Quark model calculations of N to Roper resonance EM transition amplitudes

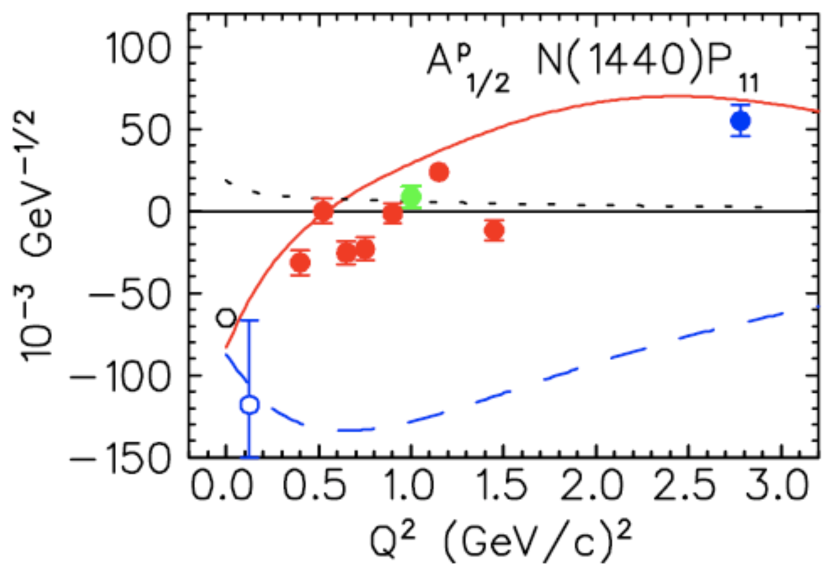
EM transition form factors

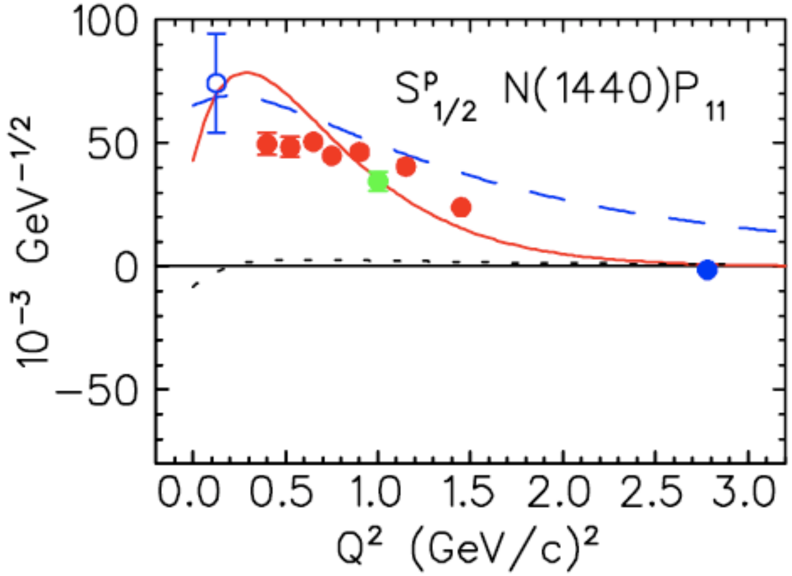
- Rigorous approaches underway:
  - Schwinger-Dyson Bethe-Salpeter studies
  - Lattice QCD
- Relativistic quark-model calculations
  - Most reliable use light-front dynamics to improve one-body current
    - Terent'ev, Weber, Dziembowski, Chung & Coester, Schlumpf, Aznauryan, Rome group, Julia-Diaz, Riska and Coester, Miller
  - Relativistic effects are large
    - Need to remove interaction dependence of boosts
    - Minimize effect of ignored two-body currents
  - Can also use point, instant forms

Giannini and Santopinto-2004

- Tiator et al. EPJ A (2004) **19**, s01, 55
- NR model using hyper-central CQM
  - $V(x) = -\tau/x + \alpha\, x, \;\; x = (\rho^2+\lambda^2)^{\frac{1}{2}}$
  - Hyperfine interaction, isospin-dependent terms
  - Fit τ & α and hyperfine strength to spectrum, use wave functions in a non-relativistic calculation of the EM transition form factors

Giannini and Santopinto-2004





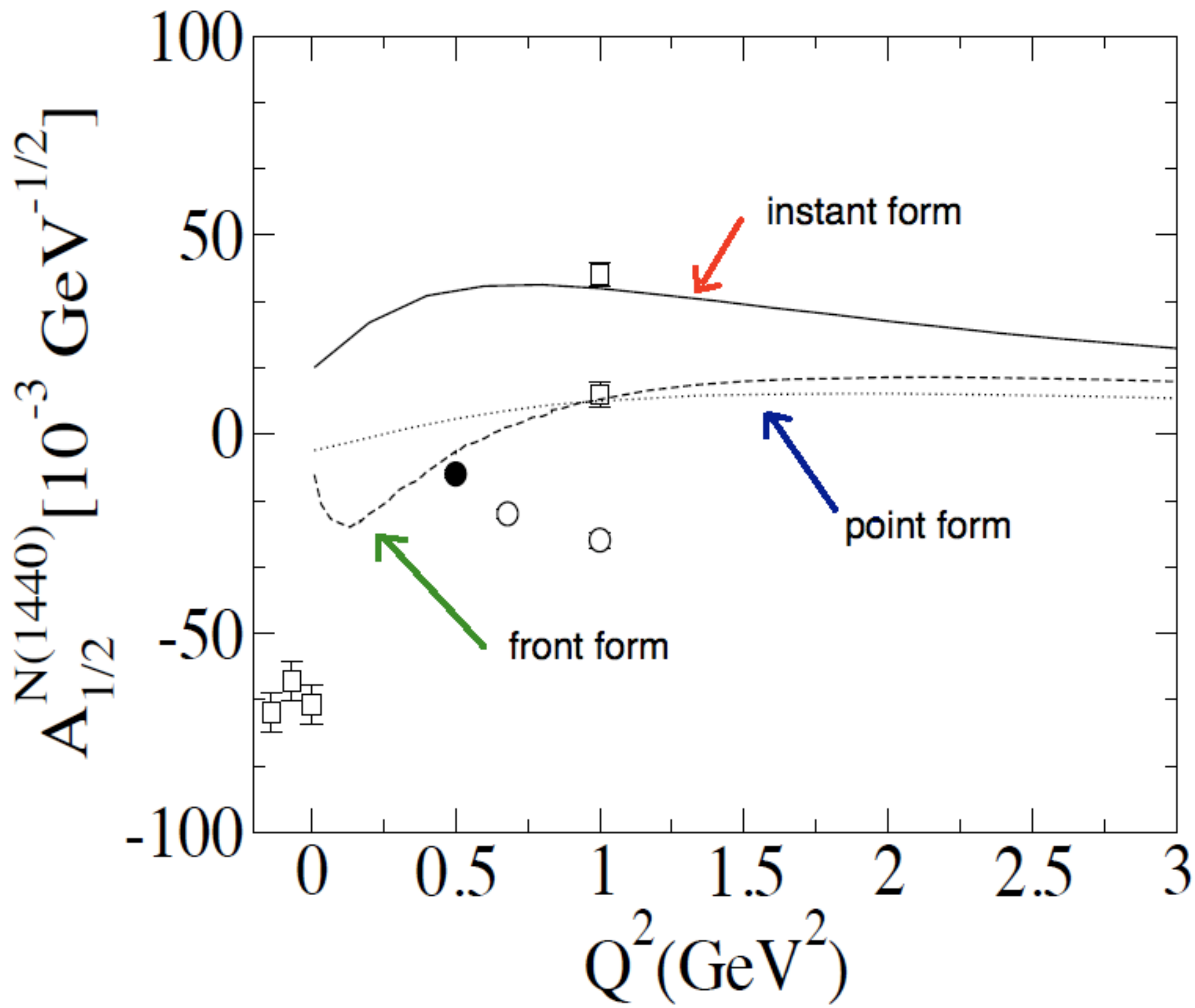
MAID fit, π cloud, model

- No calculation of Nπ sign
- Note $A^p_{\frac{1}{2}}$ does not cross zero

Julia-Diaz, Riska and Coester-2004

- Julia-Diaz, Riska and Coester, PRC **69** (035212) 2004
  - use simple wave functions depending on hyperspherical momentum $\mathbf{P}^2 \sim \mathbf{p}_\rho{}^2+\mathbf{p}_\lambda{}^2$ (perm. symmetric and Lorentz invariant)
    - Nucleon $\phi_0(P) = N\,(1+P^2/4b^2)^{-a}$,
    - b and a: range and shape parameters
    - Roper $\phi_1(P)$ orthogonal, normalized, FT has a single node
  - Use point form, front form & instant form of relativistic kinematics to evaluate vector EM current
    - change a and b to fit nucleon elastic form factors separately for each form

Julia-Diaz, Riska and Coester-2004



Inna Aznauryan-2007

- I.G. Aznauryan, PRC **76** (025212) 2007
  - Light-front relativistic quark model
    - Wave functions depend on the sum of the totally-symmetric invariant mass squared of the quarks, M_0^2 (expressed in light-cone coordinates

      $p^\mu = [p^+ = p_0 + p_3,\ p^- = (m^2 + \mathbf{p}_T^2)/p^+,\ \mathbf{p}_T];\ p^2 = p^+ p^- - \mathbf{p}_T^2$

    - 3 denotes (spin) quantization axis, $\mathbf{p}_T = (p_1, p_2)$
    - Distribution of invariant momentum fractions $x_i = p_i^+/P^+$ ($P = \Sigma_i\, p_i$ for + and transverse components) can be measured in high-energy DIS and elastic scattering

Inna Aznauryan-2007

- Relative four-momenta for three-body system (see Weber)

  $k = (x_2\, p_1 - x_1\, p_2)/(x_1+x_2)$

  $K = (x_1+x_2)\, p_3 - x_3(p_1+p_2)$

  - Space-like, since $k^+ = K^+ = 0$ so $k^2 = -\mathbf{k}_T^2$, $K^2 = -\mathbf{K}_T^2$
  - In static limit ($|\mathbf{p}| \ll m$), $x_i \to m_q/m_N \sim 1/3$
    - $k \sim \mathbf{p}_\rho$, $K \sim \mathbf{p}_\lambda$ (usual three-body Jacobi coordinates)
  - Volume element in momentum space is 6D

$$d\Gamma = (dx_1/x_1)\,(dx_2/x_2)\,(dx_3/x_3)\,\delta(1-x_1-x_2-x_3)\,d\mathbf{k}_T^2\,d\mathbf{K}_T^2\,/\,(16\pi^3)^2$$

Inna Aznauryan-2007

- Nucleon and Roper wave functions:

  $\phi_N(M_0^2) \sim \exp(-M_0^2/6\alpha_{HO}^2)$

  $\phi_R(M_0^2) = N\,(\beta^2 - M_0^2)\phi_N(M_0^2)$

- Depend on totally symmetric invariant mass squared of three-body system

  $M_0^2 = -k^2(1-x_3)/(x_1x_2) - K^2/[x_3(1-x_3)] + \sum_i m_q^2/x_i$

- Normalized and orthogonal over six-dimensional phase-space volume
  - Parameters are quark mass (0.22 GeV) and α_{HO} = 0.38 GeV (fit to nucleon static properties)

Inna Aznauryan-2007

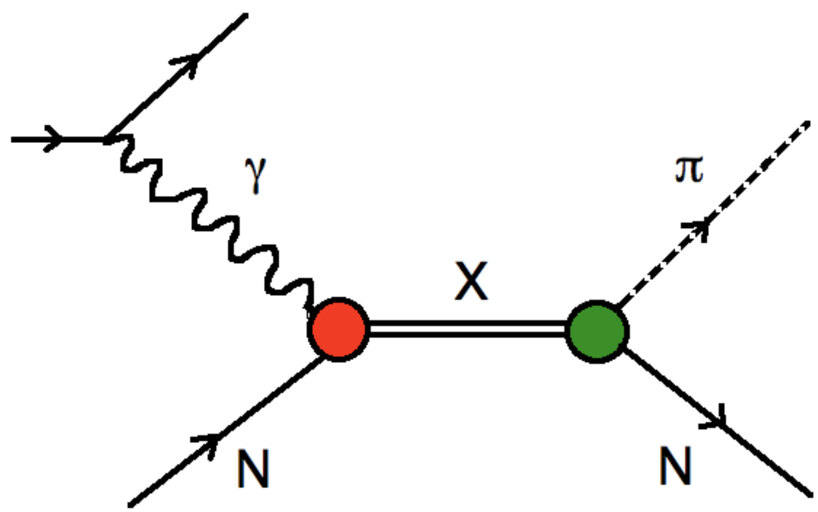
- Resulting form factors are integrals over six-dimensional phase space of:

  (kinematic factors) $\phi_N(M_0^2)\ \phi_R(M_0'^2)\ d\Gamma$

  - Signs of $N\pi$ decay amplitudes found using PCAC argument

Electro/photo-production amplitude signs

- Experiments measure interference of products of amplitudes $A^{\dagger}_{X\text{-}\gamma N}\, A_{X\text{-}N\pi}$ with nucleon Born term and/or each other
- Phase of either depends on sign conventions in N and X wave fns
- Phase of product does not!

