Resonance properties in heavy-meson decays

EBAC workshop on

Extractions and interpretations of hadron resonances and multi-meson production reactions with 12 GeV upgrade

May 27– 28, 2010

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I. Observation (CLEO, Belle and BaBar) of scalar and vector resonances in heavy-meson decays corroborates two-decade old LASS results on kaon-pion scattering.

II. Heavy effective field theory, QCD factorization and hadronix matrix elements.

III. Description of pion-kaon pair creation in S- and P-waves by appropriate *strange* scalar and vector form factors.

IV. Pion-kaon interactions from threshold to 2 GeV divided into a

low-energy domain: *form factors obtained from chiral perturbation predictions.*

larger energies: *parametrization of pion-kaon scattering with a coupled-channel T-matrix model that includes any appropriate resonances.*

V. Localization of poles in complex kaon-pion energy plane (Riemann sheets) allows clean separation of resonance from pion-kaon background contributions.

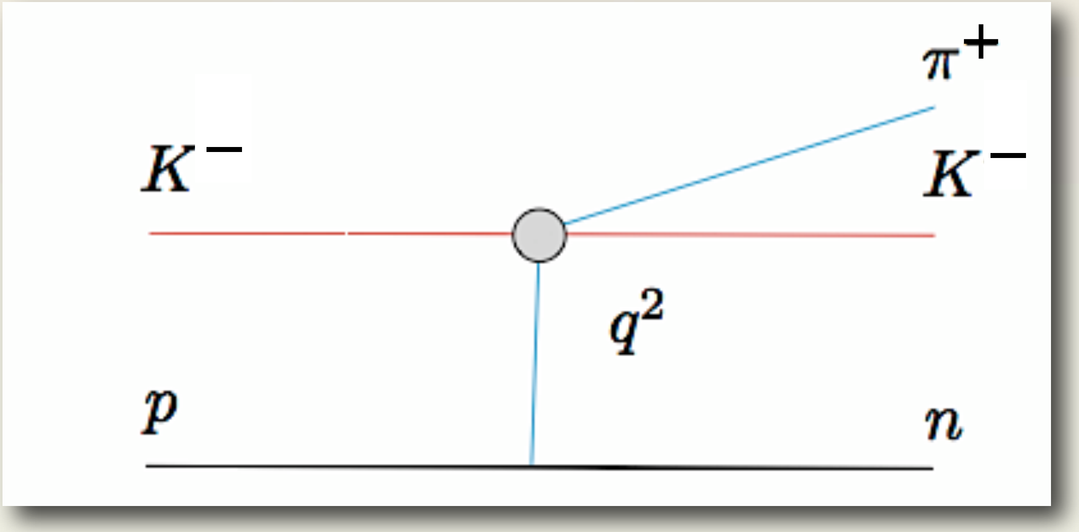
VI. Make use of the form factors to obtain the $K_0^*(1430)$ and $K^*(892)$ decay constants.

Scalar and vector kaons in experiment

(LASS, CLEO, Belle & BaBar)

Scalar and vector kaons in pion-kaon scattering

Knowledge about phase shifts and inelasticities from pion and kaon production experiments (LASS data)



High statistics experiments: Estabrooks (1979), Hyams (LASS 1988)

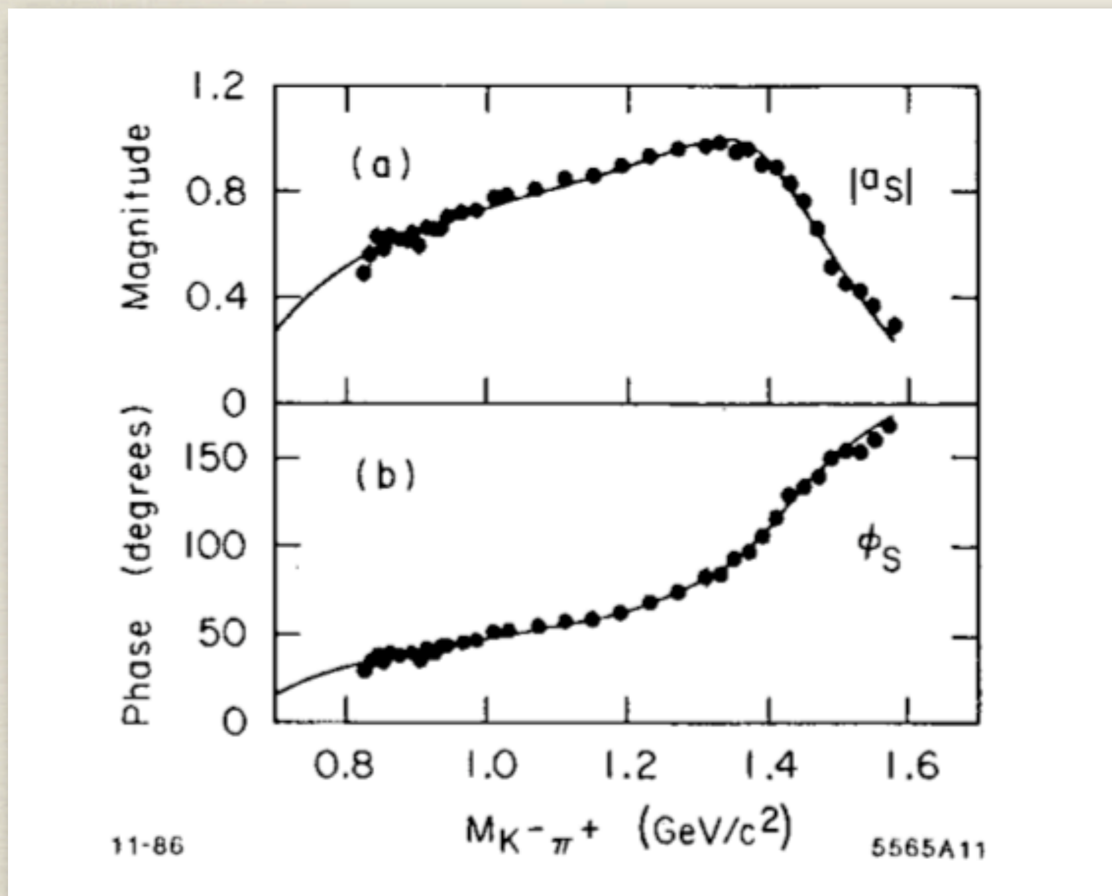
In the case of $\pi K \rightarrow \pi K$: δ_l and η_l determined for $l = 0,1, \ldots 5$

