

# Extraction of $P_{11}$ resonances from $\pi N$ data and its stability

(arXiv:1001.5083)

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## Introduction

Extraction of  $N^*$  from  $\pi N$  data is important !

\* Understanding spectrum and structure of  $N^*$  within QCD

Steps to extract  $N^*$

1. Construct a reaction model through analysis of data
2. From the constructed model, resonance properties (pole position, vertex form factor) are extracted with analytic continuation

Extraction of  $N^*$  properties is inevitably model-dependent !

Several different approaches : EBAC, GWU/VPI, Jülich and more

Existence of some  $N^*$  is controversial

**Question !**

How much extracted  $N^*$  parameters depend on

1. model
2. precision of data

## What we study

(I) **Stability of pole structure** of the Roper resonance [ $N_{11}(1440)$ ] against

- \* Large variation of parameters of EBAC-DCC model

- \* Including bare nucleon state

- \* Variation of  $P_{11}$  amplitude for  $1.6 < W < 2$  GeV

(II) Dependence of higher mass  $P_{11}$  resonances on these variations

(III) Stability of residues of the poles

## Model and Method

EBAC-DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007)

Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

$$\{a, b, c\} = \gamma^{(*)}N, \pi N, \eta N, \pi\pi N(\pi\Delta, \sigma N, \rho N)$$

$$K\Lambda, K\Sigma, \omega N$$

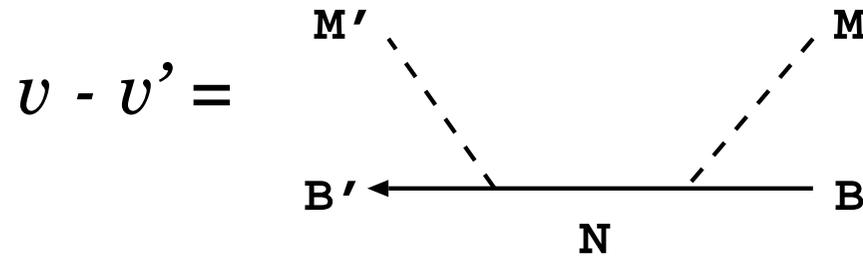
$$V_{ab} = v_{ab} + \sum_{N^*} \frac{\Gamma_{N^*,a}^\dagger \Gamma_{N^*,b}}{E - m_{N^*}^0}$$

# Bare Nucleon Model

Pearce and Afnan, PRC **34**, 991 (1986)

## \* Potentials

$$V_{ab} = v'_{ab} + \frac{\Gamma_{N,a}^\dagger \Gamma_{N,b}}{E - m_N^0} + \sum_{N^*} \frac{\Gamma_{N^*,a}^\dagger \Gamma_{N^*,b}}{E - m_{N^*}^0}$$



## \* Nucleon Pole Condition

- (i)  $T(E \sim m_N) \sim \frac{\bar{\Gamma} \bar{\Gamma}}{E - m_N}$
- (ii)  $\bar{\Gamma} = F_{\pi NN}^{\text{phys}}$

## Extraction of $N^*$ information with analytic continuation

Suzuki et al., PRC **79**, 025205 (2009)

$\pi N$  scattering amplitude near a pole ( $E \sim M_R$ )

$$F_{\pi N}(E) \sim \frac{\bar{\Gamma}(M_R) \bar{\Gamma}(M_R)}{E - M_R} + (\text{regular terms})$$