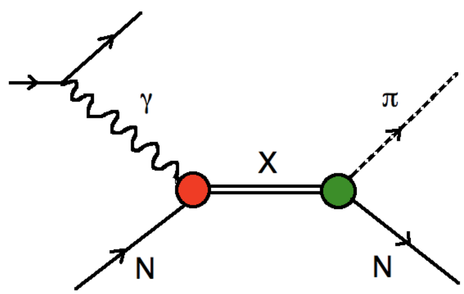


Electro/photo-production amplitude signs…

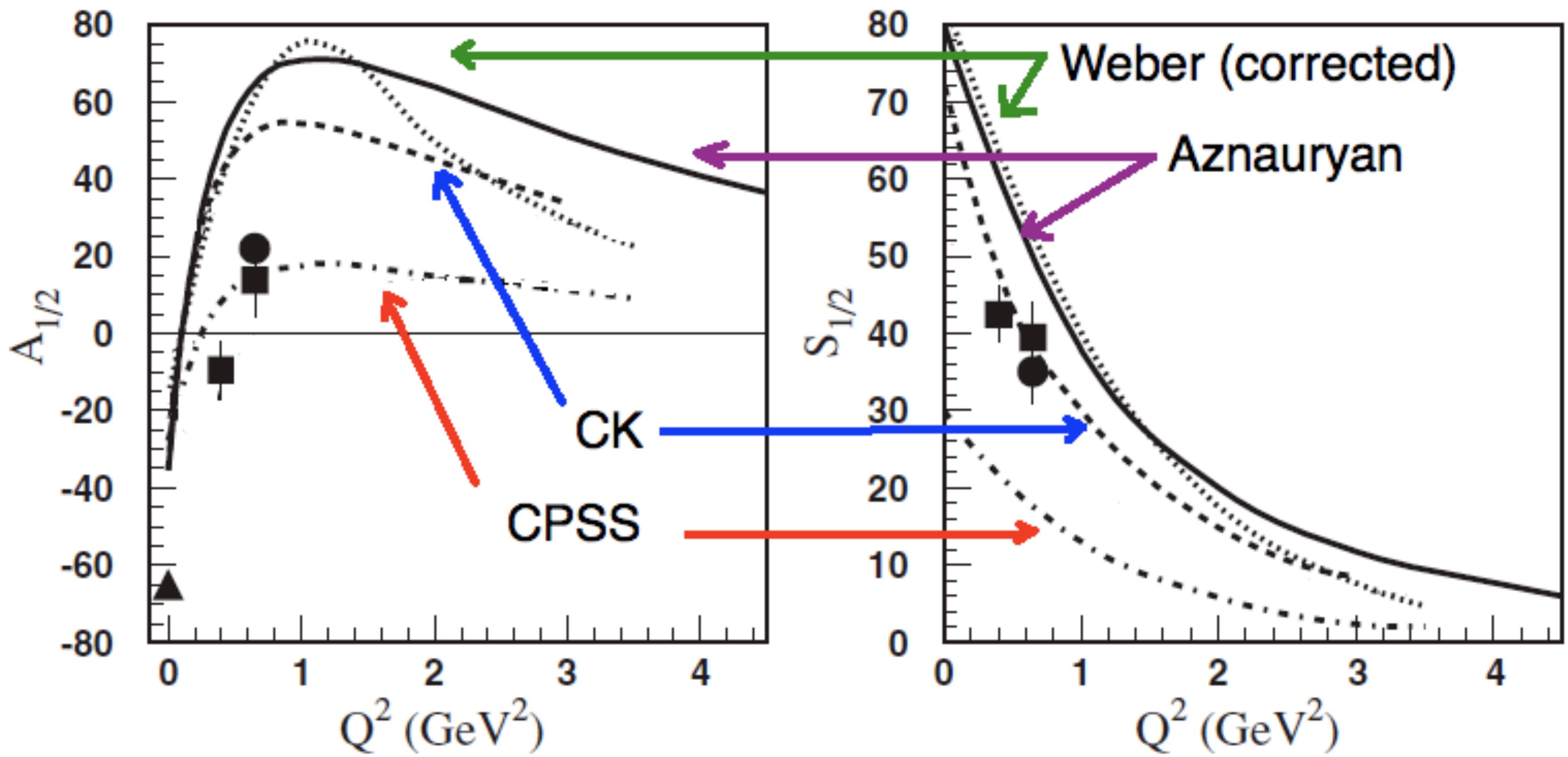
- Photo- and electro-production amplitudes quoted in analyses are the products

  $$A^{\dagger}_{X\text{-}\gamma N}\, A_{X\text{-}N\pi} / |A_{X\text{-}N\pi}|$$

  - Phase of $A_{X\text{-}N\pi}$ not measurable in $N\pi$ elastic scattering
  - Theorists must calculate $A_{X\text{->}N\pi}$ with exactly the same X and N wave functions used to calculate $A_{X\text{->}\gamma N}$
  - We use $^3P_0$ model



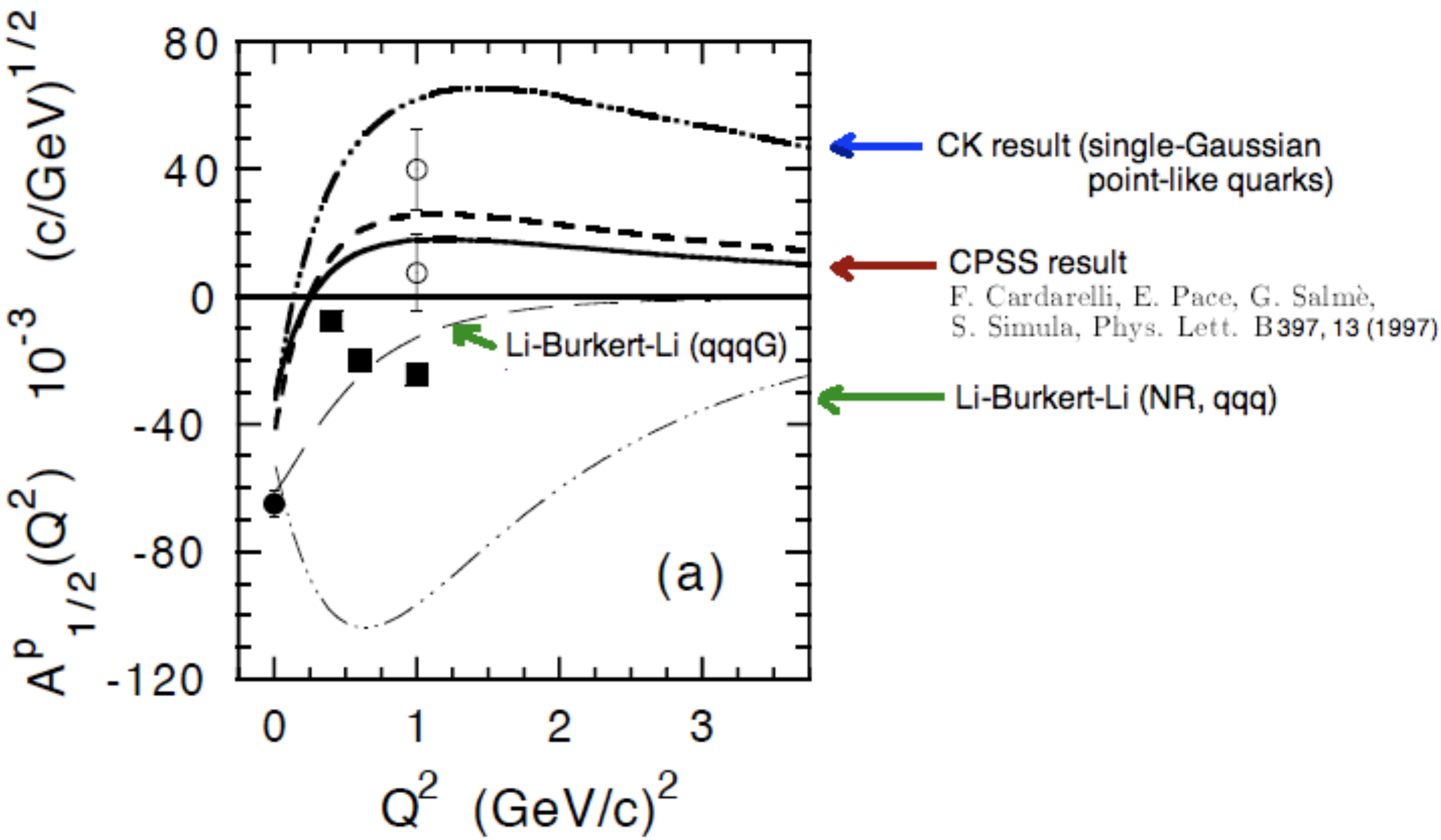
Inna Aznauryan-2007



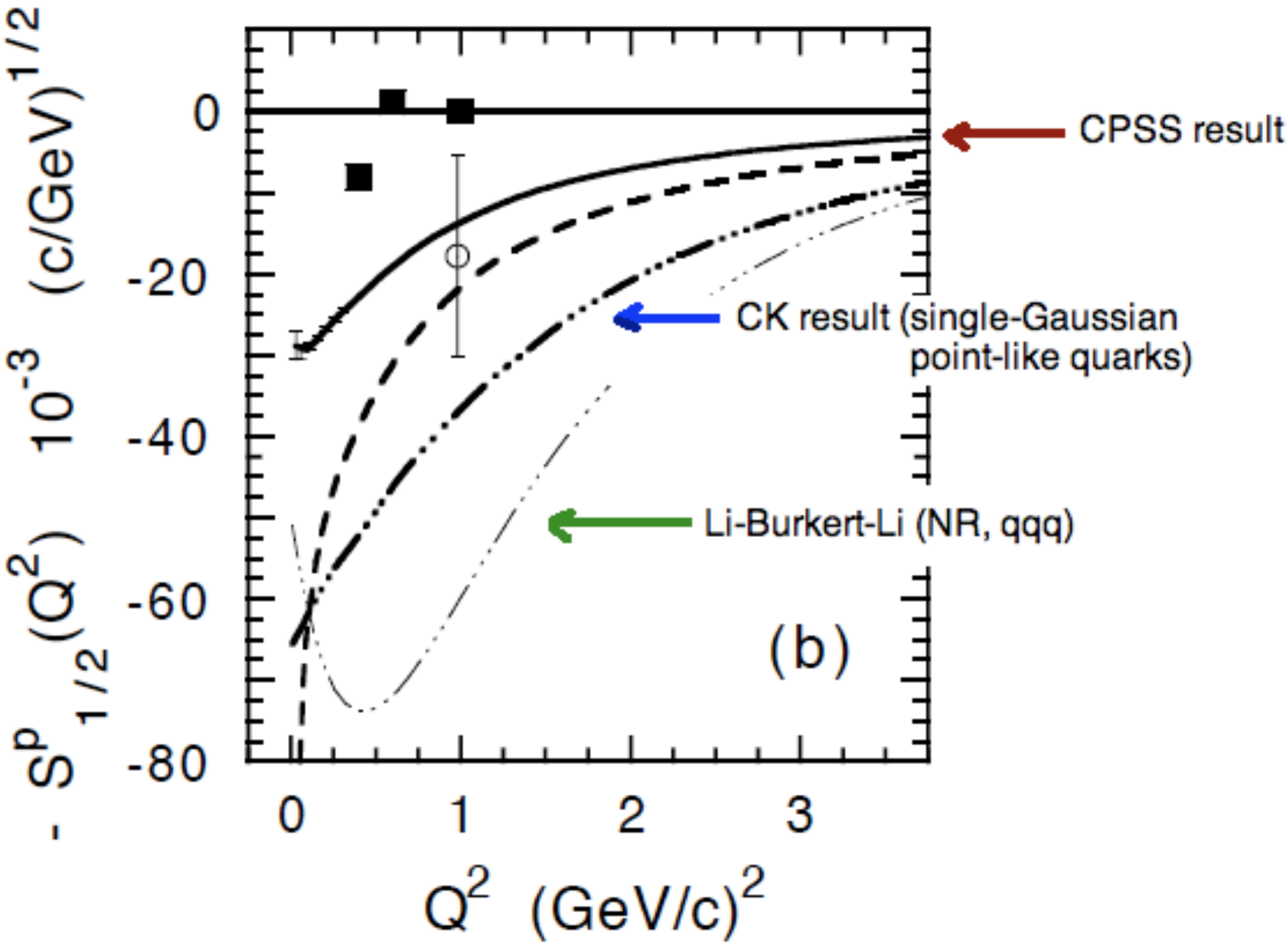
Light-front calculations-Rome group-1997