Energy domain: 0.8 GeV $< E <$ 2.5 GeV

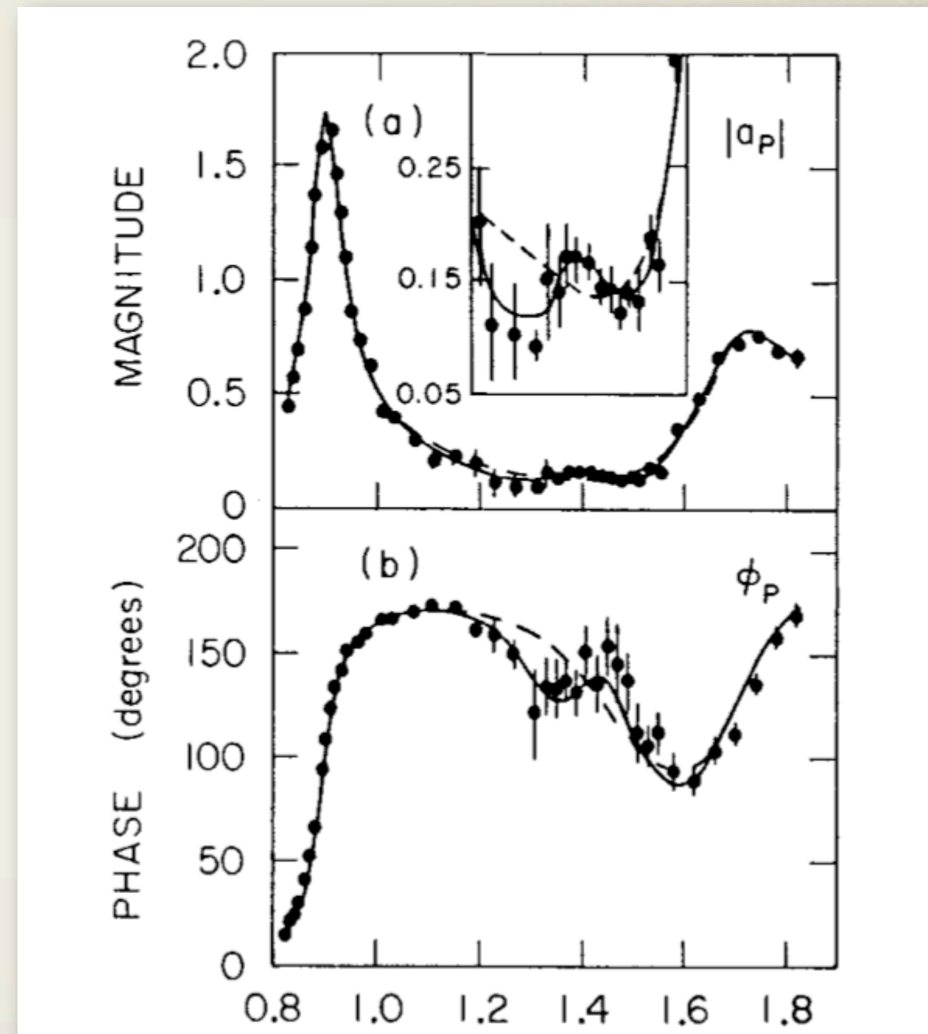
More recent data on $D \rightarrow \pi K \nu l$: $\delta_0 - \delta_1$ determined (FOCUS collaboration)

LASS data

D. Aston *et al.* in Nucl. Phys. **B296**, 493 (1988)



S-wave



P-wave

Emergence of vector kaons in D-decays

CLEO Collaboration, arXiv:1004.1954 [hep-ex] (2010)

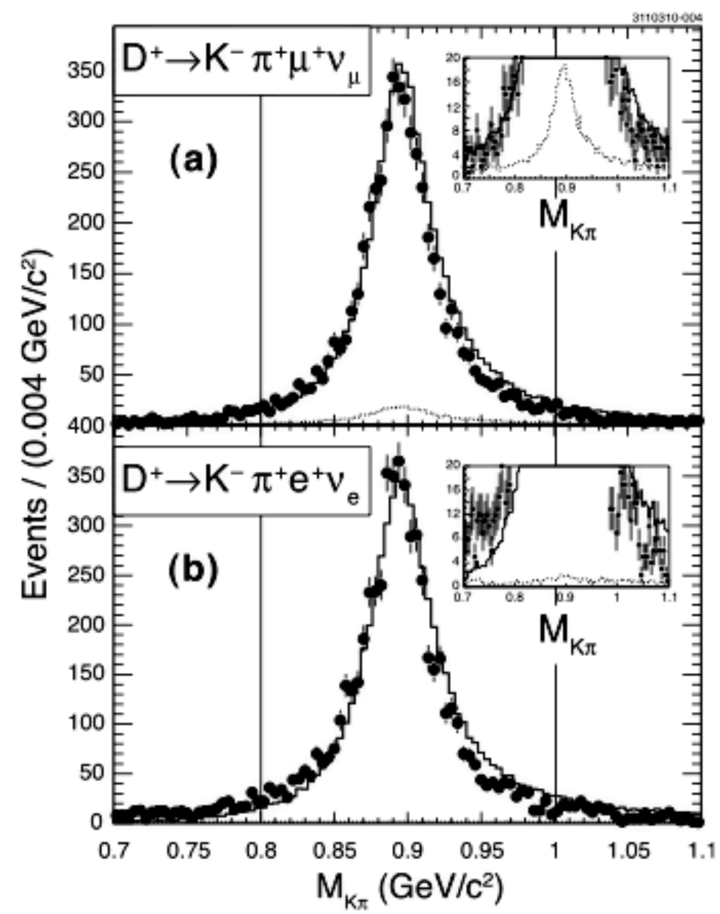
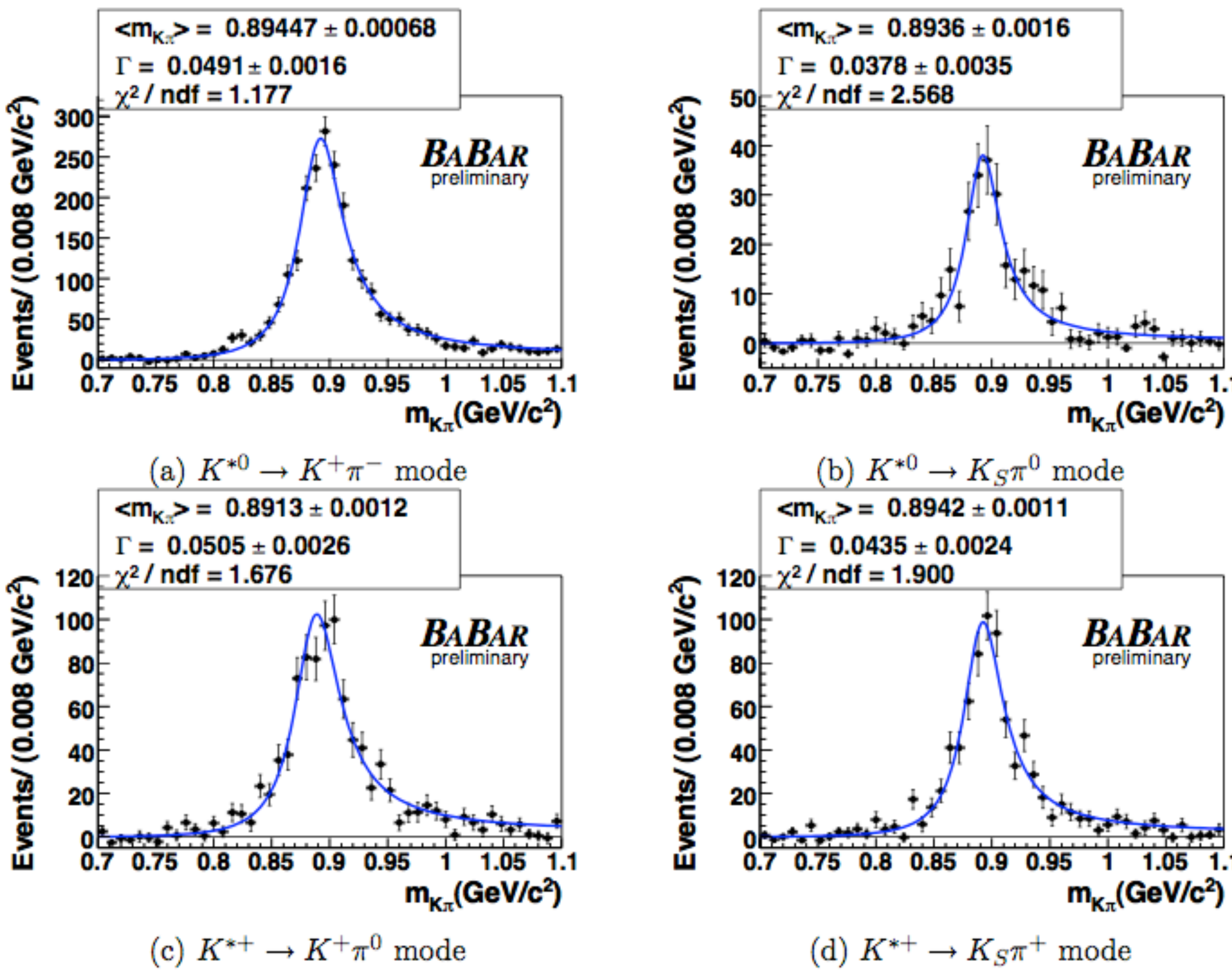