### Parameters characterizing Resonance

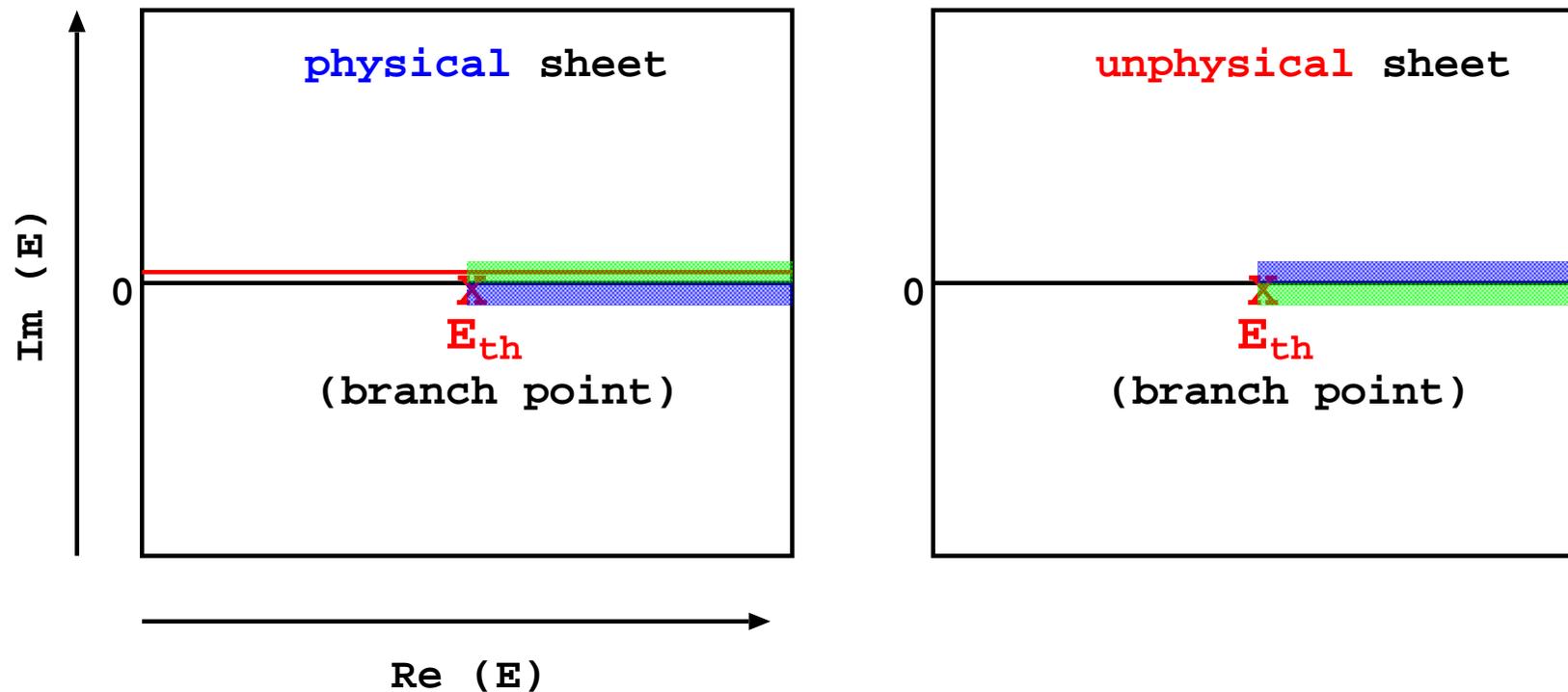
- \* Pole position of amplitude :  $M_R$
- \*  $N^* \rightarrow MB$  decay vertex :  $\bar{\Gamma}(M_R)$

## Multi-layered structure of scattering amplitudes

e.g., single-channel meson-baryon scattering

$$T(p', p; E) = V(p', p) + \int dq q^2 V(p', q) G(q, E) T(q, p; E)$$

Scattering amplitude is a **double-valued function of  $E$**  !

