{s}) \sim 1 \text{ MeV}$$

$$1^{-+}(b\bar{b}) \to \chi_b \sigma(s\bar{s}) \sim 60 \text{ MeV}$$

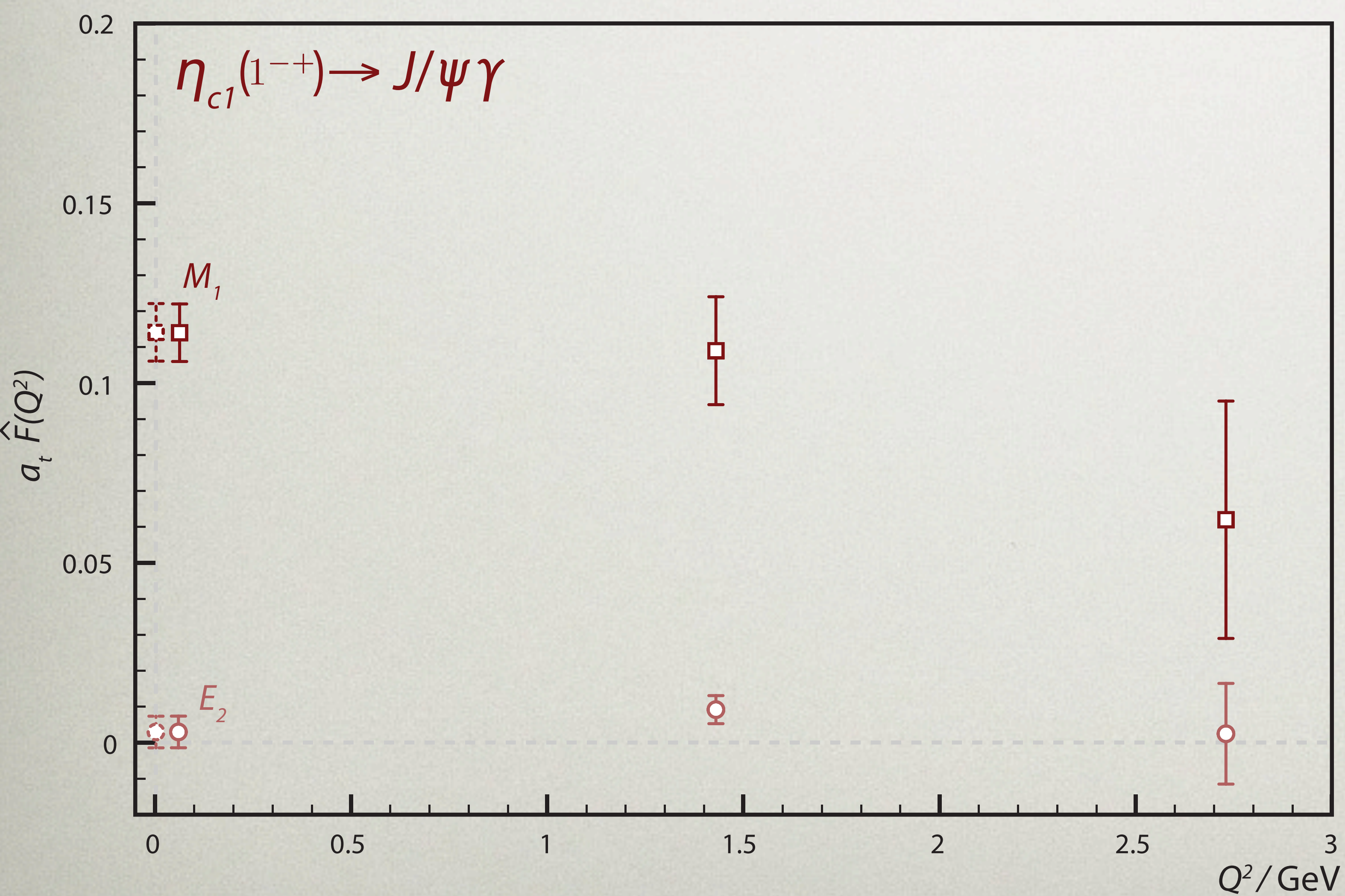
			alt	10.9 GeV hybrid	standard	IKP	reduced
2^{-+}	B^*B	P	.1	0	.5	3	44
1^{-+}	B^*B	P	.1	0	.5	3	44
0^{-+}	B^*B	P	.5	0	2	13	177
1^{--}	B^*B	P	.2	0	1.2	7	88
2^{+-}	B^*B	D	.08	.05	.25	1	22
1^{+-}	B^*B	S	.02	.1	.2	5	13
	B^*B	D	.02	.02	.15	.6	12
1^{++}	B^*B	S	.01	.05	.25	2	7
	B^*B	D	.1	.05	.5	1	24

4. Hybrid Photocoupling

JLab, PRD79, 094504 (09)

$$\Gamma(H(1^{--}) \to \eta_c \gamma) = 42 \pm 18 \text{ keV}$$

$$\Gamma(H(1^{-+}) \to J/\psi \gamma) \approx 100 \text{ keV}$$



this is an M1 decay that is comparable to an E1 decay!