FIG. 4: The $m_{K\pi}$ distributions for events satisfying our nominal $D^+ \to K^-\pi^+\ell^+\nu_\ell$ selection requirements. (a) shows the $m_{K\pi}$ distribution for $D^+ \to K^-\pi^+\mu^+\nu_\mu$ candidates, while (b) shows the $m_{K\pi}$ distribution for $D^+ \to K^-\pi^+e^+\nu_e$ candidates. Over the full displayed mass range, there are 11 801 (6 227 semielectric and 5 574 semimuonic) events satisfying our nominal selection. For this analysis, we use a restricted mass range from 0.8 – 1.0 GeV$/c^2$, which is the region between the vertical lines. In each plot, the solid histogram shows the signal plus background distribution predicted by our Monte Carlo simulation, while the dashed histogram shows the predicted background component. In this restricted region, there are 10865 (5 658 semielectric and 5 207 semimuonic) events. The inserted figures are on a finer scale to better show the estimated background contributions.

K^* in exclusive decay $B \to K^*\gamma$



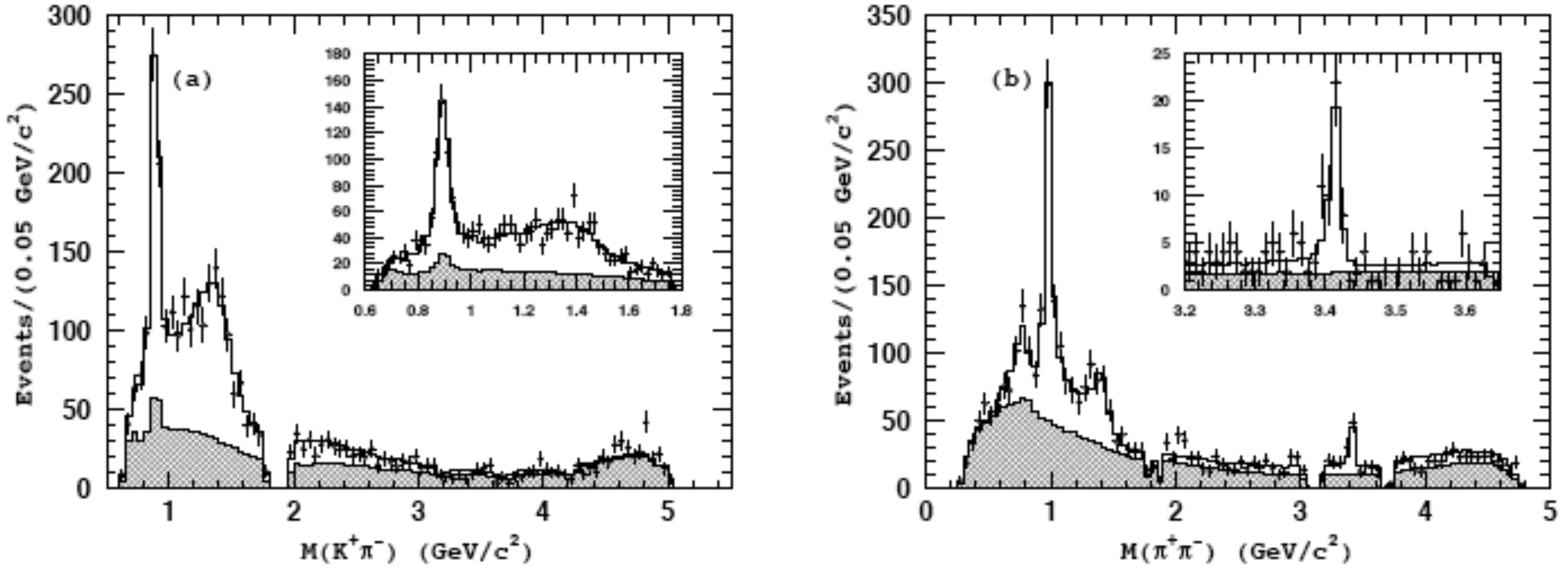
(a) $K^{*0} \to K^+\pi^-$ mode

(b) $K^{*0} \to K_S\pi^0$ mode

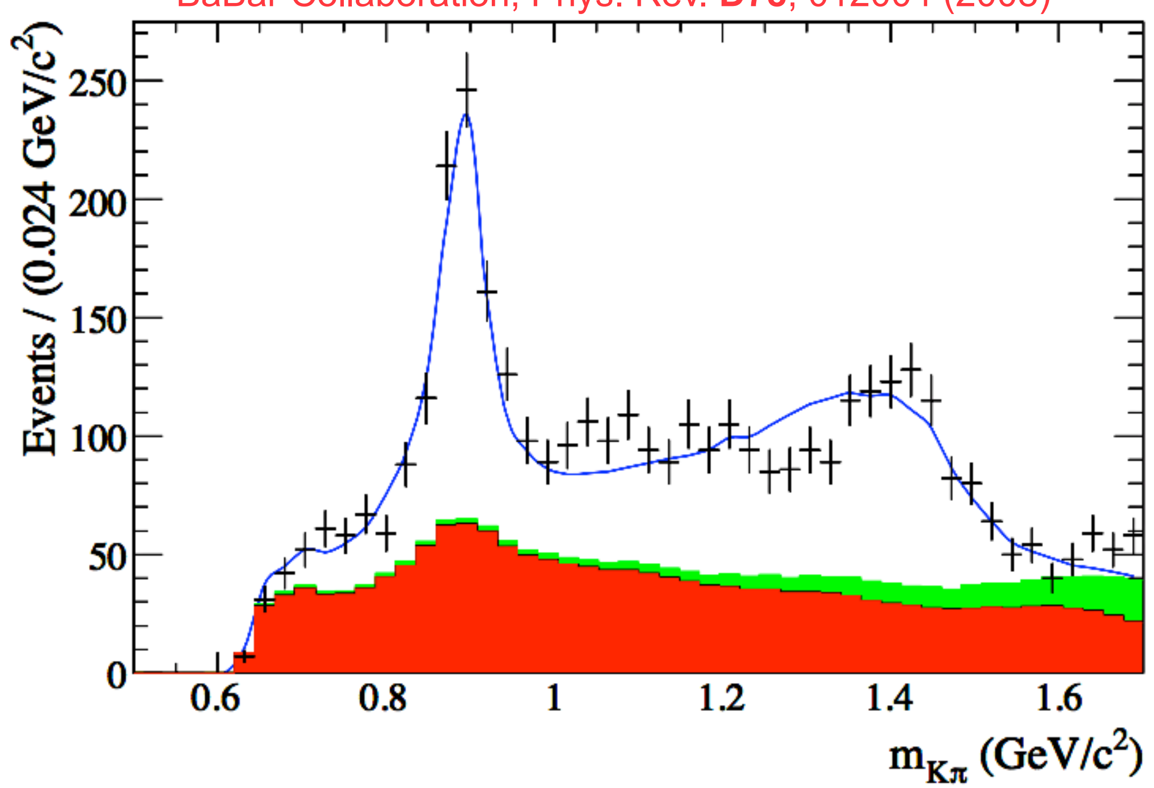
(c) $K^{*+} \to K^+\pi^0$ mode

(d) $K^{*+} \to K_S\pi^+$ mode

Belle Collaboration, hep-ex/0509001 (2005)