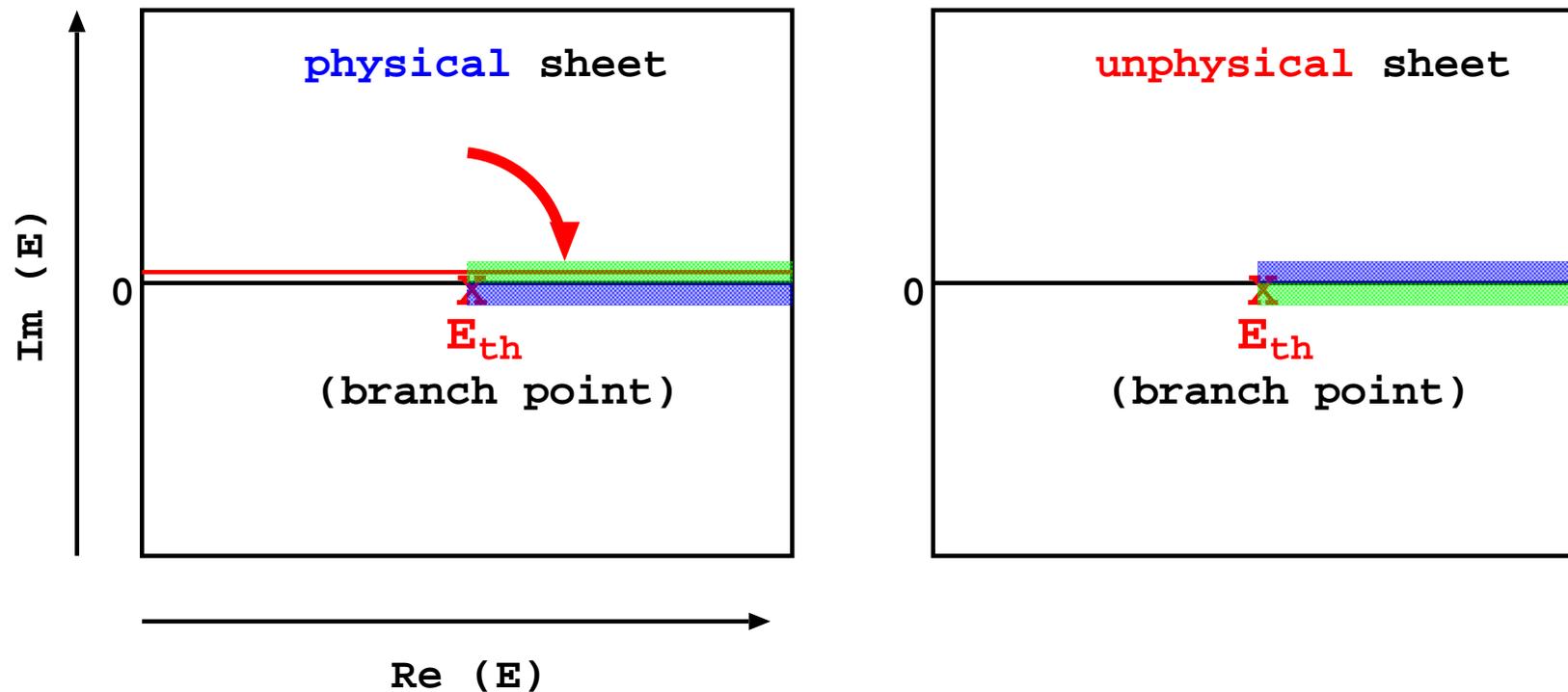


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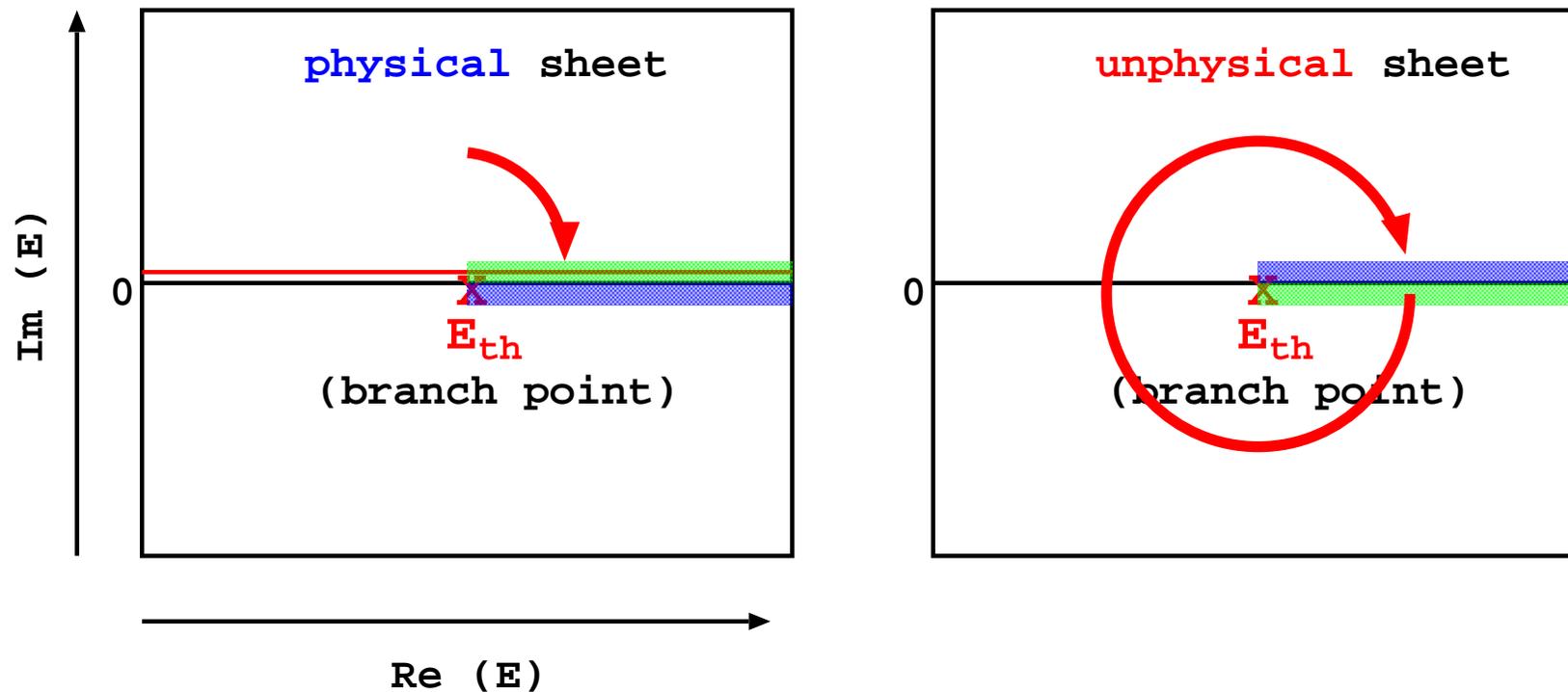


