BARYON PARTIAL WIDTHS AND HELICITY AMPLITUDES IN THE FRAMEWORK OF THE 1/Nc EXPANSION

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OUTLINE

- Introduction and general overview of 1/Nc
- Partial decay widths
- Helicity amplitudes: photo-couplings
- Comments and outlook

INTRODUCTION

A large number of years ago the 1/N expansion was invented

soon become an expansion for gauge theories ('tHooft 1974)

...and the great hope for understanding non-perturbative QCD

what happened since?

- Progress from bottom up (quark-gluon level 1/Nc) has been limited: mostly general relations and 1/Nc countings, but few quantitative results
- Luckily, $1/N_c$ expansion can be identified at hadronic level! and built into effective theories
- It could be used as a tool to study QCD phenomenology and lattice QCD results

What could 1/Nc be useful for?

- Provide one extra organizing principle to sort out effects in strong interaction observables
- Provides general relations valid at given order in 1/Nc

How well does it work in the real world with $N_c = 3$?

- No rigorous way to tell
- Hope to learn from applications to pheno and to lattice results
- Case of baryons is particularly interesting and useful

Mesons and glueballs in 1/Nc

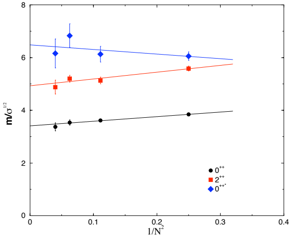
Mesons	Glueballs
$M = \mathcal{O}(N_c^0)$	$M = \mathcal{O}(N_c^0)$
$\Gamma = \mathcal{O}(1/N_c)$	$\Gamma = \mathcal{O}(1/N_c^2)$
$F_M = \mathcal{O}(\sqrt{N_c})$	$F_M = \mathcal{O}(N_c)$
OZI rule: $\Gamma_{OZI} = \mathcal{O}(1/N_c^3)$	OZI rule: $\Gamma_{OZI} = \mathcal{O}(1/N_c^2)$
$\sigma = \mathcal{O}(1/N_c^2)$	G-meson mixing$= \mathcal{O}(1/\sqrt{N_c})$
nonet symmetry	
$M_{\eta'}^2 = \mathcal{O}(N_f/N_c)$ in χ limit	

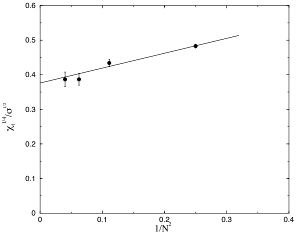
- Phenomenological evidence: multiple excited mesons with widths $\mathcal{O}(100\ \mathrm{MeV})$; issues with scalars
- OZI
- Test of $1/N_c^2$ corrections to $m_G/\sqrt{\sigma}$ in Lattice gluodynamics
- $1/N_c$ suppression of certain LECs in ChPT

- **A few nice tests**

  - Lattice gluedynamics ratios [Lucini & Teper]

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 6.43(50) - 1.5(2.6)/N_c^2 \qquad \frac{\chi_t^{1/4}}{\sqrt{\sigma}} = 0.376(20) + 0.43(10)/N_c^2$$





- Witten-Veneziano formula

$$M^2_{\eta'} = \frac{2\,N_f\;\chi_t}{F_\pi^2}$$
$$\chi_t\big|_{WV} \sim (180MeV)^3$$
$$\chi_t\big|_{latt} \sim (180MeV)^3$$

- χPT: OZI suppression of Gasser-Leutwyler LECs:
$2L_1/L_2 = 1 + \mathcal{O}(1/N_c), \qquad L_4/L_5, \;\; L_6/L_5 = \mathcal{O}(1/N_c)$