BaBar Collaboration, Phys. Rev. **D78**, 012004 (2008)



Weak decay amplitudes

Effective heavy-quark hamiltonian

&

QCD factorization

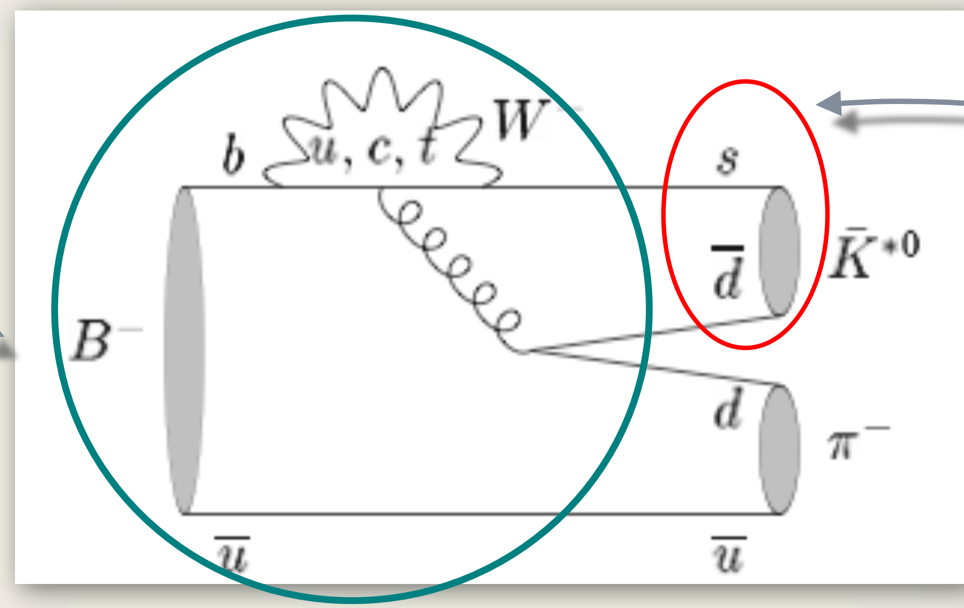
Quasi two-body approach

Examples of a *quasi two-body* decay:

$B^{\pm} \rightarrow K_0^*(1430)\pi^{\pm}$ followed by the hadronic decay

$K_0^*(1430) \rightarrow (K^{\pm}\pi^{\mp})_S$ in an *S-wave, I=1/2* state.

Weak decay amplitude,
short distance QCD corrections



Hadronic scalar and vector
form factors

➡ *no tree digrams, only QCD and electroweak penguins*

B. E. , A. Furman, R. Kamiński, L. Leśniak, B. Loiseau and B. Moussallam, Phys. Rev. D79, 094005 (2009)

Weak effective hamiltonian

Sum of local operators Q_i multiplied by short-range Wilson coefficients $C_i(\mu)$ and CKM matrix elements:

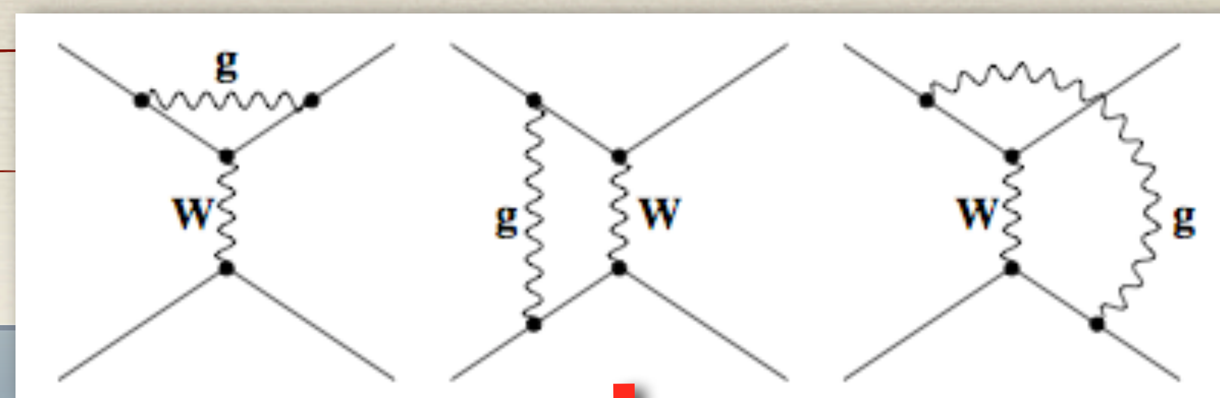
$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{us}^*(C_1(\mu)O_1^u + C_2(\mu)O_2^u) - V_{tb}V_{ts}^*\sum_{i=3}^{10} C_i(\mu)O_i\right]$$

O_1 and O_2 are left-handed current-current operators, for example:

$$O_1^u = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha \bar{u}_\beta \gamma_\mu (1-\gamma_5) b_\beta$$

O_3 …. O_{10} are QCD and electroweak penguin operators, for instance:

$$O_4 = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) b_\beta \sum_{q=u,d,s,c} \bar{q}_\beta \gamma_\mu (1-\gamma_5) q_\alpha$$



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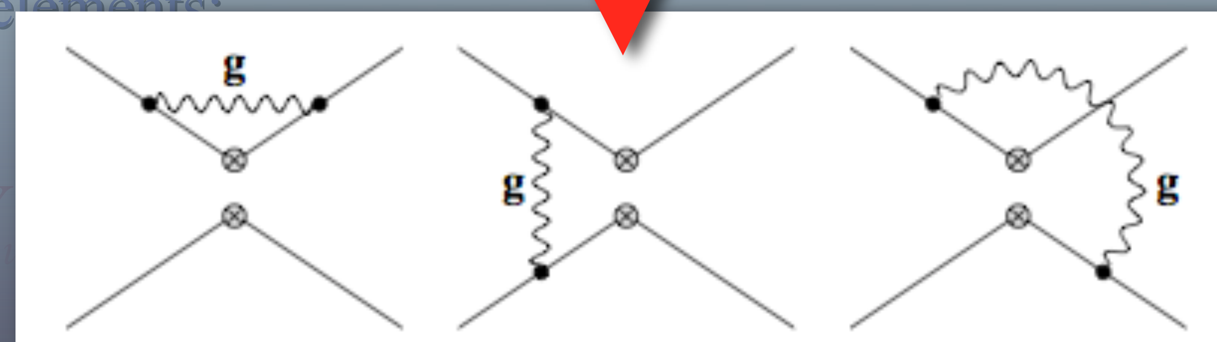

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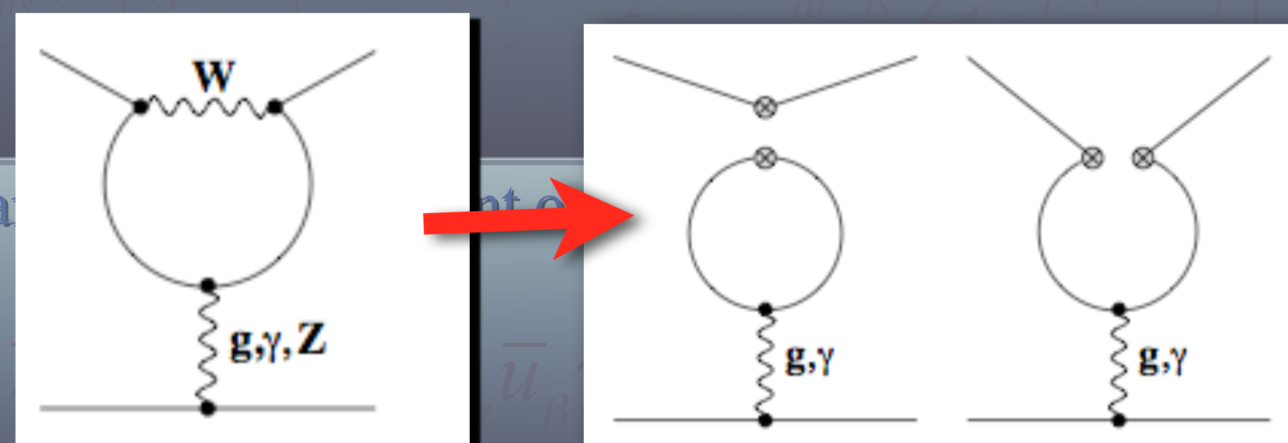