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$$T(p', p; E) = V(p', p) + \int dq q^2 V(p', q) G(q, E) T(q, p; E)$$

Scattering amplitude is a **double-valued function of  $E$**  !

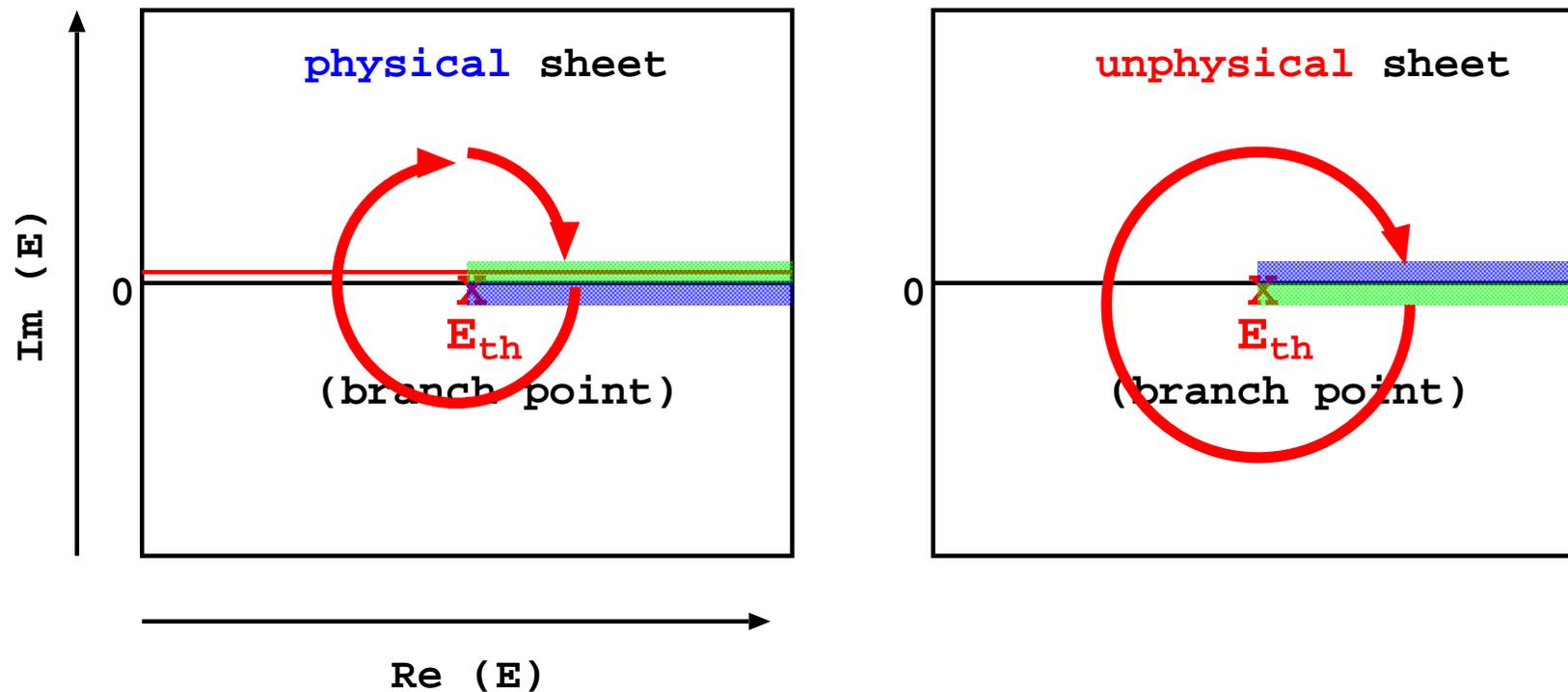


## Multi-layered structure of scattering