L_5	$0.97 \pm 0.11 \times 10^{-3}$
$2L_1/L_2$	1.17 ± 0.38
L_4	~ 0
L_6	~ 0

Baryons in $1/N_c$

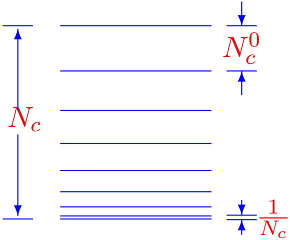
- Need for N_c valence quarks to form a color singlet
- $M_B = \mathcal{O}(N_c)$, $r_B = \mathcal{O}(N_c^0)$
- Hartree picture of baryons sufficient to figure out $1/N_c$ countings
- π-baryon couplings: $\frac{g_A}{F_\pi}\partial_i \pi_a G^{ia}$
- Axial currents G^{ia} have $\mathcal{O}(N_c)$ MEs $\Longrightarrow$ $g_{\pi BB} = \mathcal{O}(\sqrt{N_c})$
- $g_{\pi BB^*} = \mathcal{O}(N_c^0) \Longrightarrow$ $\Gamma_B = \mathcal{O}(N_c^0)$
- Model realizations of all of the above: QM, Skyrme model consistent with $1/N_c$ power counting

Contracted spin-flavor dynamical symmetry

$\pi B \to \pi B$ consistency conditions $\Rightarrow SU(2N_f)$ contracted symm $(N_f = 3)$ [Gervais & Sakita; Dashen & Manohar]

S^i, T^a, $X^{ia} \equiv G^{ia}/N_c$: generators of *contracted* $SU_C(6)$

- $SU(6)$ group with generators S^i, T^a, G^{ia} can be used to build effective theory
- Justifies vintage $SU(6)$ symmetry introduced in 1960's as a dynamical symmetry in large N_c



The O(3)$\times$SU(6) framework

Approximate $O(3)$ symmetry from phenomenology: its breaking small wrt Λ_{QCD} (small spin-orbit effects)

Classify baryons in terms of O(3)$\times$SU(6)

Will become exact $SU_C(6)$ in strict large N_c: $N_c = 3$ too small for this to happen

$N_c = 3$: $[d_{SU(6)}, \ell^P] \to [56, 0^+]$, $[70, 1^-]$, $[56, 2^+]$, etc.

Implementing the 1/Nc expansion: effective operators

- Effective operators at baryon level

$$\hat{O}_{\rm QCD} \quad \Rightarrow \quad \hat{O}_{\rm eff} = \sum \left(\frac{1}{N_c}\right)^{\nu(j)} C_j \, \hat{O}_j$$

$$\nu(j) = n_j - 1 \quad \text{for} \quad n_j - \text{body} \quad \text{operator}$$

C_j effective constants or form factors: encode relevant QCD dynamics
Operator basis $\{\hat{O}_j\}$can be ordered in powers of $1/N_c$

- Matrix elements

$$\langle B'_{GS} \mid \hat{O}_j \mid B_{GS}\rangle = \mathcal{O}(N_c^{\kappa(j)}) \; ; \quad \langle B_{GS} \mid \hat{O}_j \mid B^*\rangle = \mathcal{O}(N_c^{-\frac{1}{2}+\kappa(j)})$$

$\kappa(j)$ coherence factor of effective operator

- General form of effective operators

$$\hat{O}_j = \xi_j \otimes \mathcal{G}_j$$

ξ_j: $O(3)$ tensor operator

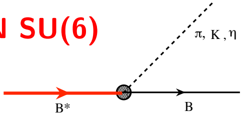
$\mathcal{G}_j$: spin-flavor tensor operator made out of product of n_j generators of $SU(6)$

$\kappa(\hat{O}_j) = \#G_{ia} \quad (a = 1, 2, 3)$ or T^8 generators factors in $\mathcal{G}_j$

- For any observable, systematic way of building bases of effective $\mathcal{G}_j$ operators ordered in $1/N_c$; numerous "reduction rules"

- In general, # operators < # observables at NLO: parameter free relations exact up to NNLO corrections

PARTIAL DECAY WIDTHS IN SU(6)



• Strong partial wave amplitudes

$$\mathcal{M}(\ell_\pi(Y_\pi, I_\pi), B, B^*) = (-1)^{\ell_\pi} \sqrt{2M_{B^*}} \frac{\sqrt{N_c}}{F_\pi} \langle B_{GS} \mid \mathbf{B}^{\ell_\pi(Y_\pi, I_\pi)} \mid B^* \rangle$$