$O_1^u =$

$O_3 \ldots O_{10}$ are QCD and electroweak penguin operators, for instance:

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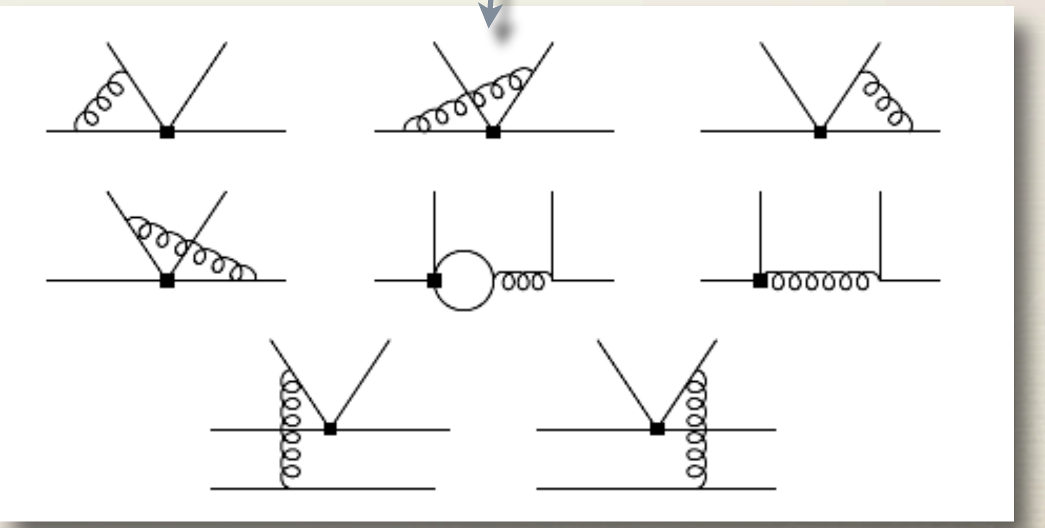
QCD factorization (two body)

Beneke, Buchalla, Neubert, Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000);
Nucl. Phys. B 606, 245 (2001); *Beneke & Neubert*, Nucl. Phys. B 675, 333 (2003).

$$\langle M_1 M_2 | Q_k(\mu) | B \rangle \sim \langle M_2 | J_1 | 0 \rangle \otimes \langle M_1 | J_2 | B \rangle \times \left[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\mathbf{QCD}}/m_b) \right]$$

Radiative vertex corrections
and hard gluon exchange
with spectator quark

Decay constant
(mostly known experimentally)



Hadronic transition form factor;
estimated with QCD sum rules,
lattice QCD, quark models ...

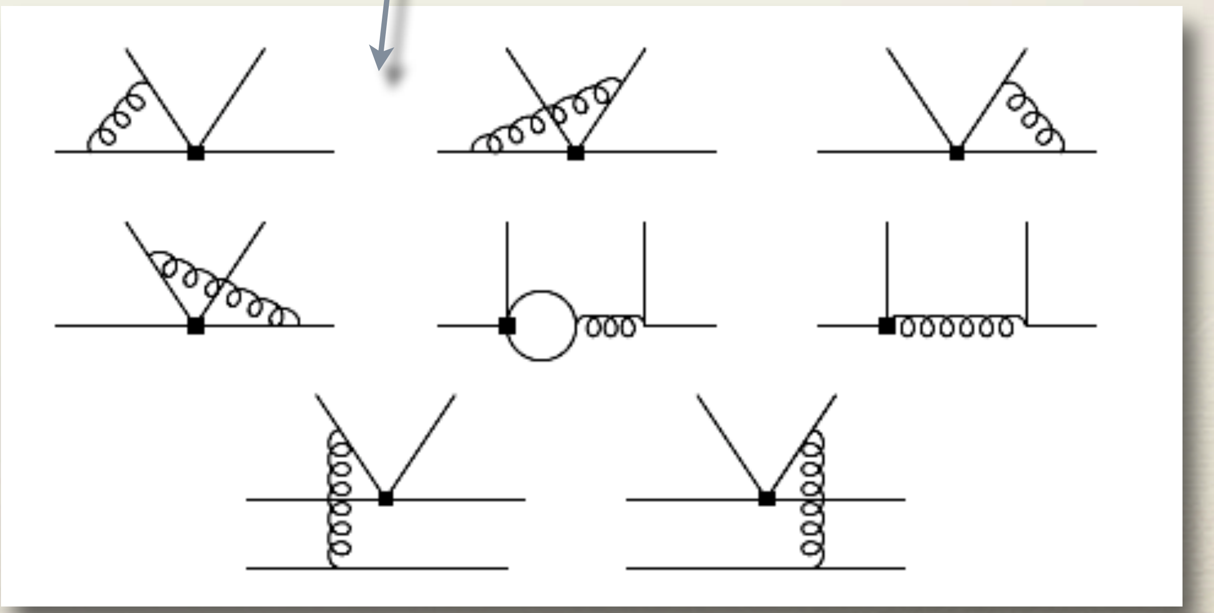
QCD factorization (quasi-two body)

B. E. , A. Furman, R. Kamiński, L. Leśniak, B. Loiseau and B. Moussallam, Phys. Rev. D79, 094005 (2009)

$$\langle (M_1 M_2)_{S,P} M_3 | Q_k(\mu) | B \rangle \sim \langle (M_1 M_2)_{S,P} | J_1 | 0 \rangle \otimes \langle M_3 | J_2 | B \rangle \times \left[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\mathbf{QCD}}/m_b) \right]$$

Radiative vertex corrections and hard gluon exchange with spectator quark

Scalar or vector form factor; their definition allows for inclusion of pion-pion and kaon-pion form factors and the calculation of a "resonance decay constant"



Strange scalar & vector form factors

Definition of scalar and vector πK form factors

The hadronic matrix element that describes the creation of a kaon-pion pair can be written in terms of the usual Lorentz invariants:

$$\langle K^-(p_{K^-})\pi^+(p_{\pi^+})|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle = f_+^{K^-\pi^+}(q^2)\ (p_{K^-}+p_{\pi^+})_\mu + f_-^{K^-\pi^+}(q^2)\ (p_{K^-}-p_{\pi^+})_\mu$$

with $t=(p_k-p_\pi)^2$ and which can be re-expressed in terms of a scalar $F_1\,(t=q^2)$ and a vector $G_1\,(t=q^2)$ form factor, such as:

in *S*- and *P*-wave

$$\langle K^-(p_{K^-})\pi^+(p_{\pi^+})|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle =$$
$$= \left[(p_{K^-}-p_{\pi^+})_\mu - \frac{m_K^2-m_\pi^2}{q^2}q_\mu\right] G_1^{K^-\pi^+}(q^2) + \frac{m_K^2-m_\pi^2}{q^2}q_\mu\, F_1^{K^-\pi^+}(q^2)$$

Two sets of form factors are related by: $F_1(t)=\sqrt{2}\left[f_+^{K^-\pi^+}(t)+\frac{t}{m_K^2-m_\pi^2}f_-^{K^-\pi^+}(t)\right]$

$$G_1(t)=\sqrt{2}f_+^{K^-\pi^+}(t)$$

These form factors also appear in semileptonic decays $\tau\to K\pi\nu_\tau$ and $K\to\pi\ell\nu_\ell$