amplitudes

e.g., single-channel meson-baryon scattering

$$T(p', p; E) = V(p', p) + \int_C dq q^2 V(p', q) G(q, E) T(q, p; E)$$

Scattering amplitude is a **double-valued function of  $E$**  !

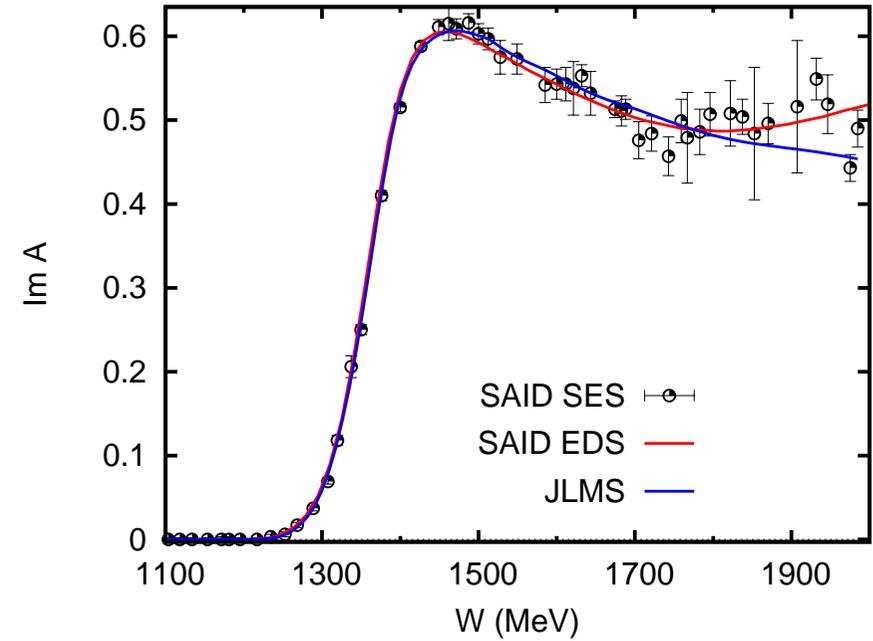
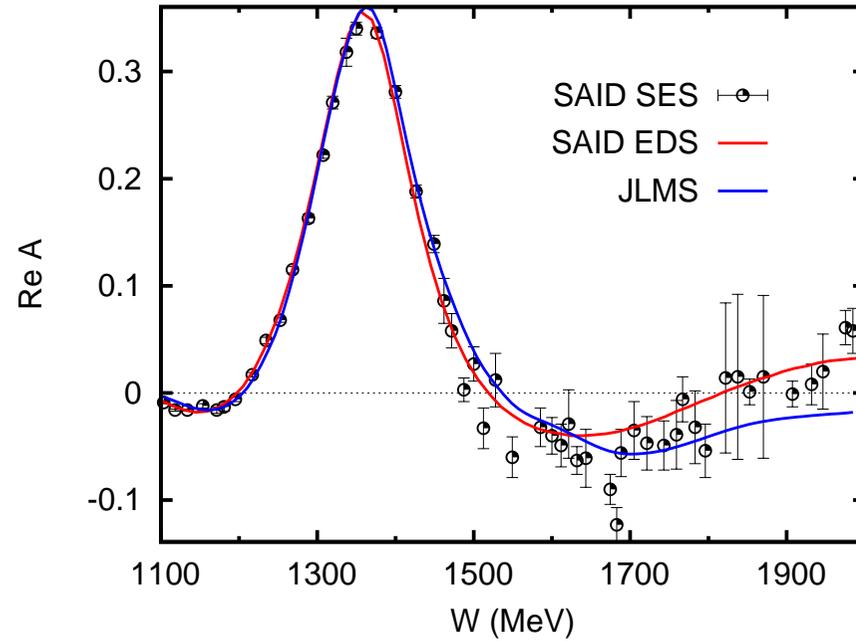