- Cardarelli, Pace, Salme and Simula
  - Used CI wave functions and light-front techniques to evaluate transition amplitudes
    - Quarks have f_1 and $f_2(Q^2)$ form factors
      - κ_u = +0.085, κ_d = -0.153 fit to nucleon moments
      - f_{1q} linear combination of monopole and dipole
      - f_{2q}/κ linear combination of dipole and quadrupole
      - Different Λ^2 values for each flavor of quark and type of form factor
        - » 12 parameters (in addition to anomalous moments) fit to nucleon and pion elastic form factors

Light-front calculations-Rome group-1997



Light-front calculations-Rome group-1997



Work with Brad Keister

- Calculations of EM transition form factors from N to N*
  - Light-cone (relativistic) quark model fit to nucleon elastic form factors
  - Baryon wave functions found by solving a three-quark Hamiltonian
  - Calculate strong-decay signs using pair-creation ($^3P_0$) model

Light-cone model of EM form factors

- Construct baryon wave functions in baryon CM frame in terms of free-particle light-front spinors
  - Bakamjian-Thomas construction
- Evaluate matrix elements of one-body EM current using these wave functions
- Find helicity amplitudes for EM transitions in terms of reduced matrix elements

Light-front dynamics

- Light-front Hamiltonian dynamics
  - Constituents are treated as particles rather than fields
  - Certain combinations of boosts and rotations are independent of the interactions which govern quark dynamics
    - Simplifies calculations of matrix elements in which composite baryons recoil with large momenta
  - Use complete orthonormal set of basis states
    - Composed of three constituent quarks
    - Satisfy rotational covariance

Calculation scheme

- Bakamjian and Thomas scheme:
  - Three-body relativistic bound-state problem is solved for the wave functions of baryons with the assumption of three interacting constituent quarks
  - Wave functions used to calculate the matrix elements of one (and in principle, two, and three)-body electromagnetic current operators

Calculational details

- Expand in sets of free-particle states:
  - Evaluate I^+ (EM) current matrix element by expanding baryon wave function in terms of light-front spinors for the quarks

$$\langle M'j;\tilde{\mathbf{P}}'\mu'|I^+(0)|Mj;\tilde{\mathbf{P}}\mu\rangle=$$

$$(2\pi)^{-18}\int d\tilde{\mathbf{p}}'_1\int d\tilde{\mathbf{p}}'_2\int d\tilde{\mathbf{p}}'_3\int d\tilde{\mathbf{p}}_1\int d\tilde{\mathbf{p}}_2\int d\tilde{\mathbf{p}}_3\sum\langle M'j';\tilde{\mathbf{P}}'\mu'|\tilde{\mathbf{p}}'_1\mu'_1\tilde{\mathbf{p}}'_2\mu'_2\tilde{\mathbf{p}}'_3\mu'_3\rangle$$

$$\times\langle\tilde{\mathbf{p}}'_1\mu'_1\tilde{\mathbf{p}}'_2\mu'_2\tilde{\mathbf{p}}'_3\mu'_3|I^+(0)|\tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3\rangle\langle\tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|Mj;\tilde{\mathbf{P}}\mu\rangle.$$

  - Need baryon state vectors written in terms of wave functions

Calculational details…

- Expand in sets of free-particle states:

$$\langle \tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|Mj;\tilde{\mathbf{P}}\mu\rangle =$$

$$\left|\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\mathbf{k}_1,\mathbf{k}_2)}\right|^{-1/2}(2\pi)^3\delta(\tilde{\mathbf{p}}_1+\tilde{\mathbf{p}}_2+\tilde{\mathbf{p}}_3-\tilde{\mathbf{P}})\langle\tfrac{1}{2}\bar{\mu}_1\tfrac{1}{2}\bar{\mu}_2|s_{12}\mu_{12}\rangle\langle s_{12}\mu_{12}\tfrac{1}{2}\bar{\mu}_3|s\mu_s\rangle$$
$$\times\langle l_\rho\mu_\rho l_\lambda\mu_\lambda|L\mu_L\rangle\langle L\mu_L s\mu_s|j\mu\rangle Y_{l_\rho\mu_\rho}(\hat{\mathbf{k}}_\rho)Y_{l_\lambda\mu_\lambda}(\hat{\mathbf{K}}_\lambda)\Phi(k_\rho,K_\lambda)$$
$$\times D^{(1/2)\dagger}_{\bar{\mu}_1\mu_1}[\underline{R}_{cf}(k_1)]D^{(1/2)\dagger}_{\bar{\mu}_2\mu_2}[\underline{R}_{cf}(k_2)]D^{(1/2)\dagger}_{\bar{\mu}_3\mu_3}[\underline{R}_{cf}(k_3)].$$

Calculational details…

- Cluster expansion of electromagnetic current operator $$I^{\mu}(x) = \sum_j I^{\mu}_j(x) + \sum_{j<k} I^{\mu}_{jk}(x) + \cdots .$$
- We evaluate only one-body matrix elements and assume struck quark has EM current of free Dirac particle

$$\langle \tilde{\mathbf{p}}'\mu' | I^+(0) | \tilde{\mathbf{p}}\mu \rangle = F_{1q}(Q^2)\delta_{\mu'\mu} - i(\sigma_y)_{\mu'\mu}\frac{Q}{2m_i}F_{2q}(Q^2)$$

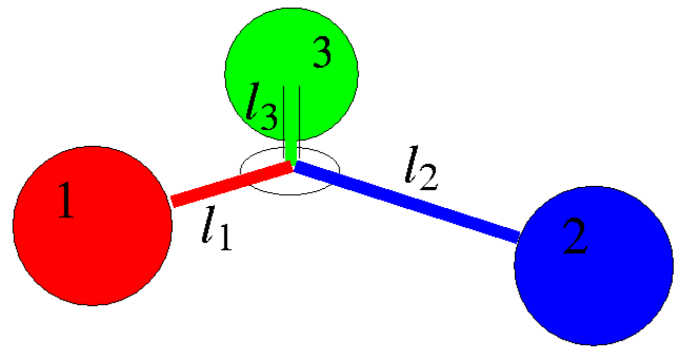
  - Result is a 6D integral that we evaluate using numerical techniques [quasi-random number (Sobol) sequences]

Light-cone model…

- Wave functions expanded in h.o. basis up to N=6 or 7 ($\hbar\omega$)
  - e.g. 50 components for N and Roper, 70 for N(1535)S_{11}
- Requires simultaneous calculation of strong-decay amplitudes
  - Calculate $N\pi$ sign using $^3P_0$ model using *identical* wave functions
- Fit quark EM form factors to nucleon EM form factors (moments and Q^2 dependence)
  - Similar to calculations performed by Rome group (Cardarelli, Pace, Salme, Simula), but simpler F_{1q}, F_{2q}

Model of spectrum and wave functions

- Confinement:
  - Flux tubes, combined with adiabatic approx.
  - minimum length string:
    $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sigma(l_1+l_2+l_3)=\sigma L_{min}$
  - linear at large q-junction separations



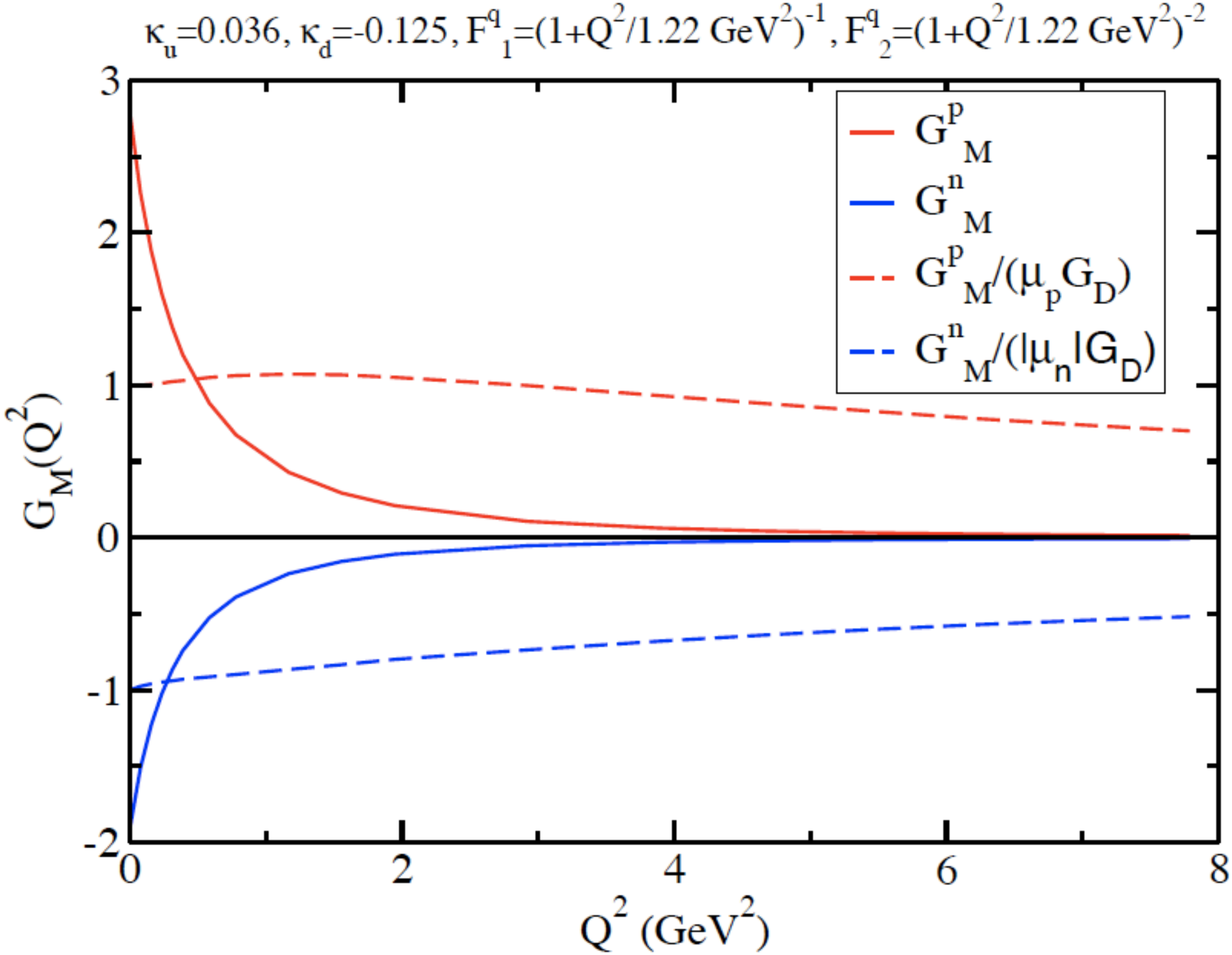
- Short-range interactions:
  - Ground-state spectrum suggests flavor-dependent short-range (contact) interactions
  - Use OGE (other possibilities: OBE, instanton-induced interactions)