$\mathbf{B}^{\ell_\pi,(Y_\pi,I_\pi)}$ baryon operator carrying the meson quantum numbers

• 1/Nc expansion

Expand $\mathbf{B}$ in effective operators

$$\mathbf{B}^{\ell_\pi,(Y_\pi,I_\pi)} = \left(\frac{k_\pi}{\Lambda}\right)^{\ell_\pi} \sum_n C_n \, O_n^{\ell_\pi(Y_\pi,I_\pi)}$$

$$O_n^{\ell_\pi(Y_\pi,I_\pi)} = \left[\xi^\ell \, \mathcal{G}_n^{j_n(Y_\pi,I_\pi)}\right]^{\ell_\pi(Y_\pi,I_\pi)}$$

C_n effective coefficients to be fitted, $\mathcal{G}_n^{j_n I_\pi}$ spin-flavor tensor operators
ξ^ℓ $O(3)$ tensor, ℓ $O(3)$ quantum number of excited baryon

$[70,1^-]$ Partial Widths

[SU(4): JLG, Schat & Scoccola; SU(6): Jayalath, JLG & Scoccola]

70-plet: 30 isospin multiplets; 18 known with *** and ****

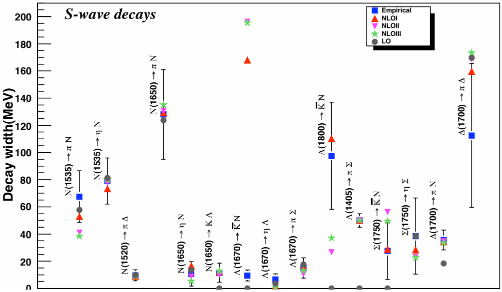
State	Mass PDG	[MeV] $1/N_c$
$N_{1/2}$	1538 ± 18	1541
$\Lambda_{1/2}$	1670 ± 10	1667
$\Sigma_{1/2}$	(1620)	1637
$\Xi_{1/2}$	–	1779
$N_{3/2}$	1523 ± 8	1532
$\Lambda_{3/2}$	1690 ± 5	1676
$\Sigma_{3/2}$	1675 ± 10	1667
$\Xi_{3/2}$	1823 ± 5	1815
$N'_{1/2}$	1660 ± 20	1660
$\Lambda'_{1/2}$	1785 ± 65	1806
$\Sigma'_{1/2}$	1765 ± 35	1755
$\Xi'_{1/2}$	–	1927
$N'_{3/2}$	1700 ± 50	1699
$\Lambda'_{3/2}$	–	1864
$\Sigma'_{3/2}$	–	1769
$\Xi'_{3/2}$	–	1980

State	Mass PDG	[MeV] $1/N_c$
$N_{5/2}$	1678 ± 8	1671
$\Lambda_{5/2}$	1820 ± 10	1836
$\Sigma_{5/2}$	1775 ± 5	1784
$\Xi_{5/2}$	–	1974
$\Delta_{1/2}$	1645 ± 30	1645
$\Sigma''_{1/2}$	–	1784
$\Xi''_{1/2}$	–	1922
$\Omega_{1/2}$	–	2061
$\Delta_{3/2}$	1720 ± 50	1720
$\Sigma''_{3/2}$	–	1847
$\Xi''_{3/2}$	–	1973
$\Omega_{3/2}$	–	2100
$\Lambda''_{1/2}$	1407 ± 4	1407
$\Lambda''_{3/2}$	1520 ± 1	1520

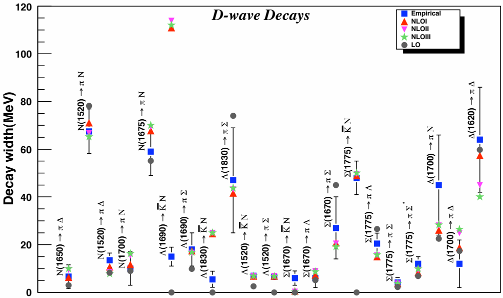
- **Operator basis (restricted to 2-body)**