# Results 1 : Stability of pole positions

## Benchmarks

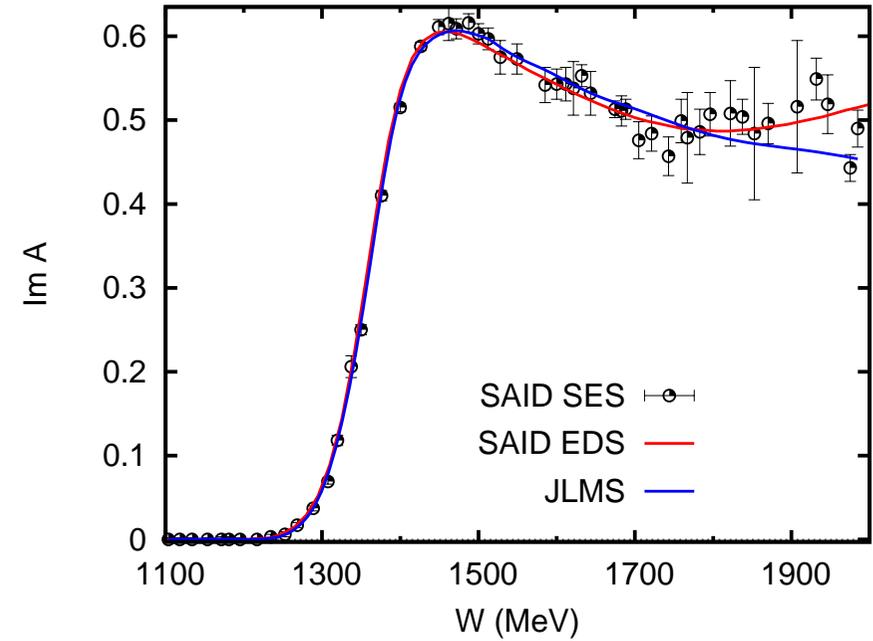
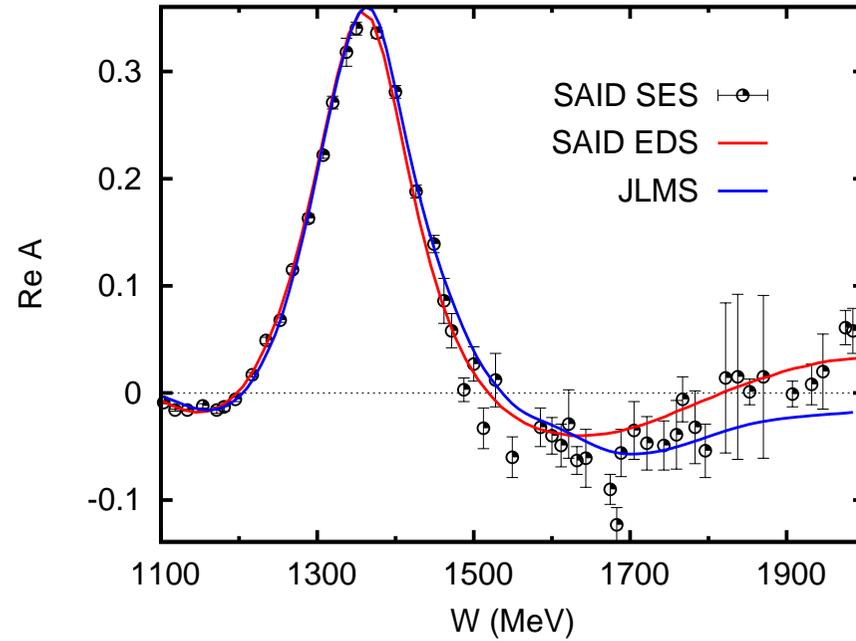
- \* SAID SES (single energy solution) (SP06)
  - ⇐ Parameters of all models are fitted
- \* SAID EDS (energy-dependent solution) (SP06)
- \* JLMS (Julía-Díaz et al., PRC **76**, 065201 (2007) )