Wave functions

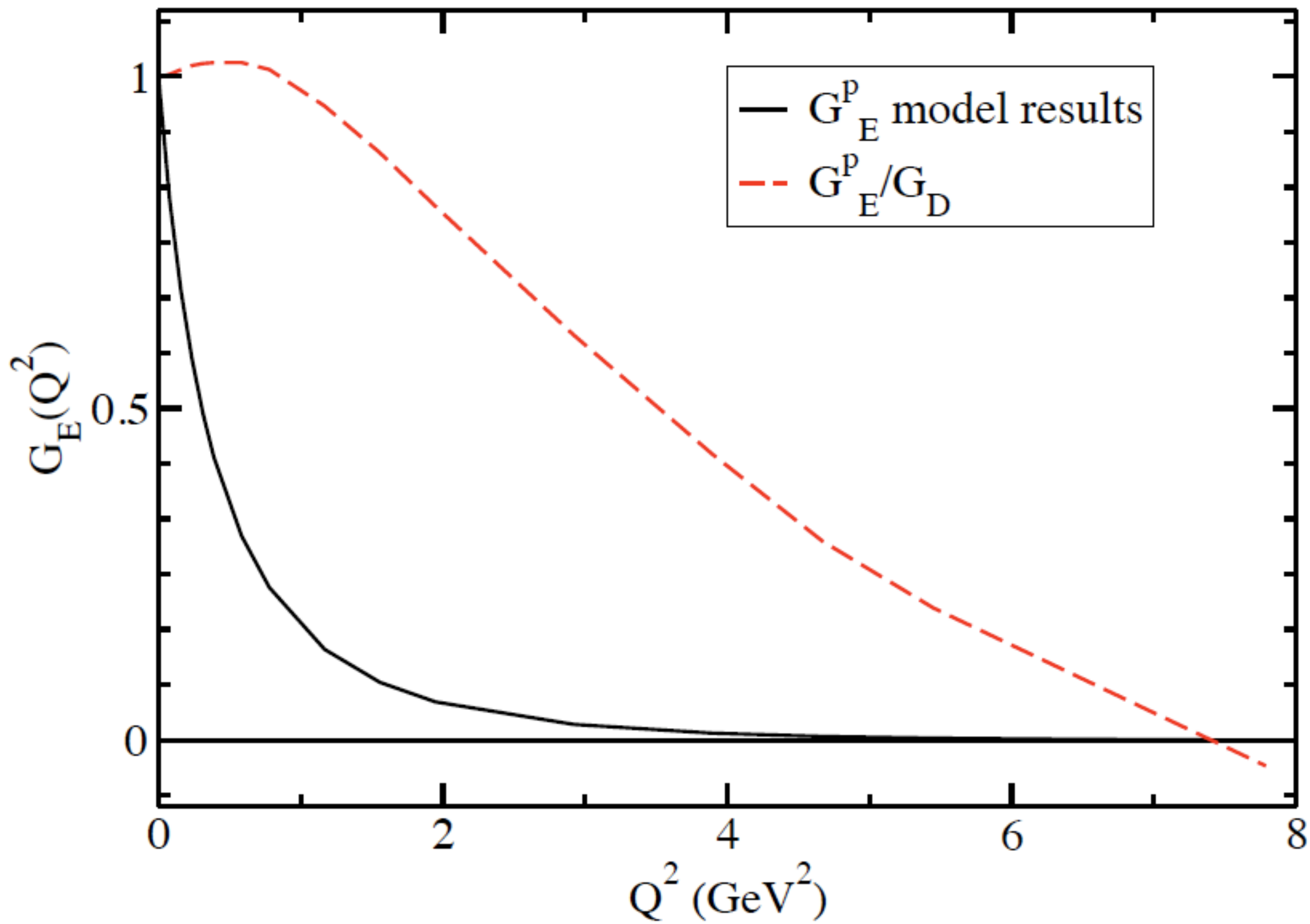
- Variational calculation in large HO basis (SC, N. Isgur)
  - String confinement, plus associated spin-orbit
  - Include OGE Coulomb, contact, tensor, spin-orbit
  - Relativistic KE, relativistic corrections in potentials, e.g.

$$\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}} \frac{8\pi}{3}\alpha_s(r_{ij})\frac{2}{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\left[\frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}}e^{-\sigma_{ij}^2 r_{ij}^2}\right]\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}}$$

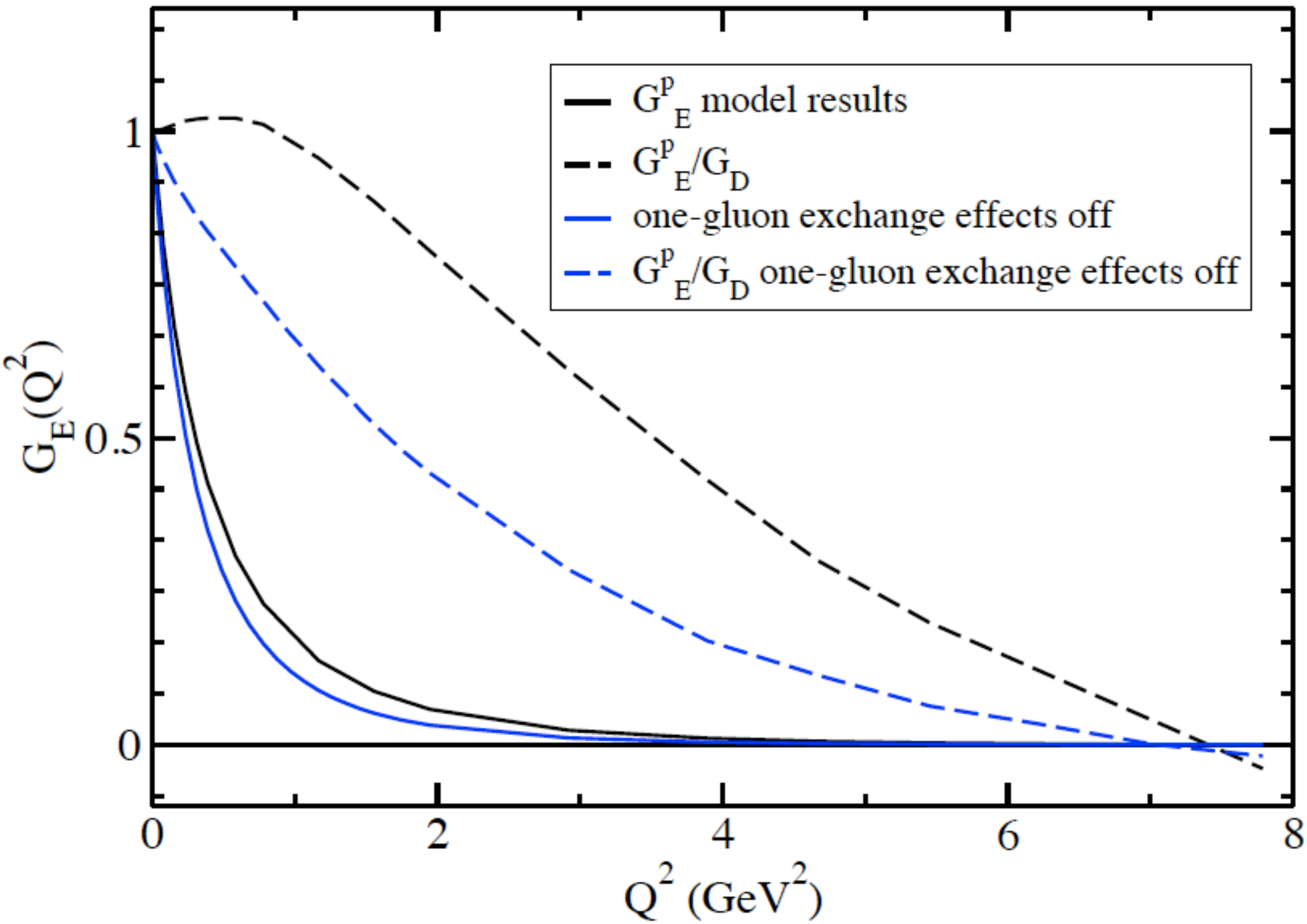
  - Contact interaction smeared with Gaussian form factor, σ_{ij} depends on quark flavor (1.8 GeV for light quarks)


κu=0.036, κd=-0.125, Fq1=(1+Q2/1.22 GeV2)-1, Fq2=(1+Q2/1.22 GeV2)-2
GpM
GnM
GpM/(μpGD)
GnM/(|μn|GD)
GM(Q2)
Q2 (GeV2)
3
2
1
0
-1
-2
0
2
4
6
8