Operator	n-bodyness	$1/N_c$ order
$\mathcal{O}_0 = (\xi g)^{[\ell,8]}$	1	0
$\mathcal{O}_1 = \frac{1}{N_c}\left(\xi\,(sT^c)^{[1,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_2 = \frac{1}{N_c}\left(\xi\,(tS^c)^{[1,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_3 = \frac{1}{N_c}\left(\xi\,(sG^c)^{[1,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_4 = \frac{1}{N_c}\left(\xi\,(gS^c)^{[1,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_5 = \frac{1}{N_c}\left(\xi\left(f^{abd}g_b T^c_d\right)^{[1,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_6 = \frac{1}{N_c}\left(\xi\left(d^{abd}g_b T^c_d\right)^{[1,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_7 = \frac{1}{N_c}\left(\xi\,(sG^c)^{[2,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_8 = \frac{1}{N_c}\left(\xi\,(gS^c)^{[2,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}_9 = \frac{1}{N_c}\left(\xi\left(f^{abd}g_b G^c_d\right)^{[2,8]}\right)^{[\ell,8]}$	2	1
$\mathcal{O}^{SB}_1 = \left(\xi\left(d^{8ab} - \frac{\delta^{ab}}{\sqrt{3}}\right) g\right)^{[\ell,8]}$	1	0
$\mathcal{O}^{SB}_2 = \left(\xi f^{8ab} g\right)^{[\ell,8]}$	1	0

• Fits to NLO



S-wave decays
Decay width(MeV)
Empirical
NLOI
NLOII
NLOIII
LO



D-wave Decays
Decay width(MeV)
Empirical
NLOI
NLOII
NLOIII
LO

	LO	NLO		
		Fit 1	Fit 2	Fit 3
χ^2_{dof}	1.16	1.05	1.38	1.84
dof	13	11	17	25
C_{S1}	13.75(1.22)	11.26(0.42)	12.66(0.40)	13.10(0.40)
C_{S2}	...	7.28(2.06)	11.86(2.09)	11.56(2.07)
C_{S3}	...	-9.48(1.96)	-8.41(2.27)	-6.10(2.35)
C_{S4}	...	-13.06(2.39)	-15.09(2.36)	-13.25(2.37)
C_{S5}	...	7.32(1.42)	2.17(1.51)	1.03(1.48)
C_{S6}	...	7.22(2.45)	-3.46(2.15)	-7.95(2.09)
B_{S1}	-16.64(5.30)	-11.64(3.08)	-5.50(3.13)	-5.85(3.03)
C_{D1}	1.94(0.12)	1.95(0.06)	2.07(0.05)	2.09(0.05)
C_{D2}	...	-1.67(0.20)	-1.94(0.19)	-2.03(0.19)
C_{D3}	...	0.95(0.48)	1.34(0.46)	1.45(0.46)
C_{D5}	...	2.04(0.27)	2.06(0.26)	2.04(0.26)
C_{D6}	...	-1.36(0.23)	-1.89(0.22)	-2.02(0.22)
C_{D8}	1.25(0.14)	1.22(0.09)	0.94(0.08)	0.85(0.08)
C_{D9}	...	-0.16(0.45)	-0.60(0.37)	-0.78(0.36)
B_{D1}	...	-2.71(0.23)	-2.69(0.23)	-2.69(0.23)
θ_{N_1}	0.48(0.07)	0.48(0.06)	0.63(0.24)	0.68(0.19)
θ_{N_3}	2.76(0.04)	2.81(0.04)	3.01(0.87)	3.07(0.94)
θ_{Λ_1}	0.81(0.10)	0.68(0.07)	0.52(0.35)	0.58(0.29)
θ_{Λ_3}	2.56(0.49)	2.57(1.53)	3.03(0.89)	3.08(0.95)
θ_{Σ_1}	0.82(1.12)	0.95(0.13)	0.76(0.12)	0.79(0.09)
θ_{Σ_3}	2.55(0.77)	3.00(0.38)	2.97(0.84)	3.05(0.93)