B. Moussallam, Eur.Phys.J.C53:401-412 (2008); M. Jamin, J. Oller & A. Pich, Nucl.Phys.B622:279-308 (2002).

Conditions and constraints on scalar/vector form factors

- Limitation to domains of Dalitz plots in which the the $\pi\, K^*$ interaction with the (collinear pion) is suppressed.
- The form factors $F_1(q^2)$ and $G_1(q^2)$ are analytical functions in the complex plane with a cut along the real axis $(m_\pi + m_K)^2 \leq t < \infty$.
- Construction of a unitary, time-reversal invariant scattering matrix parametrization which reproduces LASS data in the energy range 0.9 GeV ≲ E ≲ 2.5 GeV.
- Inelasticity in S-wave scattering sets in at $K\eta'$ threshold and is saturated by this channel; three observable quantities: $\eta_{K\pi}, \delta_{K\pi}$ and $\delta_{K\eta'}$

$$S = \begin{pmatrix} \eta_{K\pi} e^{2i\delta_{K\pi}} & \sqrt{1-(\eta_{K\pi})^2} e^{i(\delta_{K\pi}+\delta_{K\eta'})} \\ \sqrt{1-(\eta_{K\pi})^2} e^{i(\delta_{K\pi}+\delta_{K\eta'})} & \eta_{K\pi} e^{2i\delta_{K\eta'}} \end{pmatrix}$$

$$S_{mn} = \delta_{mn} + 4i \left(\frac{q_m(s) q_n(s)}{s} \right)^{\frac{1}{2}} T_{mn}; \quad m,\ n = 1,\ 2 \text{ with } s = (p_K + p_\pi)^2 \equiv m_{K\pi}^2$$

- The P-wave is dominated by the $K^*\pi$ and $K^*\rho$ channels $\Rightarrow$ 3×3 T-matrix.
- At *low energies*, 0.9 GeV≲ E, constraints from χPT and Roy-Steiner Eqns. (unitarity + crossing).

Dominant inelastic channels for $E \lesssim 2.5$ GeV: **LASS (1987), LASS (1984)**

$- l = 0$	$K\eta'$ dominant	
$- l = 1$	$K^*\pi$	via $K^*(1410)$
	$K^*\pi,\ K\rho$	via $K^*(1680)$

Remark: little $l = 1$ coupling via resonances in $K\eta$ and $K\eta'$ $\Rightarrow$ *two* more form factors:

$$\langle K^{*+}(p_V,\lambda)|\bar{u}\gamma_\mu s|\pi^0(p_\pi)\rangle = \epsilon_{\mu\nu\alpha\beta}\, e^{*\nu}(\lambda) p_V^\alpha p_\pi^\beta \; H_2(t)$$
$$\langle \rho^0(p_V,\lambda)|\bar{u}\gamma_\mu s|K^-(p_K)\rangle = -\epsilon_{\mu\nu\alpha\beta}\, e^{*\nu}(\lambda) p_V^\alpha p_K^\beta \; H_3(t)$$

Mushkelishvili-Omnès Equations

Analyticity and asymptotic conditions: ***dispersion relation without subtraction***

$$\mathrm{Re}\ F_1(t) = \frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} \frac{\mathrm{Im}\ F_1(t')}{t'-t}\,dt'$$ (scalar form factor)

$$\mathrm{Re}\ G_1(t) = \frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} \frac{\mathrm{Im}\ G_1(t')}{t'-t}\,dt'$$ (vector form factor)

Unitarity equations and T-invariance:

$$\mathrm{Im}\, F_m(t) = \frac{1}{2}\sum_n T^*_{mn}(t) F_n(t) \qquad \mathrm{Im}\, G_m(t) = \frac{1}{2}\sum_n T^*_{mn} G_n(t)$$

with the approximation of the truncation $|n\rangle = |K\pi\rangle, |K\eta'\rangle$ for the scalar $F_1(t)$ and $|n\rangle = |K\pi\rangle, |K^*\pi\rangle, |K\rho\rangle$ for the vector $G_1(t)$.

Some technical remarks

- The form factors satisfy a set of n coupled, homogeneous singular integral equations with a kernel linear in the T-matrix ($n = 2$ for S-wave and $n = 3$ for P-wave).

- The number of independent solutions N is given by the index of the integral operator which can be expressed in terms of the sum of the eigenphases $\delta_j(t)$ of the S-matrix:

$$\sum_{j=1}^{n}[\delta_j(\infty) - \delta_j(0)] = N\pi$$

- N is the number of independent conditions that one must impose on the form factors in order to determine them from the integral equations.

- Impose chiral symmetry conditions at $t = 0$ and near $t = 0$ and asymptotic condition.

Chiral symmetry constraints in *S*-wave

$t = 0$

$$F_1(0) = 0.961, \quad F_1(m_K^2 - m_\pi^2) = \frac{f_K}{f_\pi} - 3.1 \times 10^{-3}$$

Cheng-Dashen point

Test variations

* $t = 0$ Chiral symmetry constraints in *P*-wave

$G_1(0)$: $= 1 + O(\,(m_s - m)^2\,)$ Ademollo-Gatto (1964)

$= 0.987 + O(\,(m_s - m)^3\,)$ Gasser-Leutwyler(1985)

$H_2,\ H_3$: $H_2(0) = (1.54 \pm 0.08)\ GeV^{-1}$ Chiral limit $(\,\rho^+ \to \gamma\pi^+\,)$

$H_3(0) = (-1.54 \pm 0.08\ GeV^{-1}$

$H_2(0) = (1.41 \pm 0.09 - 65.4\,a)\ GeV^{-1}$ with $O(m_s, \hat{m})$ correc.

$H_3(0) = (-1.34 \pm 0.07 - 65.4\,a)\ GeV^{-1}$ a= $O(10^{-3})$

* $t = \infty$ QCD, Brodsky-Lepage (1980)

$$G_1(-Q^2)\Big|_{Q^2\to\infty} \sim \frac{16\pi\sqrt{2}\,\alpha_s(Q^2)F_\pi^2}{Q^2}$$

Pole contributions on 2nd Riemann sheet and bound states

We want to define the extrapolation of the scalar form factor to the 2nd Riemann sheet in t.
Scattering is elastic up to the $K\eta'$ threshold. The discontinuity across the cut is:

$$F_1^{K\pi}(t+i\epsilon) - F_1^{K\pi}(t-i\epsilon) \;=\; -2\sigma_{\pi K}(t+i\epsilon)T_{11}^{S,P}(t+i\epsilon)F_1^{K\pi}(t-i\epsilon)$$

$$\text{with} \;\; \sigma_{\pi K}(t) \;=\; 1/t\sqrt{((m_K+m_\pi)^2-t)(t-(m_K-m_\pi)^2)}$$

$$T_{11}^{S,P}(t+i\epsilon) - T_{11}^{S,P}(t-i\epsilon) = -2\sigma_{\pi K}(t+i\epsilon)T_{11}^{S,P}(t+i\epsilon)T_{11}^{S,P}(t-i\epsilon)$$

This allow us to find the extension of $F_1^{K\pi}$ on the 2nd Riemann sheet : $F_1^{\mathrm{II}}(t) = \dfrac{F_1^{K\pi}(t)}{1-2\sigma_{\pi K}(t)T_{11}^{S}(t)}$

which, by definition, must satisfy $F_1^{\mathrm{II}}(t-i\epsilon) = F_1^{K\pi}(t+i\epsilon)$ along the cut.