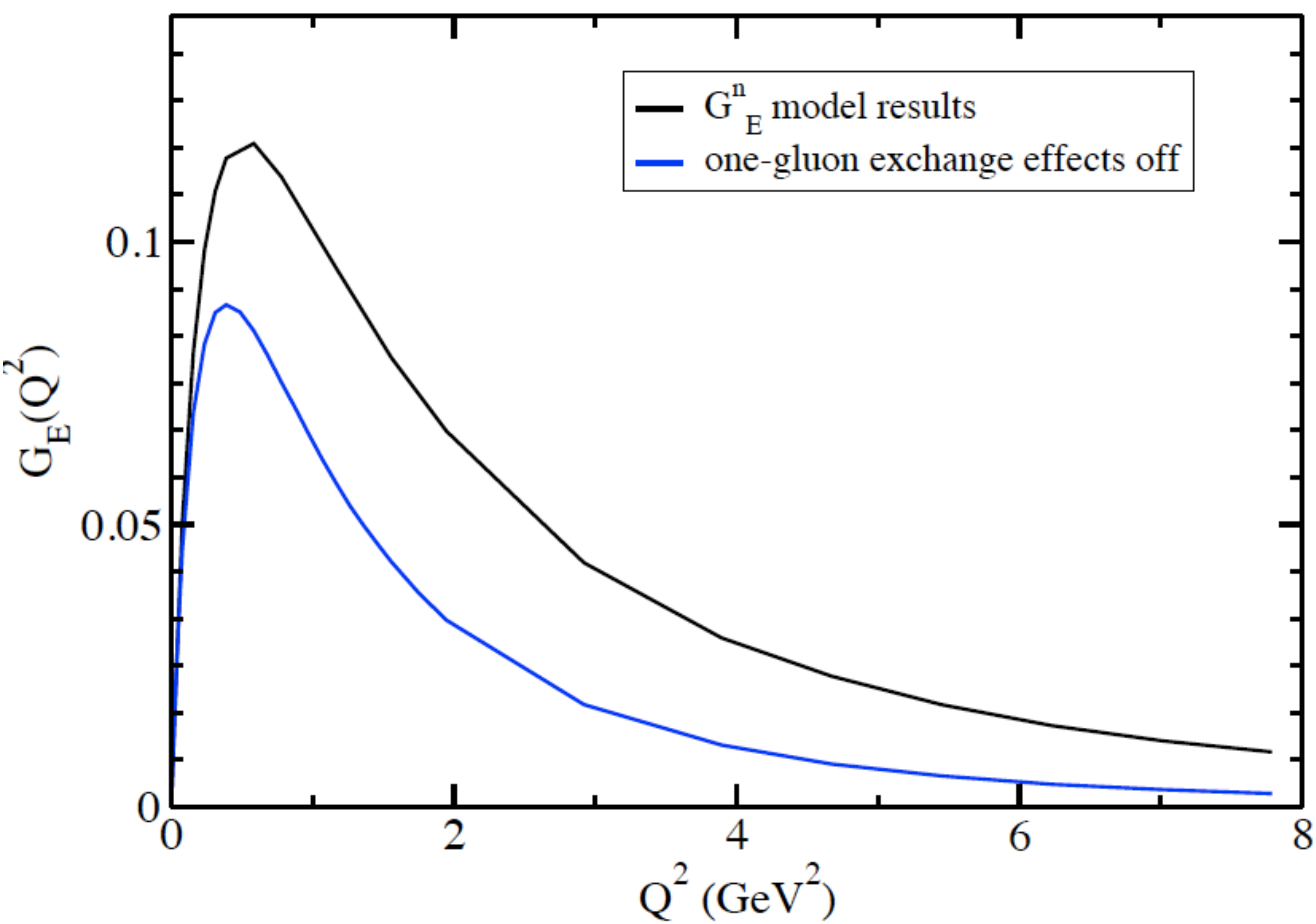
Proton electric form factor



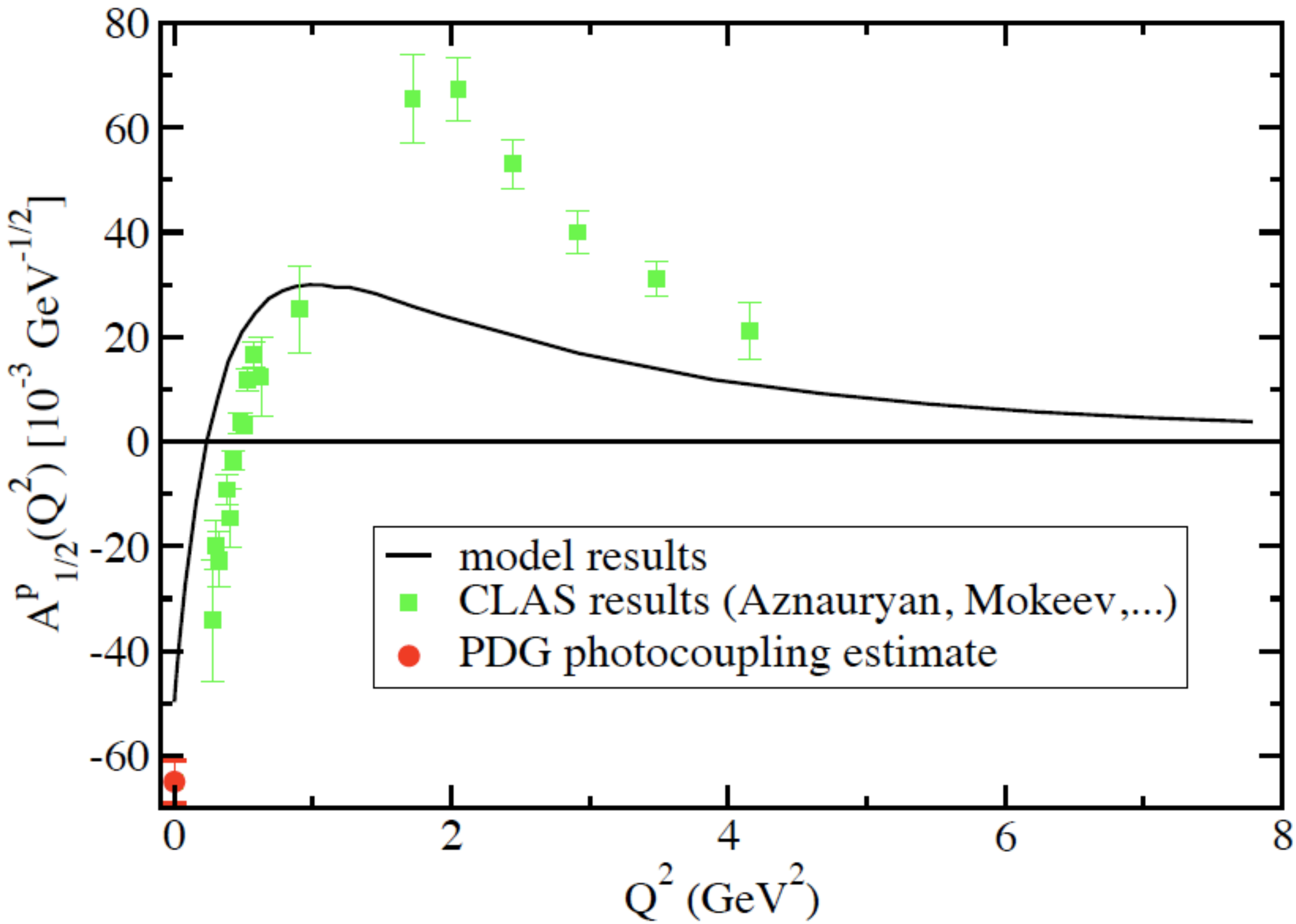
Proton electric form factor



Neutron electric form factor



Roper resonance transverse amplitude

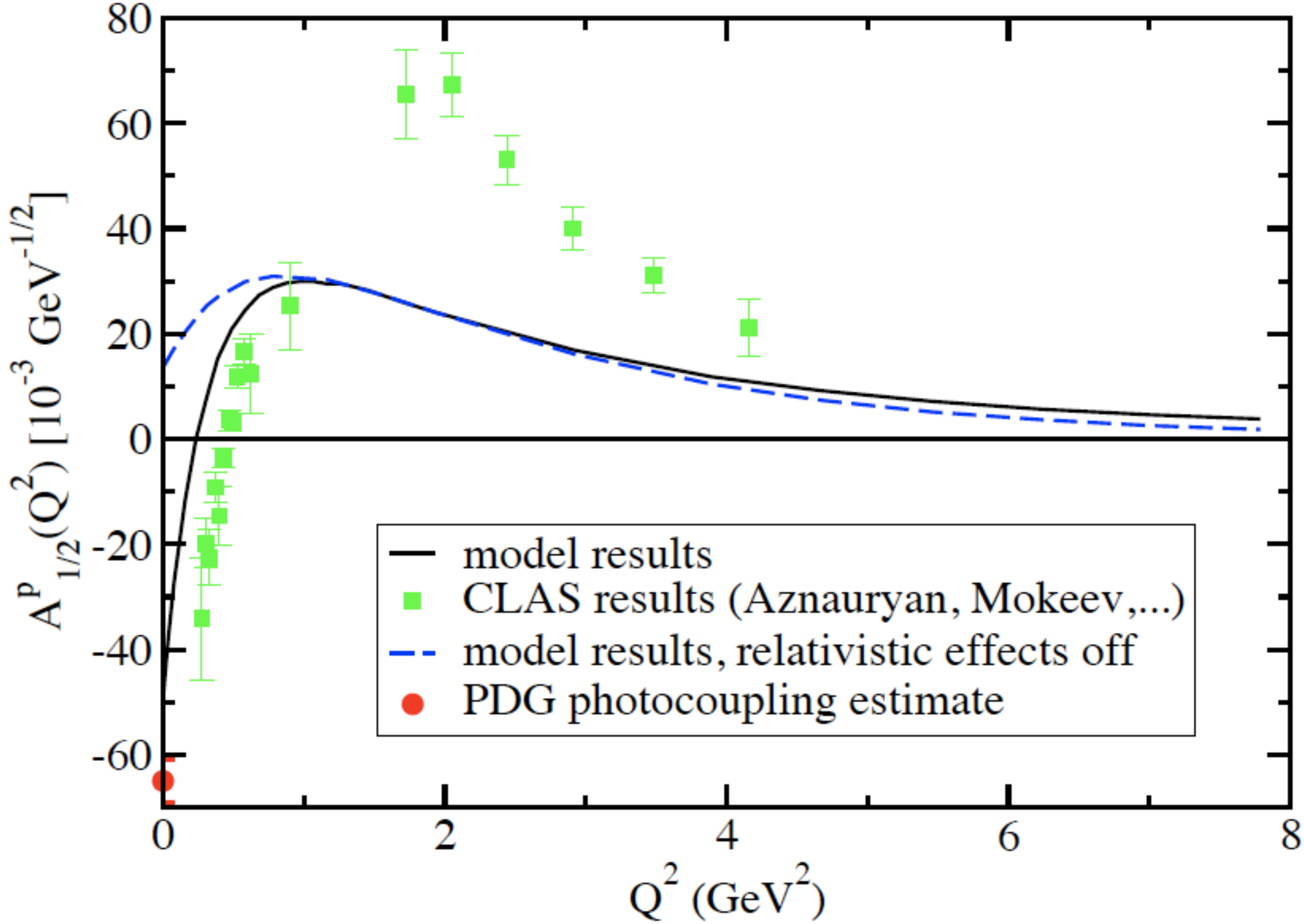


NR limit of light-front calculations?

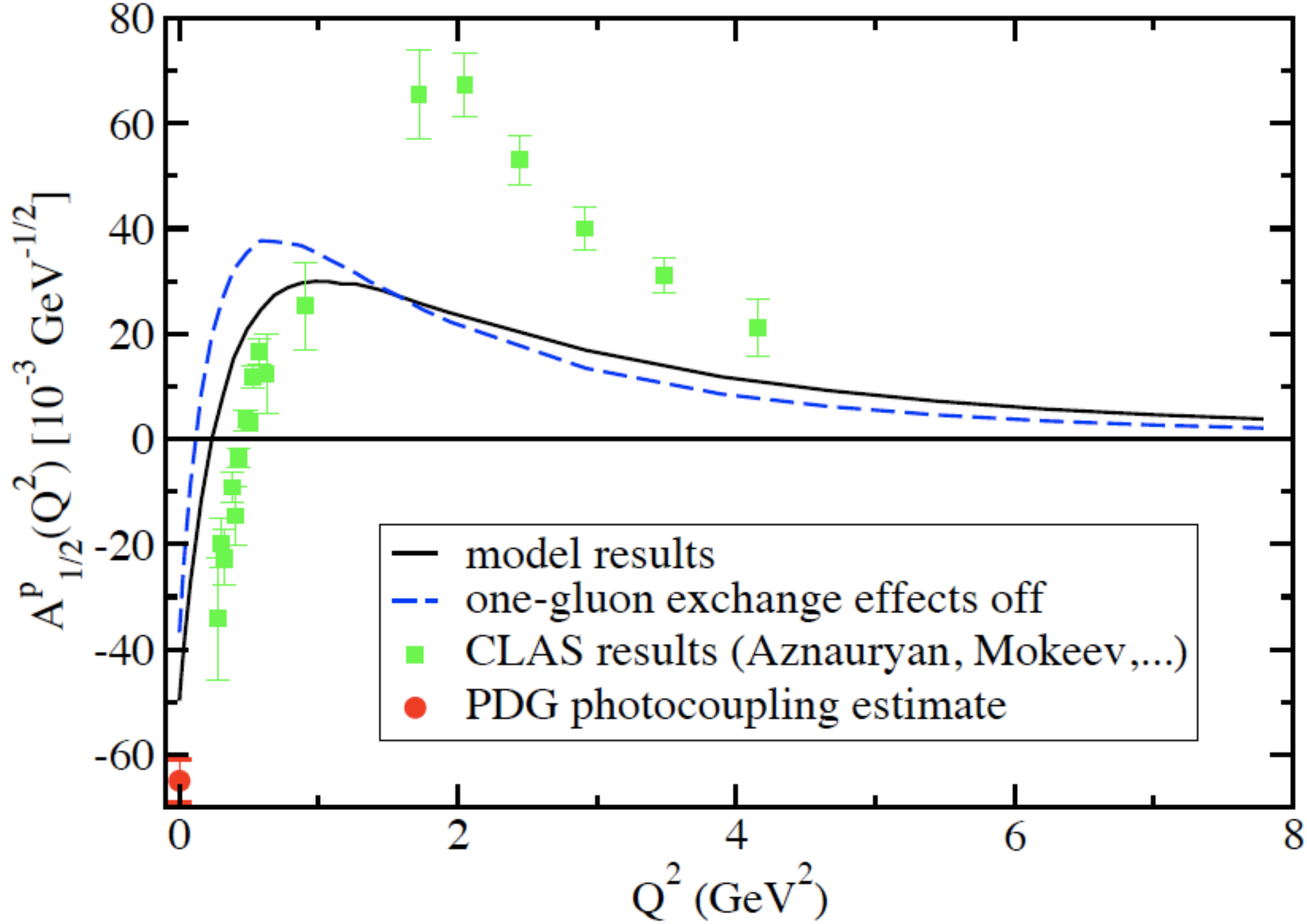
- NR: turn off Jacobians, Melosh rotations, relativistic kinematics (not true NR limit!)

$$
\begin{aligned}
&\langle \tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|Mj;\tilde{\mathbf{P}}\mu\rangle = \\
&\cancel{\left|\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\mathbf{k}_1,\mathbf{k}_2)}\right|^{-1/2}} (2\pi)^3\delta(\tilde{\mathbf{p}}_1+\tilde{\mathbf{p}}_2+\tilde{\mathbf{p}}_3-\tilde{\mathbf{P}})\langle\tfrac{1}{2}\bar{\mu}_1\tfrac{1}{2}\bar{\mu}_2|s_{12}\mu_{12}\rangle\langle s_{12}\mu_{12}\tfrac{1}{2}\bar{\mu}_3|s\mu_s\rangle \\
&\times\langle l_\rho\mu_\rho l_\lambda\mu_\lambda|L\mu_L\rangle\langle L\mu_L s\mu_s|j\mu\rangle Y_{l_\rho\mu_\rho}(\hat{\mathbf{k}}_\rho)Y_{l_\lambda\mu_\lambda}(\hat{\mathbf{K}}_\lambda)\Phi(k_\rho,K_\lambda) \\
&\times\cancel{D^{(1/2)\dagger}_{\bar{\mu}_1\mu_1}[\underline{R}_{cf}(k_1)]D^{(1/2)\dagger}_{\bar{\mu}_2\mu_2}[\underline{R}_{cf}(k_2)]D^{(1/2)\dagger}_{\bar{\mu}_3\mu_3}[\underline{R}_{cf}(k_3)]}.
\end{aligned}
$$