• Parameter free relations: LO (NLO only at amplitude level)

- S-wave relations

$$\frac{\tilde\Gamma(N(1535)\to N\pi)-\tilde\Gamma(N(1650)\to N\pi)}{\tilde\Gamma(N(1535)\to N\pi)+\tilde\Gamma(N(1650)\to N\pi)} = \frac{1}{5}(3\cos 2\theta_{N_1}-4\sin 2\theta_{N_1}) \to \theta_{N_1}=0.46(10)\ or\ 1.76(10)$$

$$\frac{\tilde\Gamma(N(1535)\to N\eta)-\tilde\Gamma(N(1650)\to N\eta)}{\tilde\Gamma(N(1535)\to N\eta)+\tilde\Gamma(N(1650)\to N\eta)} = \sin 2\theta_{N_1} \to \theta_{N_1}=0.51(27)$$

$$\tilde\Gamma(N(1535)\to N\pi)+\tilde\Gamma(N(1650)\to N\pi) = \tilde\Gamma(\Delta(1535)\to\Delta\pi)$$

$$51(10)\quad vs\quad 31(15)$$

$$\frac{\tilde\Gamma(\Delta(1620)\to N\pi)}{\tilde\Gamma(\Delta(1700)\to \Delta\pi)} = 0.1\ (th)\quad vs\quad 0.29(15)\ (exp)$$

$\cdots$ many others which give predictions

- D-wave relations

$$2\tilde\Gamma(\Delta(1620)\to\Delta\pi)+\tilde\Gamma(\Delta(1700)\to\Delta\pi) = 15\tilde\Gamma(\Delta(1620)\to N\pi)+32\tilde\Gamma(\Delta(1700)\to N\pi)$$

$$5.9(1.9)\quad vs\quad 8.3(2.3)$$

$$\tilde\Gamma(N(1535)\to\Delta\pi)+\tilde\Gamma(N(1650)\to\Delta\pi)+11\tilde\Gamma(\Delta(1620)\to\Delta\pi) = 132\tilde\Gamma(\Delta(1700)\to N\pi)+90\tilde\Gamma(N(1675)\to N\pi)$$

$$32(11)\quad vs\quad 41(10)$$

$\cdots$ four more testable relations, all $\sim$ consistent!

$$\frac{\tilde\Gamma(N(1520)\to N\pi)-\tilde\Gamma(N(1700)\to N\pi)}{\tilde\Gamma(N(1520)\to N\pi)+\tilde\Gamma(N(1700)\to N\pi)} = \frac{1}{41}(39\cos 2\theta_{N_3}+4\sin 2\theta_{N_3}) \to \theta_{N_3}=0.38(22)\ or\ 3.08(22)$$