$D(t) = 0 \Rightarrow$ pole

$$\boxed{F_1^{\mathrm{pole}}(t) = \frac{F_1^{K\pi}(t_0)}{\alpha\,(t-t_0)} \qquad \alpha = dD(t)/dt|_{t=t_0}}$$

Pole positions

S-wave

Numerically, the location of the pole is found to be:

$$t_0 = (1.9487 - i\,0.3825)\ \text{GeV}^2, \quad \sqrt{t_0} = (1.4026 - i\,0.1364)\ \text{GeV}.$$

Results compare reasonably well with the values of the mass $M_R = (1.414 \pm 0.006)$ GeV and of the half-width $\Gamma_R/2 = (0.145 \pm 0.011)$ GeV of the $K^*(1430)$ given by the Particle Data Group. The other quantities needed for $F_1^{\text{pole}}(t)$ are

$$F_1^{K\pi}(t_0) = -0.3242 - i\,1.4679, \quad \alpha = (0.8381 + i\,1.1713)\ \text{GeV}^{-2}.$$

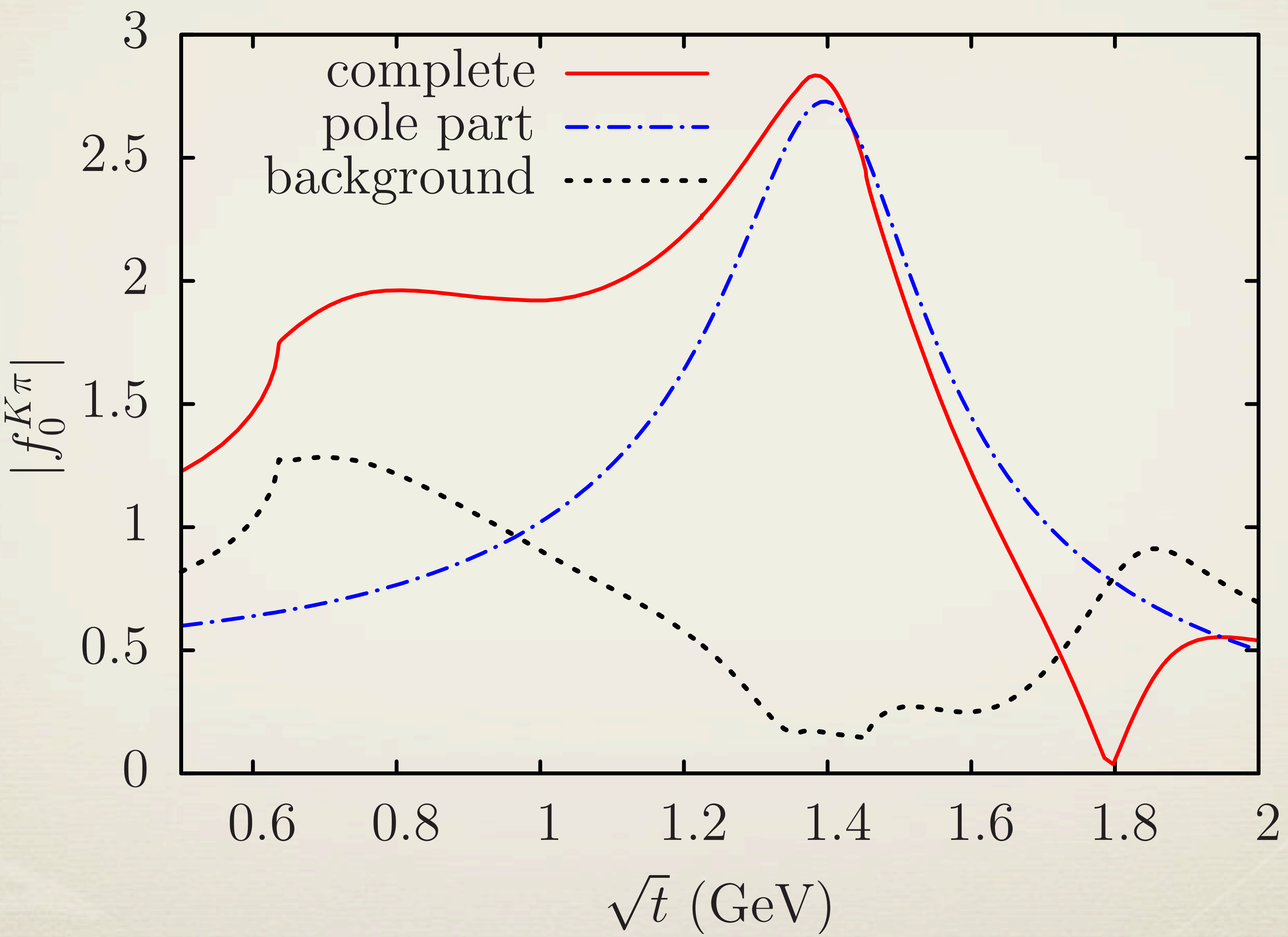
P-wave

In the case of the $K^*(892)$ the values for the pole parameters are

$$t_1 = (0.7982 - i\,0.0504)\ \text{GeV}^2, \qquad \sqrt{t_1} = (0.8939 - i\,0.0282)\ \text{GeV},$$
$$\alpha = (-1.8874 + i\,9.5726)\ \text{GeV}^2, \qquad G_1^{K\pi}(t_1) = 0.8244 - i\,9.0784.$$

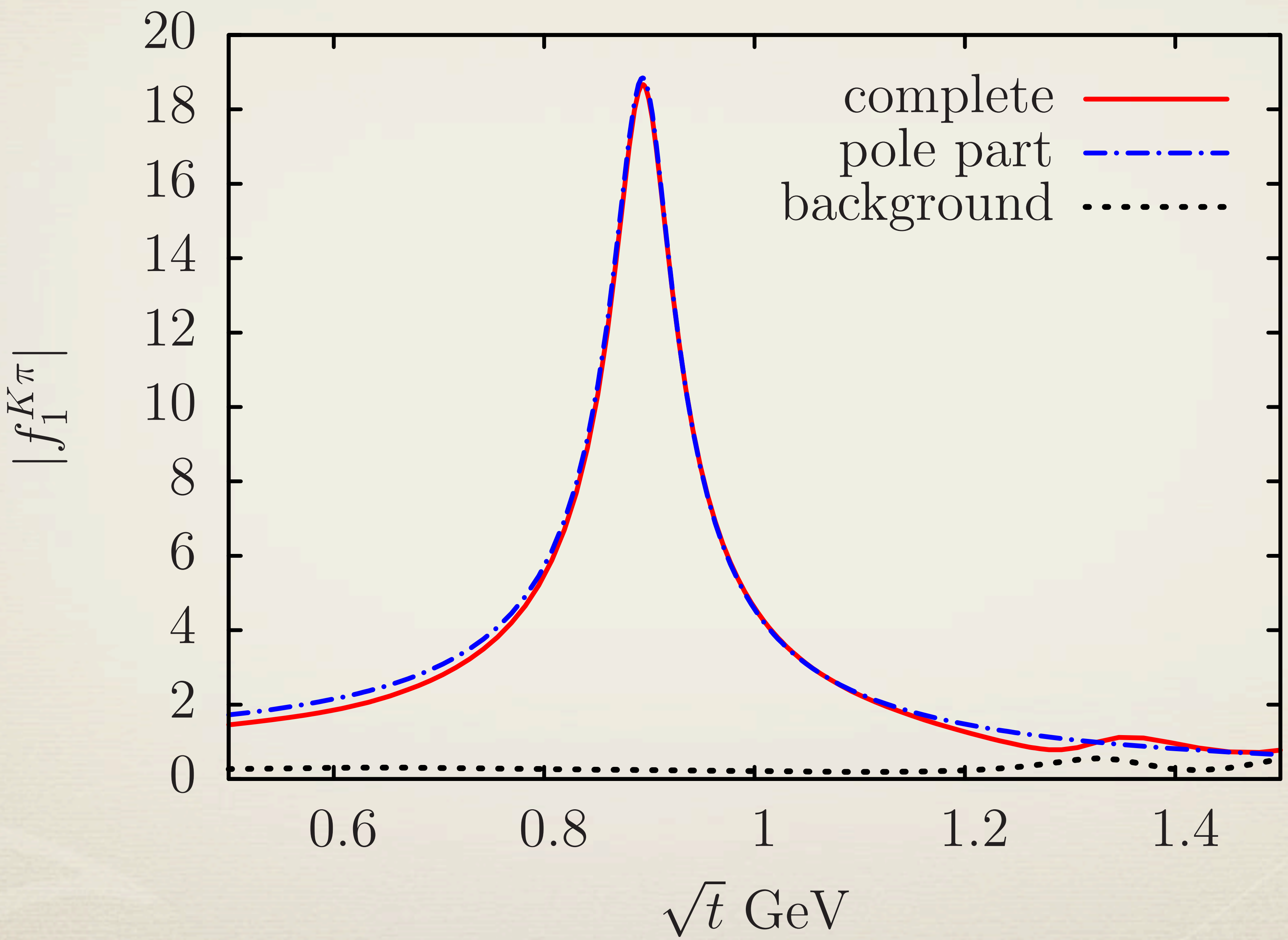
Modulus of the *scalar* form factor $F_1^{K\pi}$ compared with its pole contribution