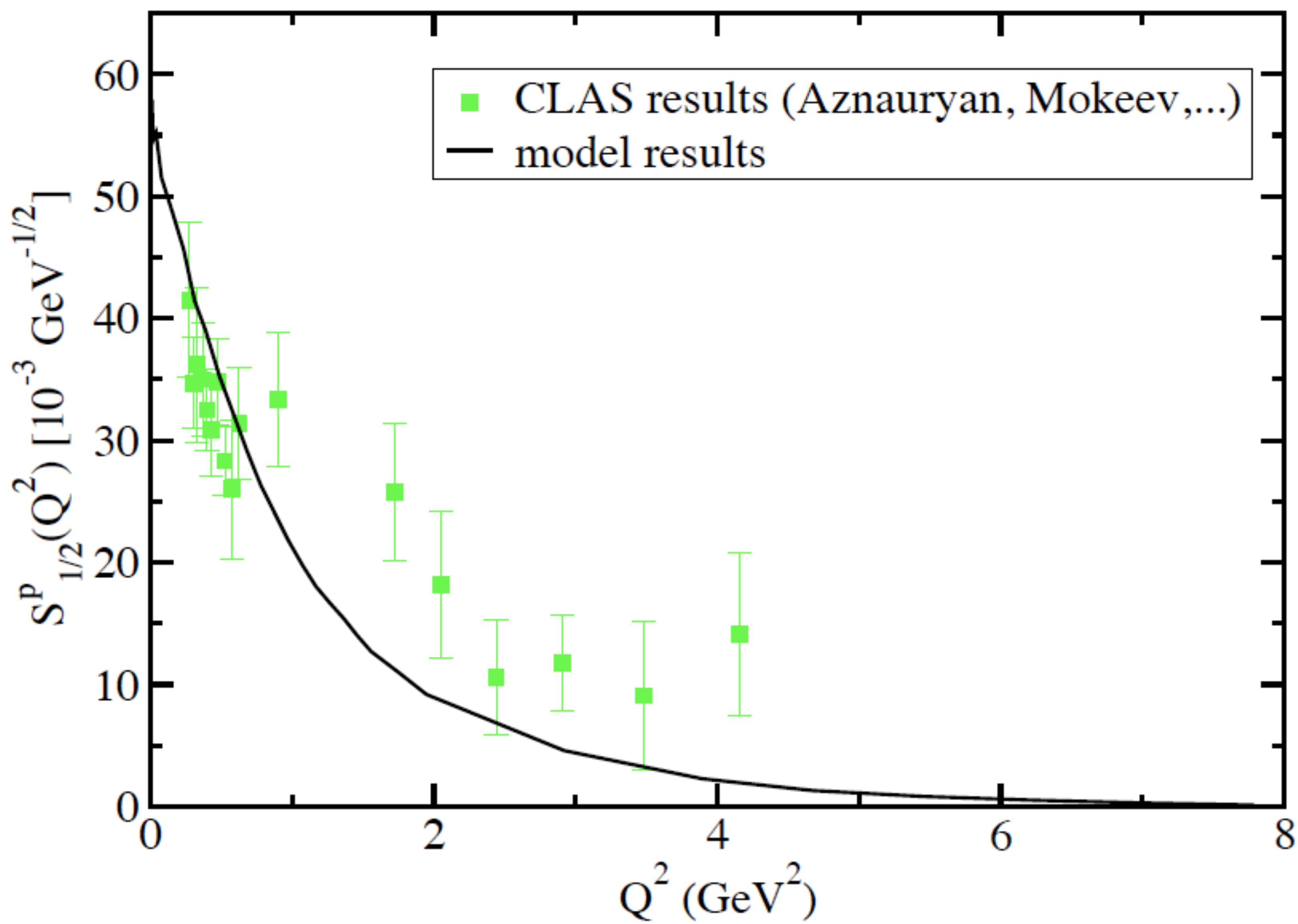
Roper resonance transverse amplitude



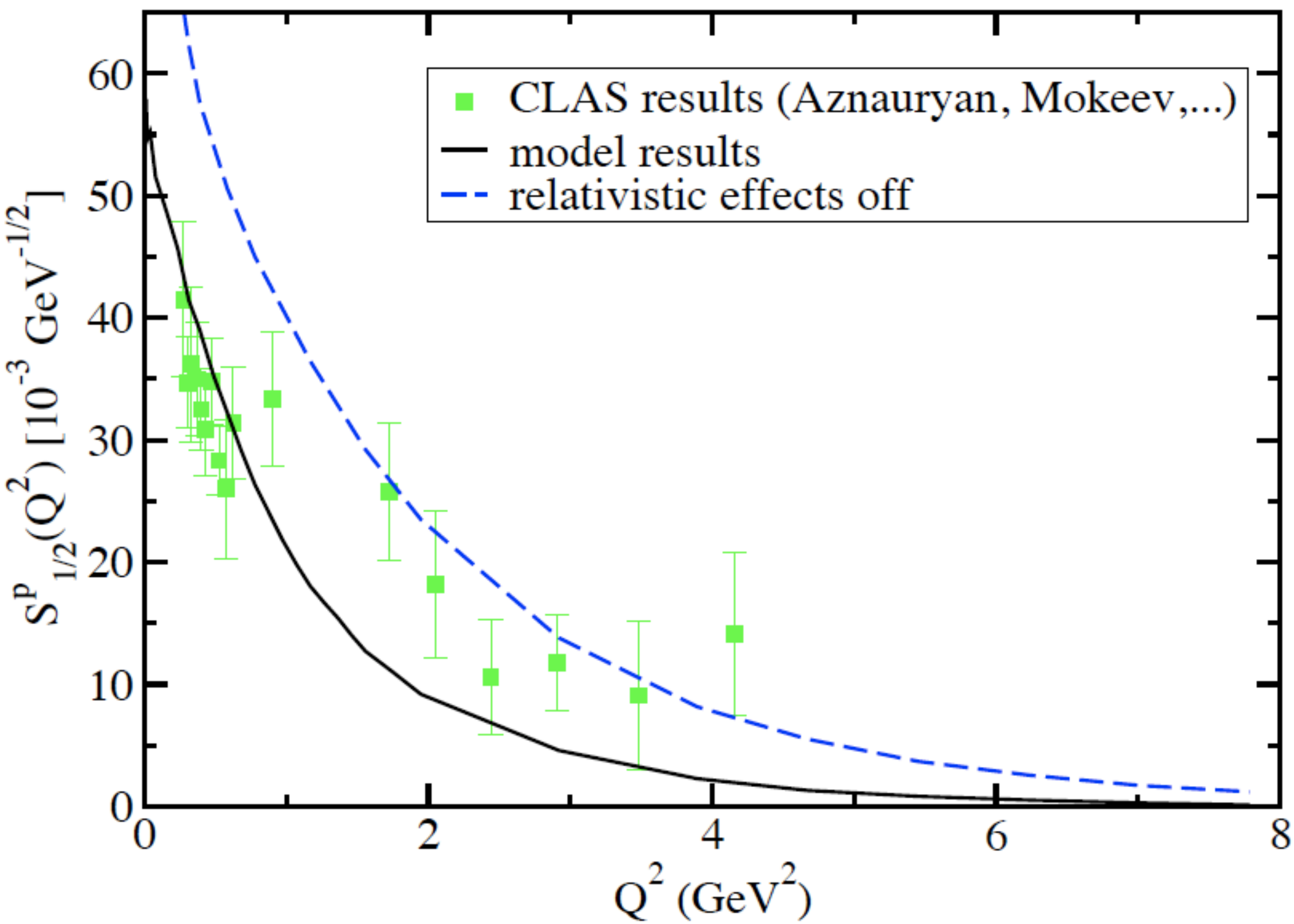
Roper resonance transverse amplitude



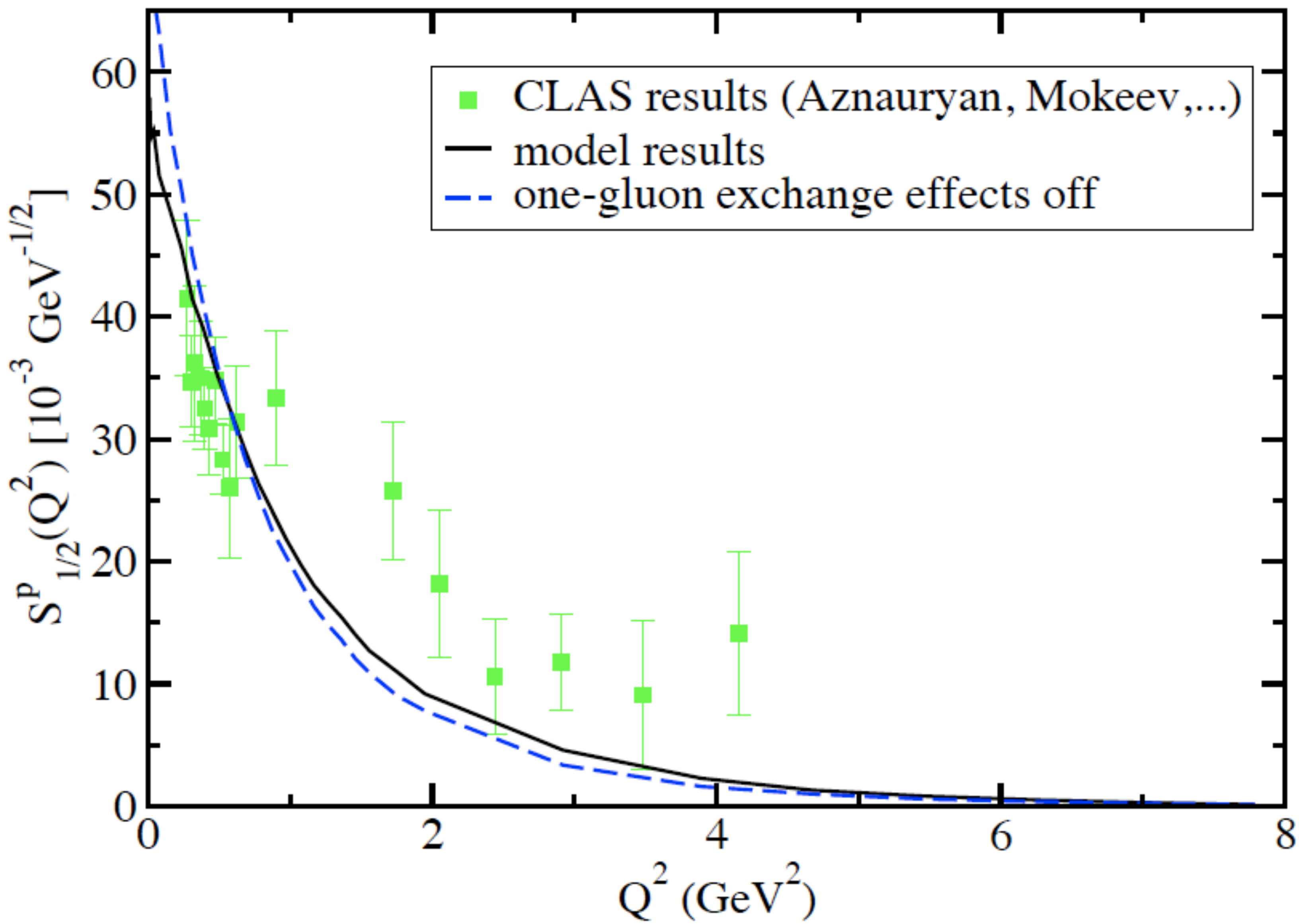
Roper resonance scalar amplitude



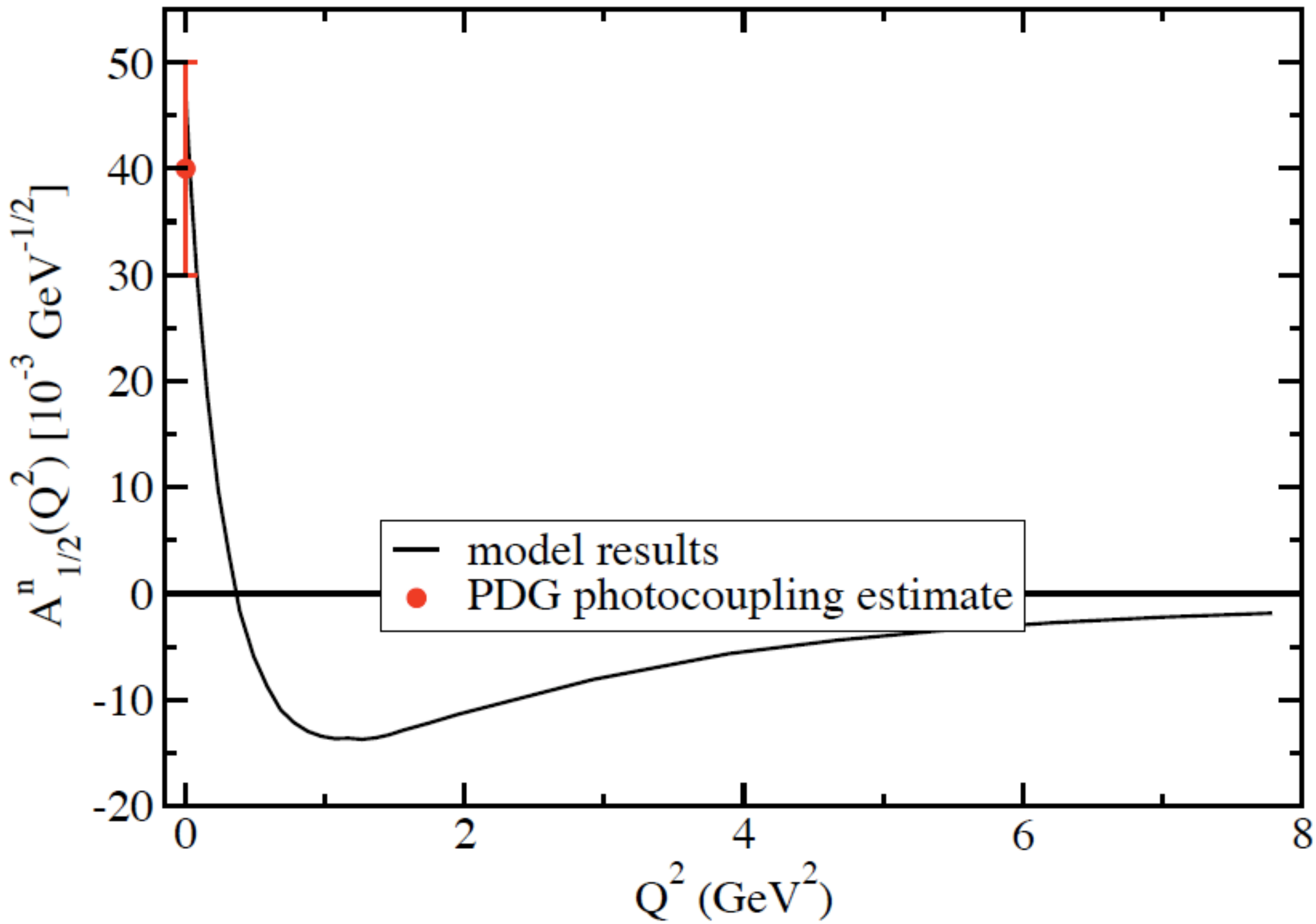
Roper resonance scalar amplitude

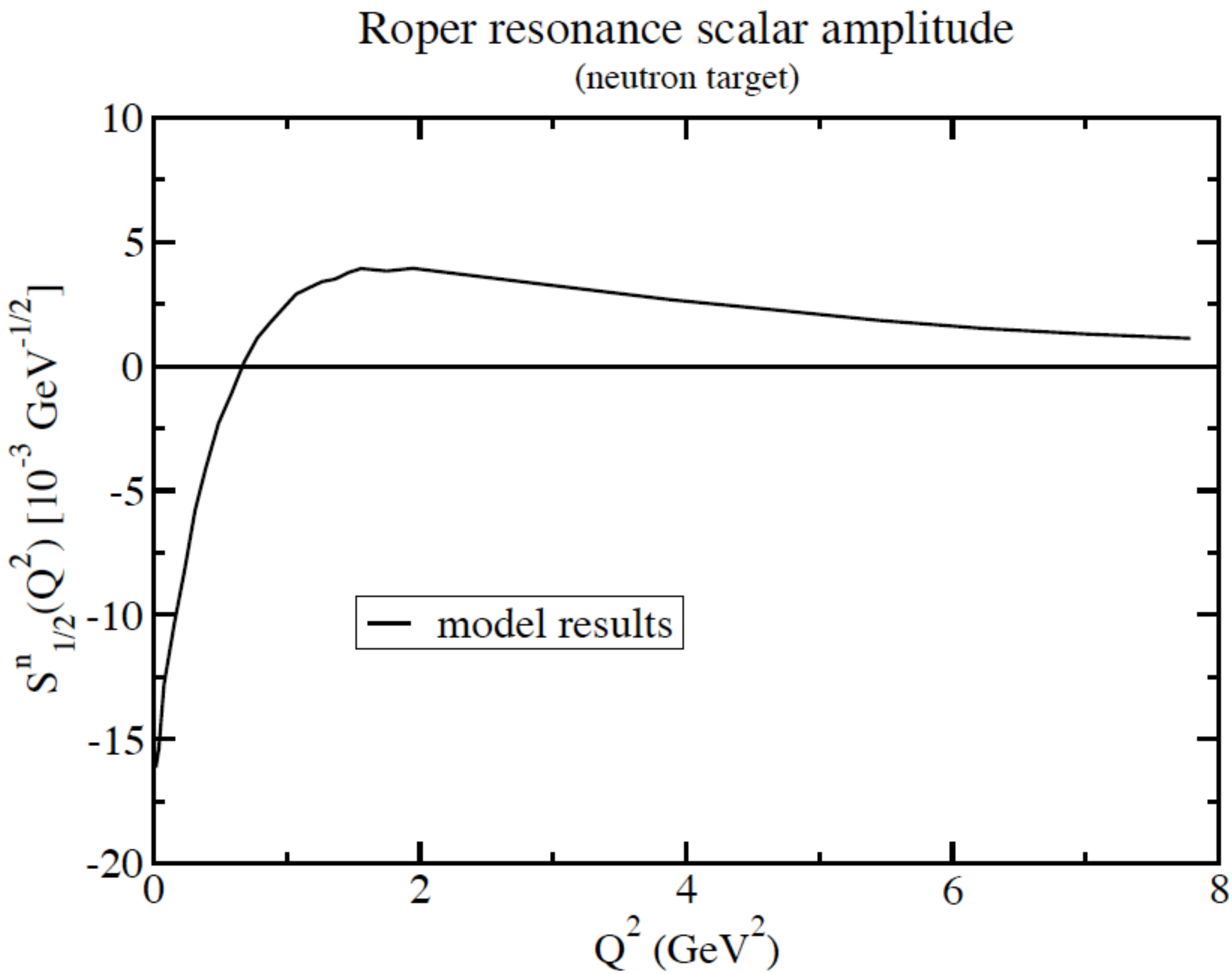


Roper resonance scalar amplitude



Roper resonance transverse amplitude
(neutron target)




Roper resonance scalar amplitude
(neutron target)
10
5
0
-5
-10
-15
-20
0
2
4
6
8
$S^n_{1/2}(Q^2)$ [10^{-3} GeV$^{-1/2}$]
Q^2 (GeV^2)
model results

Rotational covariance

- States with higher J
  - Rotations are dynamical in light-front QM
  - It is possible to quantify the violation of rotational covariance by forming a linear combination of light-front spin matrix elements which should be zero
    - E.g. for $\Delta(1232)$ there is one such combination
      - Becomes comparable to $A^p_{3/2}$, $A^p_{1/2}$ only at higher Q^2
      - Calculation of sub-dominant amplitudes (E1+, S1+) believable at Q^2 below roughly 2 GeV^2
    - Non-zero because calculation truncated at one-body currents

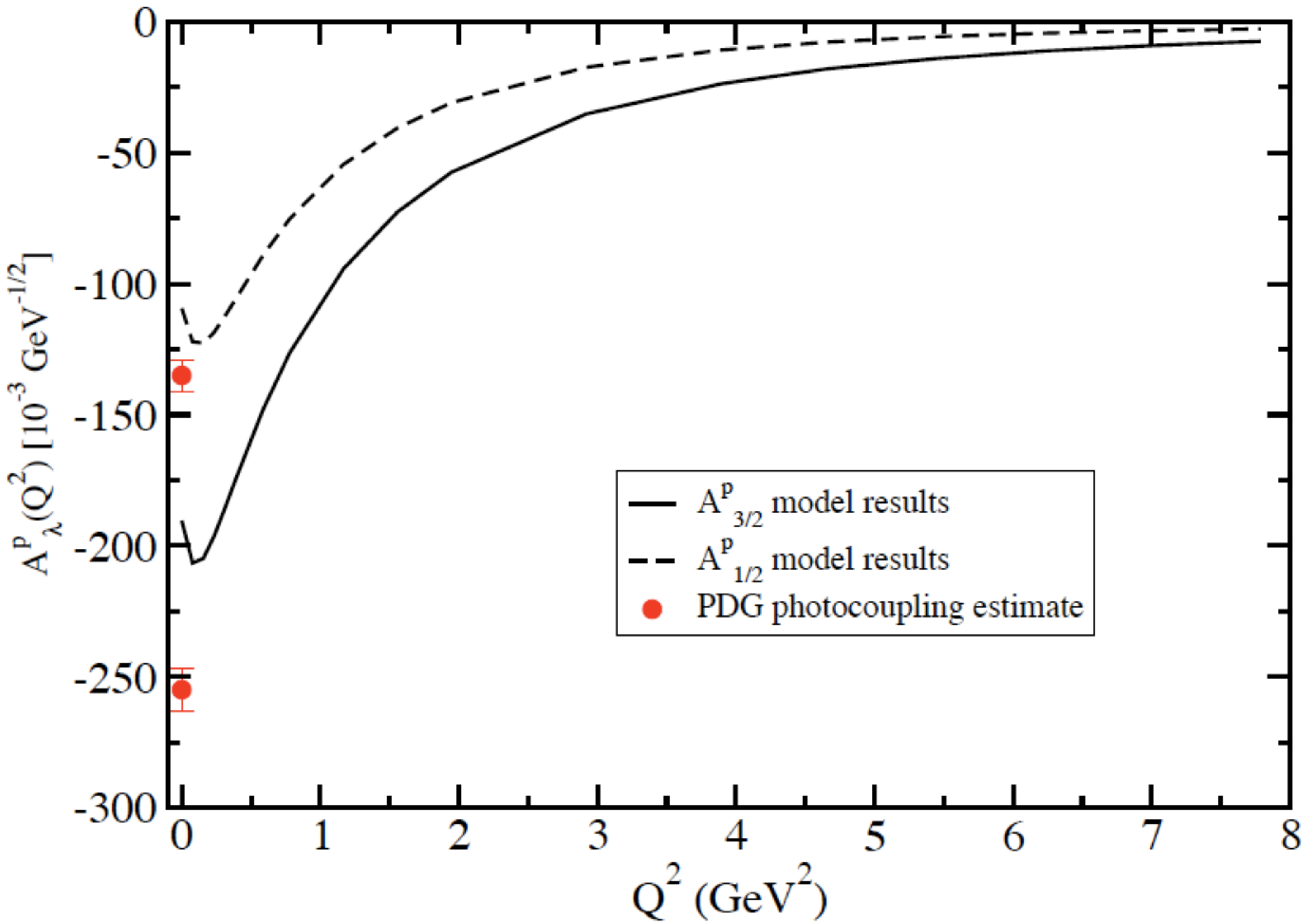
Rotational covariance…