Comparison with quark models

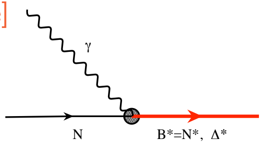
State		Mode	$\Gamma(MeV)$ Exp	KI	FKR	HLC	1/N$_c$	Comment
$N(1535)$	S_{11}	πN	67.5 ± 19	28	220	28	53	
		ηN	79 ± 17	27	71	10	73.5	
$N(1650)$	S_{11}	πN	128 ± 33	76	-	82	129	
		ηN	11 ± 9	2		2	17	
		$K\Lambda$	11.6 ± 7	9	-	5	12	
		$\pi\Delta$	7 ± 5	67	-	37	6	D-wave
$N(1520)$	D_{13}	πN	67.5 ± 9	85	105	67	71	
		$\pi\Delta$	13.5 ± 3	6	-	5	11	
		$\pi\Delta$	9.6 ± 4	45	-	19	9	S-wave
$N(1700)$	D_{13}	πN	10 ± 7	13	45	26	12	
$N(1675)$	D_{15}	πN	59 ± 10	30	36	28	68	
$\Delta(1700)$	D_{33}	πN	45 ± 21	24	30	28	26	
		$\pi\Delta$	12 ± 10	40	-	36	19	
		$\pi\Delta$	112.5 ± 53	106	-	71	160	S-wave
$\Delta(1620)$	S_{31}	πN	36 ± 7	11	25	11	34.5	
		$\pi\Delta$	64 ± 22	-	-	28	57	D-wave
$\Lambda(1405)$	S_{01}	$\bar{K}N$	50 ± 2	55	56	-	50	
$\Lambda(1520)$	D_{03}	$\bar{K}N$	7 ± 0.5	9	7	6	7	
		$\pi\Sigma$	6.6 ± 0.5	8	12	5	7	
$\Lambda(1670)$	S_{01}	$\bar{K}N$	9 ± 4	11	415	5	168	
		$\eta\Lambda$	6.6 ± 4	5	6	5	2	
		$\pi\Sigma$	15 ± 7.5	10	22	12	16	
$\Lambda(1800)$	S_{01}	$\bar{K}N$	97.5 ± 39.5	8	-	-	110	
$\Lambda(1690)$	D_{03}	$\bar{K}N$	15 ± 4	18.5	102	17	111	
		$\pi\Sigma$	18 ± 7	44	11	31	17	
$\Lambda(1830)$	D_{05}	$\bar{K}N$	5.5 ± 3	2	-	-	25	
		$\pi\Sigma$	47 ± 22	59	73	51	42	
$\Sigma(1750)$	S_{11}	$\bar{K}N$	27.5 ± 21	17	14	26	29	
		$\eta\Sigma$	38.5 ± 28	-	4	2	28.5	
$\Sigma(1670)$	D_{13}	$\bar{K}N$	6 ± 3	4	3	2	1	
		$\pi\Lambda$	6 ± 4	6	6	1	7	
		$\pi\Sigma$	27 ± 13	44	49	31	21	
$\Sigma(1775)$	D_{15}	$\bar{K}N$	48 ± 7	45	66	42	49	
		$\pi\Lambda$	20 ± 4	22	25	19	15	
		$\pi\Sigma$	4 ± 2	9	10	7	4	
		$\pi\Sigma^*$	12 ± 3	8	6	4	8	

$[70, 1^-]$ decays summary

- Accurate LO fit to π S-wave: only one parameter
- π D-wave requires NLO operators
- NLO coefficients of natural to smaller than natural size
- Mixings play central role: e.g., enhancement of $N^*(1535) \to \eta N$, suppression of $N^*(1650) \to \eta N$, etc
- Mixing angles from partial widths $\sim$ consistent with mixings from masses
- Two channels cannot be described: $\Lambda(1670) \to \bar{K}N$ and $\Lambda(1690) \to \bar{K}N$. Both states very close to $\eta\Lambda$ threshold: coupled channel. Not enough $SU(3)$ breaking operators to describe suppression
- $SU(3)$ breaking necessary but too few channels affected by it: makes predictions, e.g. Ξ decays unreliable

PHOTOPRODUCTION HELICITY AMPLITUDES

[JLG, Schat & Scoccola; Scoccola, JLG & Matagne]



- **Helicity amplitudes (PDG)**

$$\mathcal{A}_\lambda = -\sqrt{\frac{2\pi\alpha}{\omega}}\,\eta(B^*)\,\langle B^*, \lambda \mid \epsilon_+ \cdot \mathbf{J}_{EM} \mid N, \lambda - 1\rangle$$

$\eta(B^*)$ sign of strong transition reduced amplitude: determined by $1/N_c$ analyses of strong decays