$$F_1^{\text{back}}(t) = F_1^{K\pi}(t) - F_1^{\text{pole}}(t)$$



Modulus of the *vector* form factor $G_1^{K\pi}$ compared with its pole contribution

$$F_1^{\mathrm{back}}(t) = F_1^{K\pi}(t) - F_1^{\mathrm{pole}}(t)$$



Determination of $f_{K_0^*}$ and f_{K^*} in the complex pole approach

Follow definition by Maltman : $\langle 0|J^{su}(x)|K_0^*(p)\rangle = f_{K_0^*}\, m^2_{K_0^*}\, \exp(-ipx)$

$$J^{su}(x) = \partial_\mu \bar{s}(x)\gamma^\mu u(x)$$

and find residue of the pole on the 2nd Riemann sheet of the two-point correlation function

$$\Pi^{us}(t) = i\int d^4x\, \exp(ipx)\langle 0|T[J^{su}(x)(J^{su})^\dagger(0)]|0\rangle$$

$$\Rightarrow \quad f_{K_0^*} = (31.3 + i\ 7.6)\ \mathrm{MeV}$$

The result is quasi real and comparable with the value obtained by Maltman: $f_{K_0^*} = 42.2$ MeV; Phys. Lett. B 462, 14 (1999).

Similarly, for the vector meson : $f_{K^*} \approx (213.9 - i\ 13.6)\ \mathrm{MeV}$

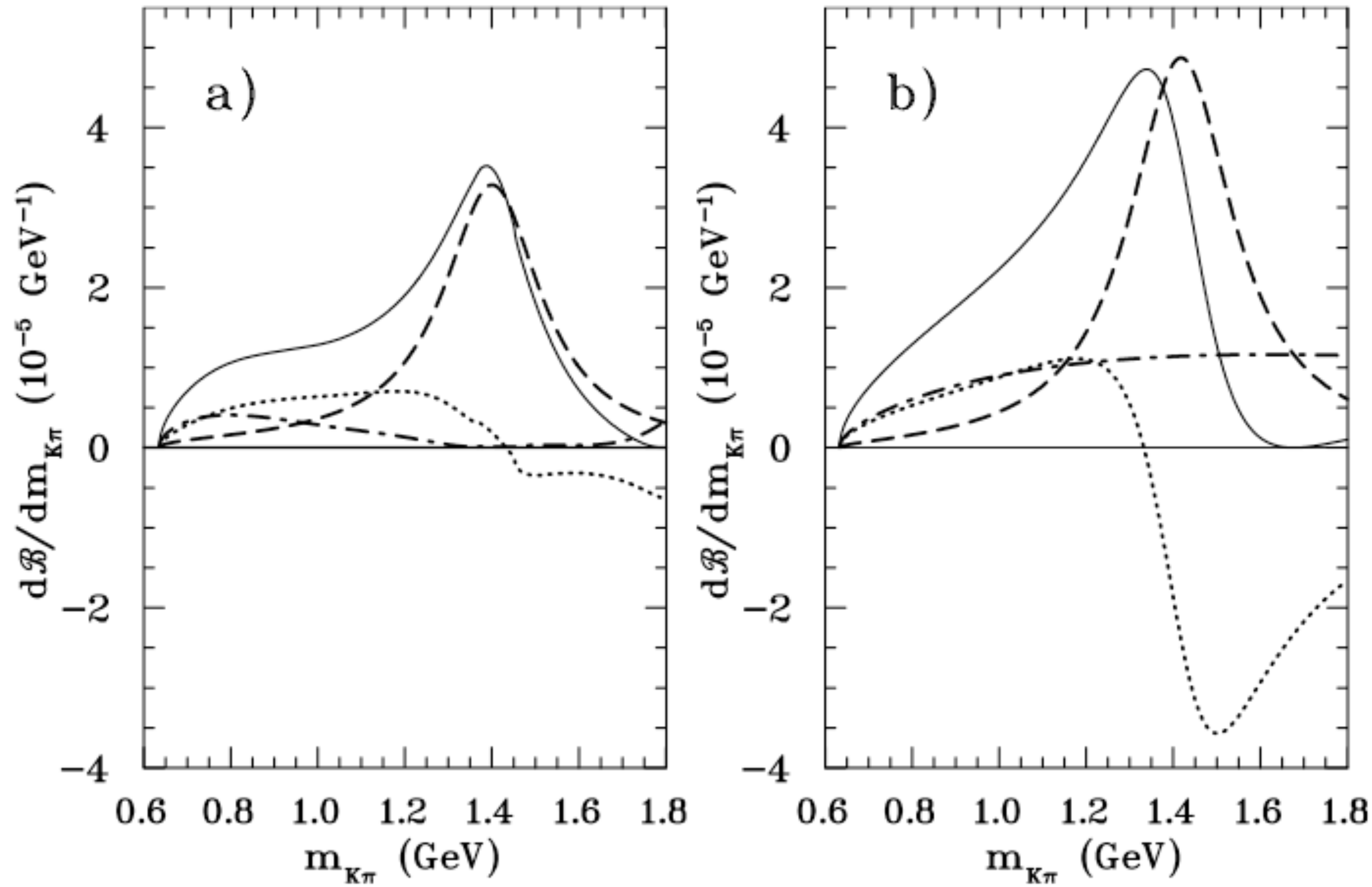


FIG. 13: Comparison of the different components of the averaged $m_{K\pi}$ distributions of the $B^{\pm} \to (K^{\pm}\pi^{\mp})_S\pi^{\pm}$ decays: a) our model, b) BaBar's LASS parametrization [11]. In this calculation our amplitude is proportional to the scalar $K\pi$ form factor but that of BaBar is the part proportional to the S-wave $K\pi$ T-matrix. The dashed lines correspond to the resonant $K_0^*(1430)$ contributions, the dotted-dashed lines to the background, dotted lines to the interference and the solid lines to their sum.

Conclusive Remarks

- Three-body decays of B (and D) mesons display a complex pattern of final-state interactions.
- Analyses of the Dalitz plots confirm the resonance spectrum in the invariant pion-kaon (and pion-pion) mass distributions observed by LASS.
- These resonances introduce strong phases in the weak decay amplitudes and important modifications in any CP-violating observables.
- It is therefore *crucial* to have a correct description of the FSI in which pole contributions and background are unambiguously separated with a unitary amplitude for S- and P-waves.
- Otherwise, the interplay of the pole, background and interference terms can be misinterpreted and some background be claimed to be a «resonance».