- For states with J=5/2 there are three linear combinations which should be zero
  - For $N5/2^+(1680)$ these may not small at 1 GeV^2
- Some authors claim to have a work around for J=1/2
  - Evaluate light-front matrix elements of other components of the EM current, take linear combinations to eliminate matrix elements which must be zero
  - But there is no free lunch for higher J!
    - If use other components of I, don't have minimal set of matrix elements which transform into each other under boosts

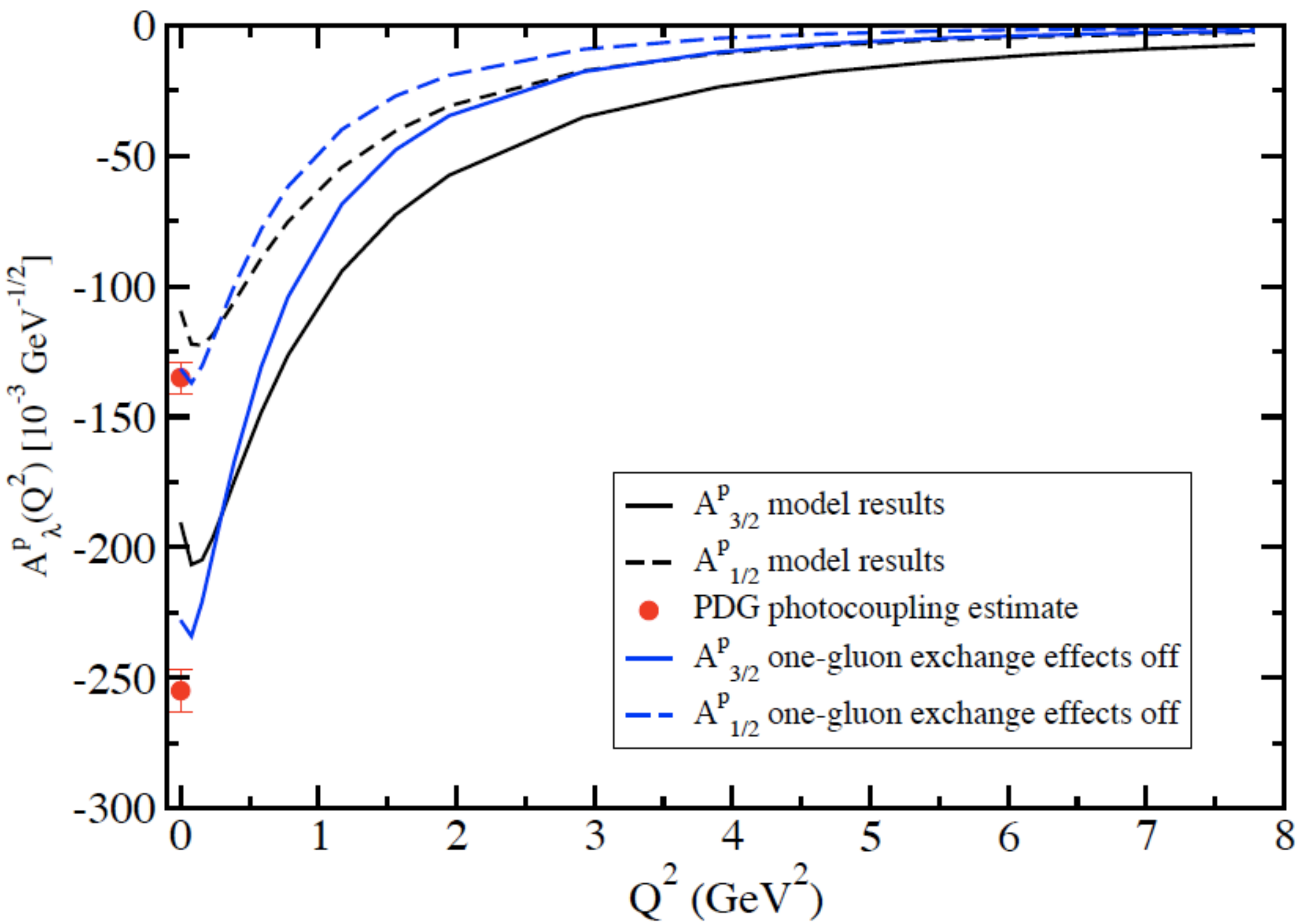
Conclusions/Outlook

- Relativistic calculations using light-front dynamics get sign of N to Roper $A^p_{\frac{1}{2}}$ to change
  - Crosses zero at lower Q^2 than amplitude extracted from data
  - Non-relativistic calculations (and light-front calculations with some relativistic effects turned off) do not see this
  - Must calculate $N\pi$ sign in model
  - Size and Q^2 dependence quite sensitive to short-range interactions between quarks
- N to Roper $S^p_{\frac{1}{2}}$ predicted a little too large, Q^2 dependence reasonable
- Neutron target amplitudes give new information
- To be believable, models should fit nucleon elastic form factors and other transition form factors!