- **$1/N_c$ expansion**

Represent EM current operator in terms of effective operators

$$\begin{aligned} \mathbf{J}_{EM} &= \sqrt{N_c}\sum_n C_n \left(k_\gamma^{[L']} O_n^{[L,I]}\right)^{[1,I]} \\ O_n^{[L,I]} &= \left(\xi^\ell \mathcal{G}_n^{(\ell_n,I)}\right)^{(L,I)} \end{aligned}$$

$$\begin{aligned}\mathcal{A}_{\lambda}^{X^{[LI]}}(I_3, J^* I^*) &= \frac{(-1)^{J^*+I^*+I+1}\, w_X(L)\, \eta(B^*)}{\sqrt{(2J^*+1)(2I^*+1)}} \\ &\times \sqrt{\frac{3\alpha N_c}{4\omega}}\, \langle L, 1; \frac{1}{2}, \lambda - 1 \mid J^*, \lambda\rangle \langle I, 0; \frac{1}{2}, I_3 \mid I^*, I_3\rangle \\ &\times \sum_n C_n(\omega)\, \langle J^* I^* \parallel O_n^{[LI]} \parallel \frac{1}{2}\rangle\end{aligned}$$

$X = E$ or M, $w_M(L) = 1)$, $w_E(L) = \sqrt{(L+1)/(2L+1)}$

Sign of strong amplitudes from our strong decay analysis

$$\eta(B^*) = (-1)^{J^* - \frac{1}{2}}\, \mathrm{sign}(\langle \ell_\pi\ N \parallel H_{\mathrm{QCD}} \parallel J^*\ I^* \rangle)$$

• $[70,1^-]$ Helicity Amplitudes [Scoccola, JLG & Matagne]

Operator	Order
$E1_1^{(0)} = (\xi s)^{[1,0]}$	1
$E1_2^{(0)} = \frac{1}{N_c}\left(\xi\,(s\;S_c)^{[0,0]}\right)^{[1,0]}$	$\frac{1}{N_c}$
$E1_3^{(0)} = \frac{1}{N_c}\left(\xi\,(s\;S_c)^{[1,0]}\right)^{[1,0]}$	$\frac{1}{N_c}$
$E1_4^{(0)} = \frac{1}{N_c}\left(\xi\,(s\;S_c)^{[2,0]}\right)^{[1,0]}$	$\frac{1}{N_c}$
$E1_1^{(1)} = (\xi t)^{[1,1]}$	1
$E1_2^{(1)} = (\xi g)^{[1,1]}$	1
$E1_3^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;G_c)^{[2,1]}\right)^{[1,1]}$	1
$E1_4^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;T_c)^{[1,1]}\right)^{[1,1]}$	$\frac{1}{N_c}$
$E1_5^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;G_c)^{[0,1]}\right)^{[1,1]} + \frac{1}{4\sqrt{3}}\;E1_1^{(1)}$	$\frac{1}{N_c}$
$E1_6^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;G_c)^{[1,1]}\right)^{[1,1]} + \frac{1}{2\sqrt{2}}\;E1_2^{(1)}$	$\frac{1}{N_c}$
$M2_1^{(0)} = (\xi s)^{[2,0]}$	1
$M2_2^{(0)} = \frac{1}{N_c}\left(\xi\,(s\;S_c)^{[1,0]}\right)^{[2,0]}$	$\frac{1}{N_c}$
$M2_3^{(0)} = \frac{1}{N_c}\left(\xi\,(s\;S_c)^{[2,0]}\right)^{[2,0]}$	$\frac{1}{N_c}$
$M2_1^{(1)} = (\xi g)^{[2,1]}$	1
$M2_2^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;G_c)^{[2,1]}\right)^{[2,1]}$	1
$M2_3^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;T_c)^{[1,1]}\right)^{[2,1]}$	$\frac{1}{N_c}$
$M2_4^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;G_c)^{[1,1]}\right)^{[2,1]} + \frac{1}{2\sqrt{2}}\;M2_1^{(1)}$	$\frac{1}{N_c}$
$E3_1^{(0)} = \frac{1}{N_c}\left(\xi\,(s\;S_c)^{[2,0]}\right)^{[3,0]}$	$\frac{1}{N_c}$
$E3_1^{(1)} = \frac{1}{N_c}\left(\xi\,(s\;G_c)^{[2,1]}\right)^{[3,1]}$	1