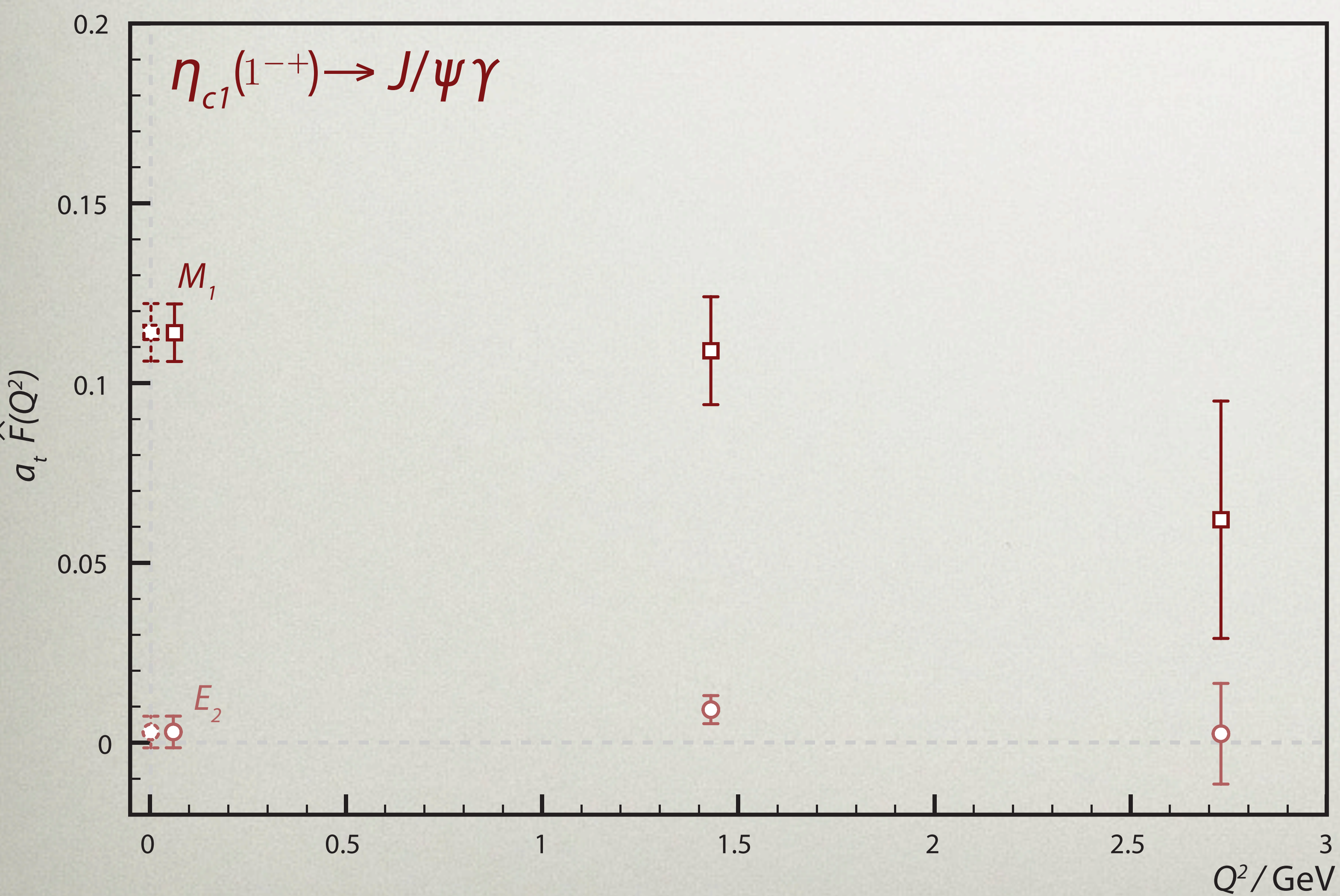
Supports the idea that 1-+ is Sqq = 1 (as in FTM)

4. Hybrid Photocoupling

JLab, PRD79, 094504 (09)

$$\Gamma(H(1^{--}) \to \eta_c\gamma) = 42 \pm 18 \text{ keV}$$

$$\Gamma(H(1^{-+}) \to J/\psi\gamma) \approx 100 \text{ keV}$$



this is an M1 decay that is comparable to an E1 decay!

Supports the idea that 1-+ is Sqq = 1 (as in FTM)

flux tube computation finds similar results (30-60 keV for 1-+)

F. Close and J.J. Dudek, PRL91, 142001 (03)

5. Hybrid-Nonexotic Mixing

T. Burch and D. Toussaint [MILC], PRD68, 094504 (03)

Compute vector meson - vector hybrid meson mixing in NRQCD on the lattice.

This mixing occurs via $\mathcal{O} = g\dfrac{\sigma \cdot B}{2M}$

Obtain
$$\Upsilon(H) \approx 0.4\% \qquad \eta_b(H) \approx 1\%$$
$$J/\psi(H) \approx 2.3\% \qquad \eta_c(H) \approx 6\%$$

Conclusions

- modelling hybrids requires much guesswork. The lattice helps.
- decay selection rule will help identify states (assuming open flavour decays)
- newer lattice results (mass, photocoupling, mixing) support older guesses

✢ ÆRIC MEC HEHT GEWYRCAN