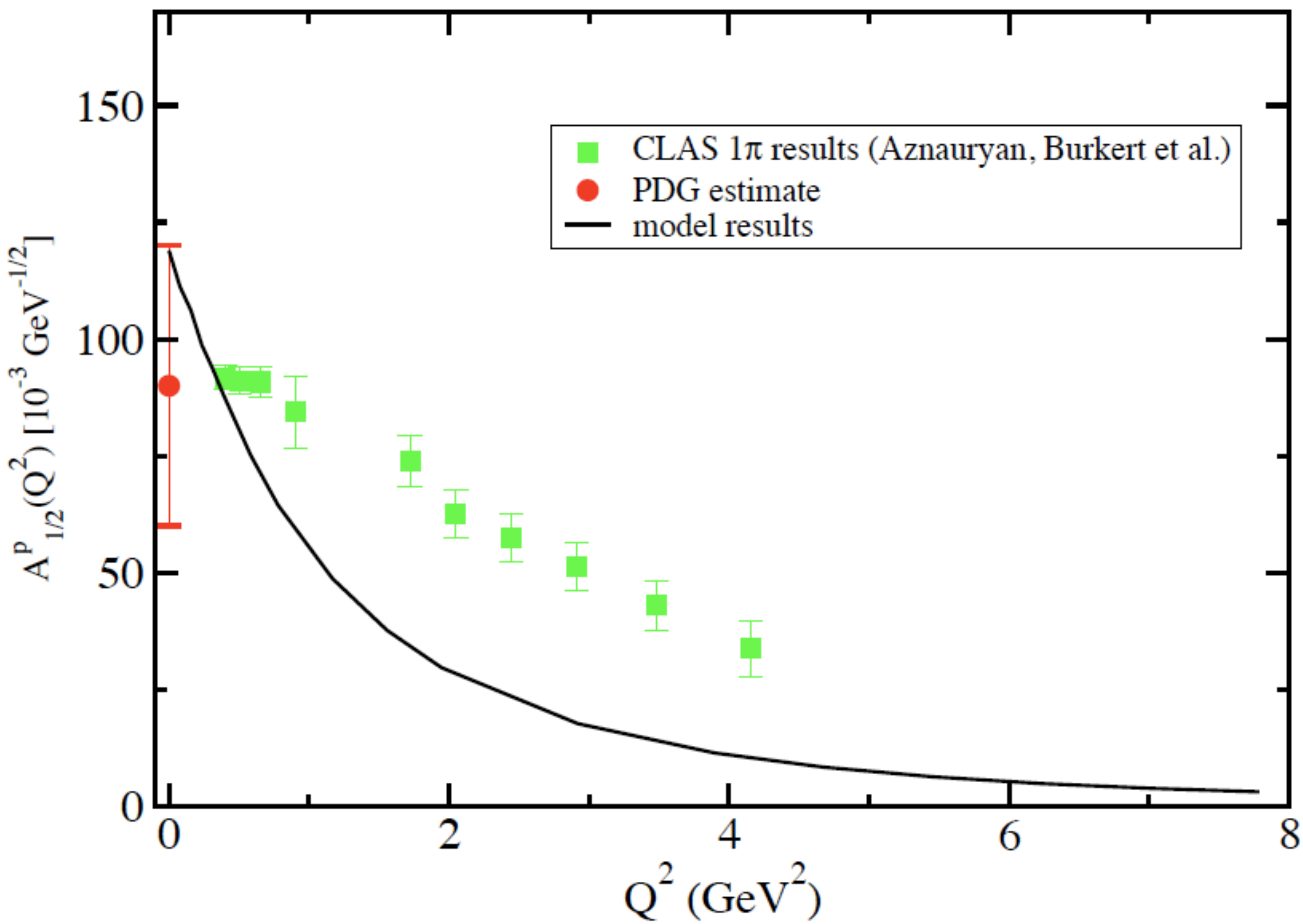
Delta resonance transverse amplitude



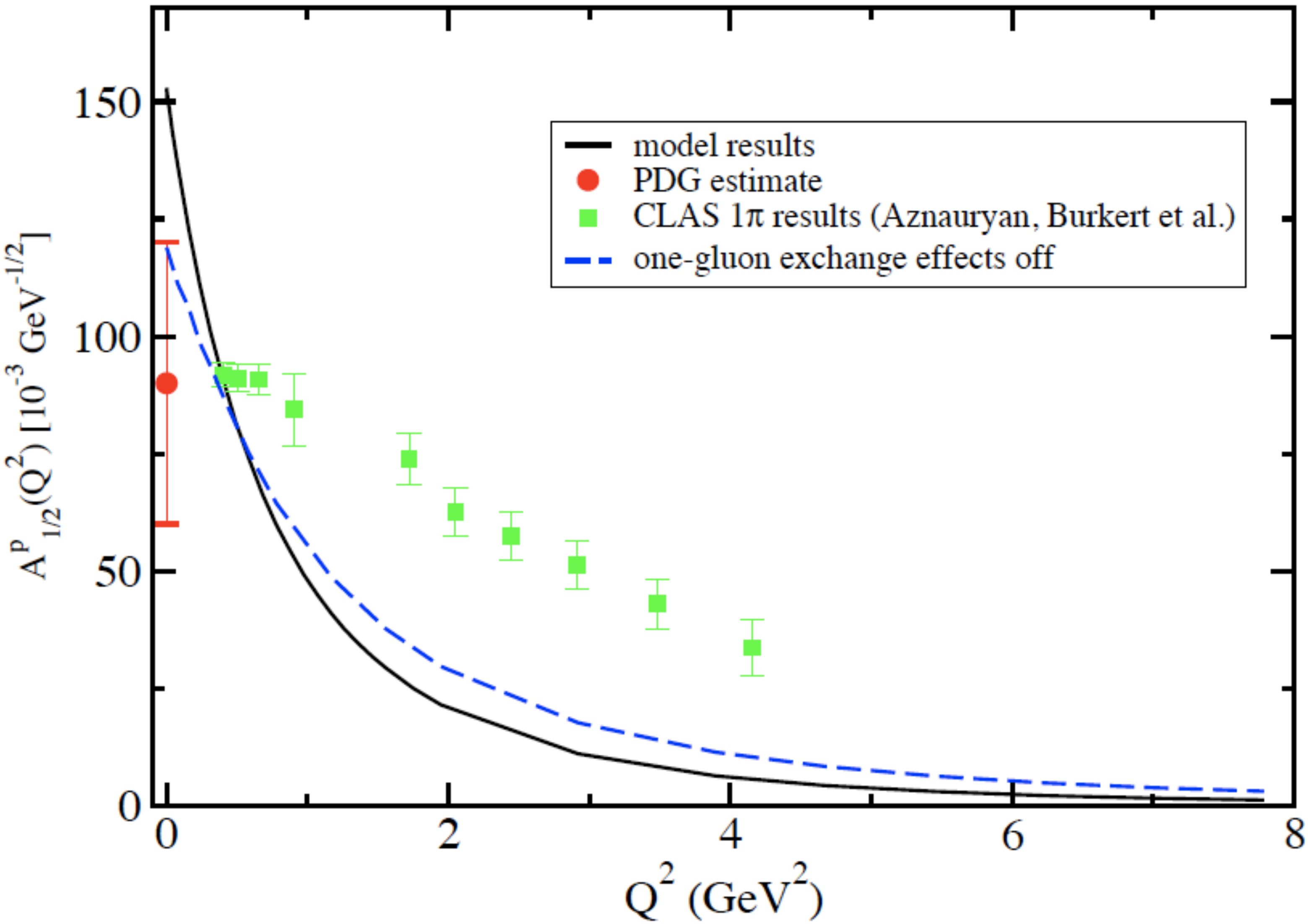
Delta resonance transverse amplitude



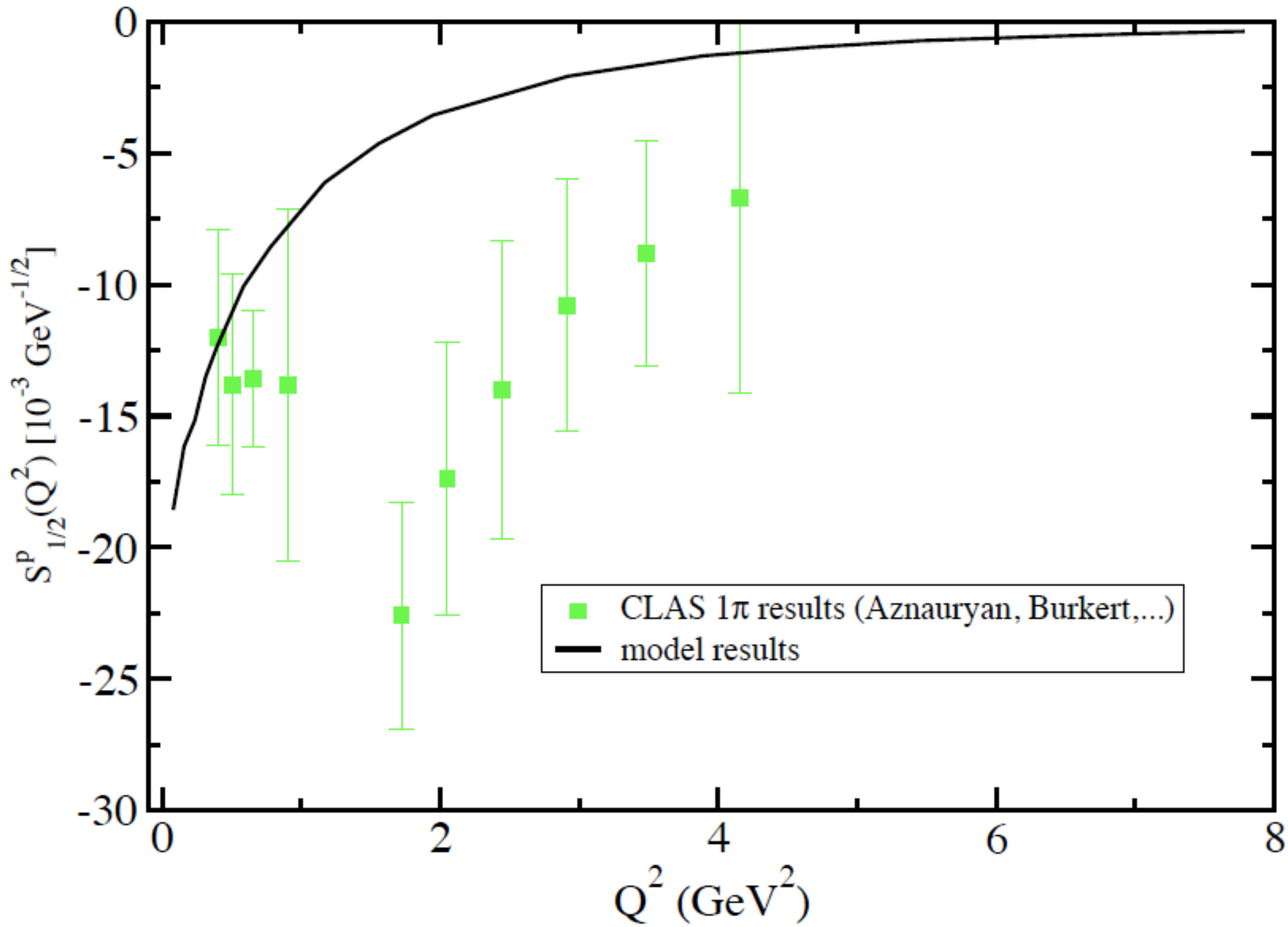
$N(1535)S_{11}$ resonance transverse amplitude



N(1535)S_{11} resonance transverse amplitude



N(1535)S_{11} resonance scalar amplitude



N(1535)S_{11} resonance scalar amplitude