Amplitude	PDG	$10^{-3}/\sqrt{GeV}$ LO	NLO*
$A^p_{1/2}[N(1535)]$	90 ± 30	$76(0.2)$	$111(0.5)$
$A^n_{1/2}[N(1535)]$	-46 ± 27	$-54(0.1)$	$-78(1.4)$
$A^p_{1/2}[N(1520)]$	-24 ± 9	$-25(0.0)$	$-20(0.2)$
$A^n_{1/2}[N(1520)]$	-59 ± 9	$-6(8.8)$	$-46(1.9)$
$A^p_{3/2}[N(1520)]$	166 ± 5	$66(4.0)$	163(0.4)
$A^n_{3/2}[N(1520)]$	-139 ± 11	$-55(4.0)$	$-143(0.1)$
$A^p_{1/2}[N(1650)]$	53 ± 16	45(0.3)	52(0.0)
$A^n_{1/2}[N(1650)]$	-15 ± 21	$-12(0.0)$	$-20(0.1)$
$A^p_{1/2}[N(1700)]$	-18 ± 13	$-18(0.0)$	$-20(0.0)$
$A^n_{1/2}[N(1700)]$	0 ± 50	$41(0.7)$	$47(0.9)$
$A^p_{3/2}[N(1700)]$	-2 ± 24	$1(0.0)$	$-10(0.1)$
$A^n_{3/2}[N(1700)]$	-3 ± 44	47(1.3)	47(1.3)
$A^p_{1/2}[N(1675)]$	19 ± 8	15 (0.3)	8(2.0)
$A^n_{1/2}[N(1675)]$	-43 ± 12	$-45(0.0)$	$-50(0.4)$
$A^p_{3/2}[N(1675)]$	15 ± 9	$10(0.3)$	$11(0.2)$
$A^n_{3/2}[N(1675)]$	-58 ± 13	$-53(0.1)$	$-71(1.0)$
$A^N_{1/2}[\Delta(1620)]$	27 ± 11	$53(5.7)$	32(0.2)
$A^N_{1/2}[\Delta(1700)]$	104 ± 15	80(0.6)	108(0.1)
$A^N_{3/2}[\Delta(1700)]$	85 ± 22	70(0.3)	112(1.5)

Summary of $[70, 1^-]$ Helicity Amplitudes

- LO OK, but misses $n_{1/2,3/2}(1520)$, $p_{3/2}(1520)$, $\Delta_{1/2}(1620)$
- LO results similar to "single quark transition model "
- NLO: LO coefficients remain stable. Naturalness not violated at NLO. One NLO 2-body $E1$ operator sufficient to have consistency
- Both $E1$ and $M2$ multipoles crucial and of similar magnitude
- E_3 multipoles marginal
- LO fit lifts ambiguity in θ_3: 2.8 ± 0.12, and fixes relative signs between S and D wave strong amplitudes
- Example where QM fails: $\Delta_{1/2}(1620)$ too big: NLO operator takes care of problem
- For a more accurate picture of NLO effects empirical helicity amplitudes need improvement to 10% level

COMMENTS & OUTLOOK

- Partial widths and helicity amplitudes have been also analyzed for 56-plets
- Recent work on electroproduction [Lebed & Yu]
- Important open issues: i) Incomplete SU(6) multiplets; ii) Incomplete SU(3) multiplets. Key for usage of SU(3) approximate symmetry and $1/N_c$ expansion
- Phenomenology of excited baryons indicates consistency with a $1/N_c$ expansion
- QCD dynamics simplifies the picture: LO spin-flavor symmetry breaking and numerous $1/N_c$ corrections are dynamically suppressed
- Mass predictions for missing states in 56- and 70-plets: e.g. Ξ's. Hope for learning more from lattice baryon spectrum
- Theoretical challenges: consistently treat the finite width of excited baryons; understand the physics behind the effective coefficients of the expansion, which contain both short and long distance dynamics
- Can $1/N_c$ be a useful tool in combination with coupled-channel models?