\*  $P_{11}$  amplitude



Model	$upuupp$	$upuppp$	$uuuuup$	$\chi_{pd}^2$
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	2.94
JLMS	(1357, -76)	(1364, -105)	(1820, -248)	3.55

\*  $P_{11}$  amplitude



\* Pole positions

$$(s_{\pi N}, s_{\eta N}, s_{\pi\pi N}, s_{\pi\Delta}, s_{\rho N}, s_{\sigma N}) = (u, p, u, u, p, p)$$

## More Models 1

\*  $2N^*-3p$

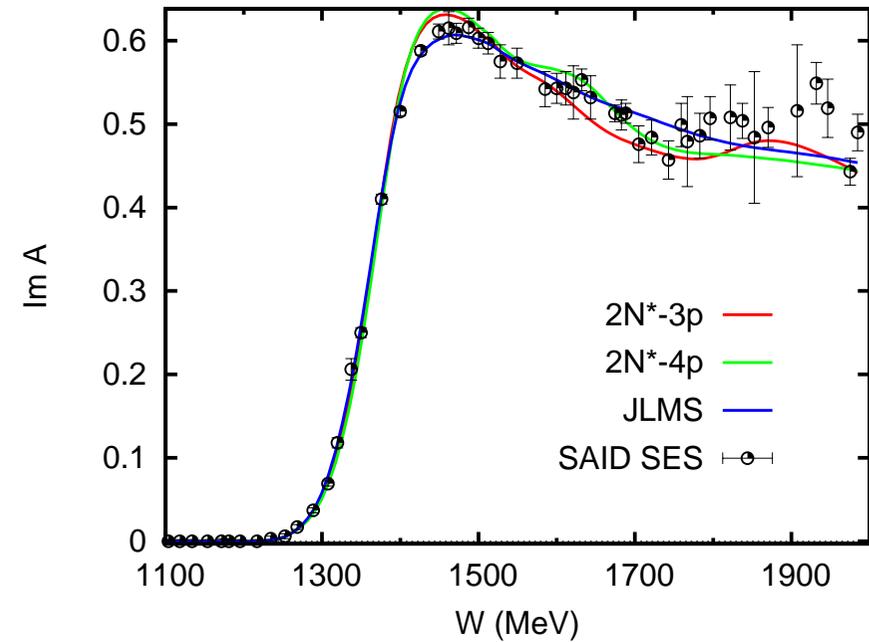
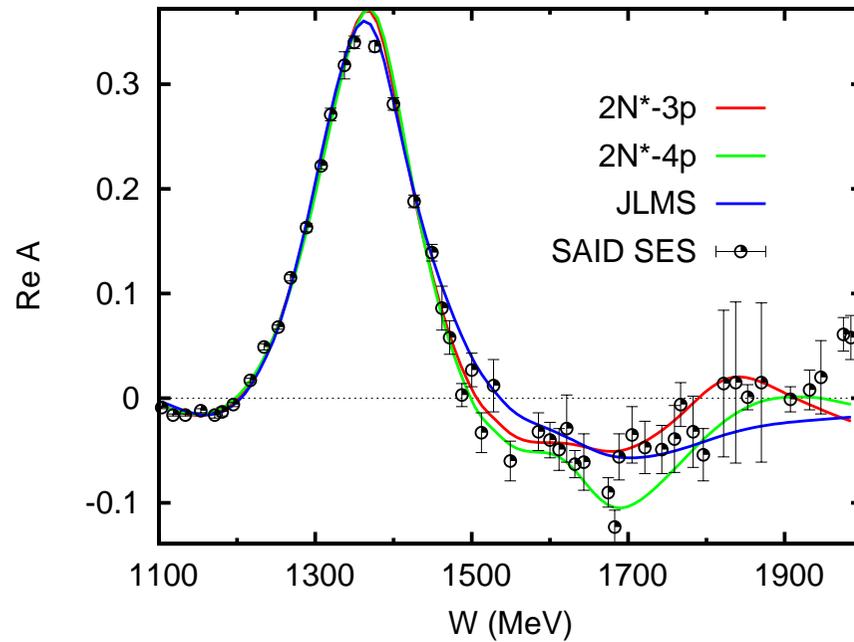
$v \neq v_{JLMS}$

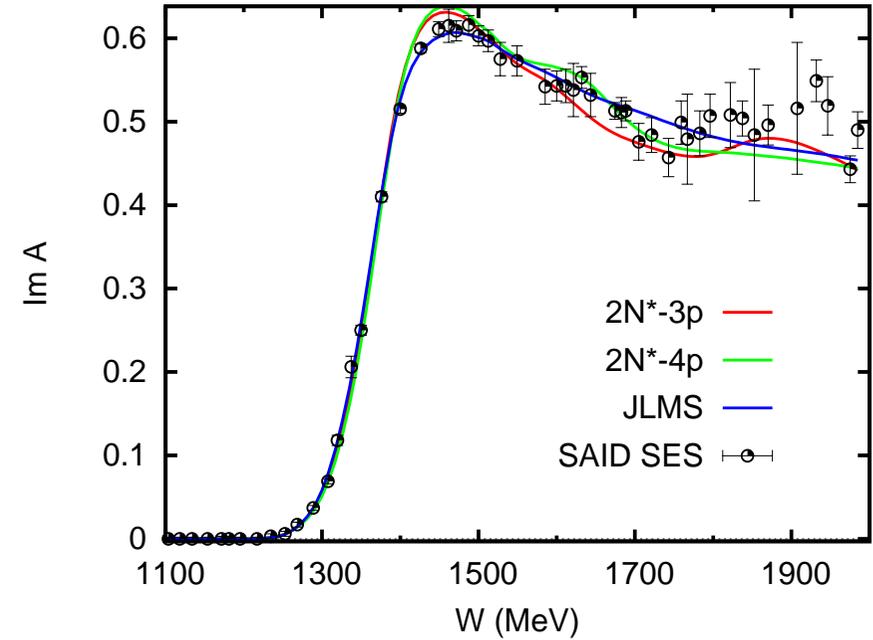
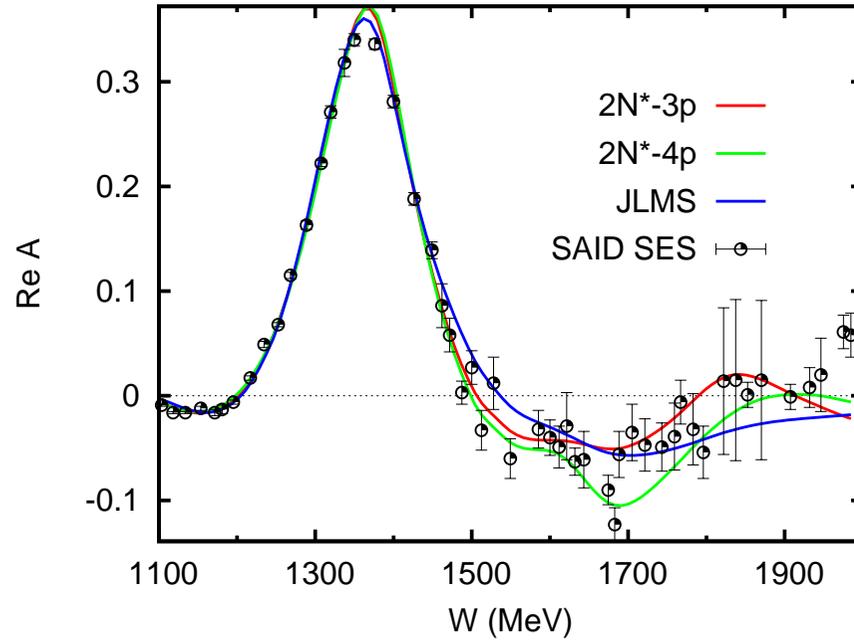
smooth fit

\*  $2N^*-4p$

$v \neq v_{JLMS}$

fitted to oscillated behavior





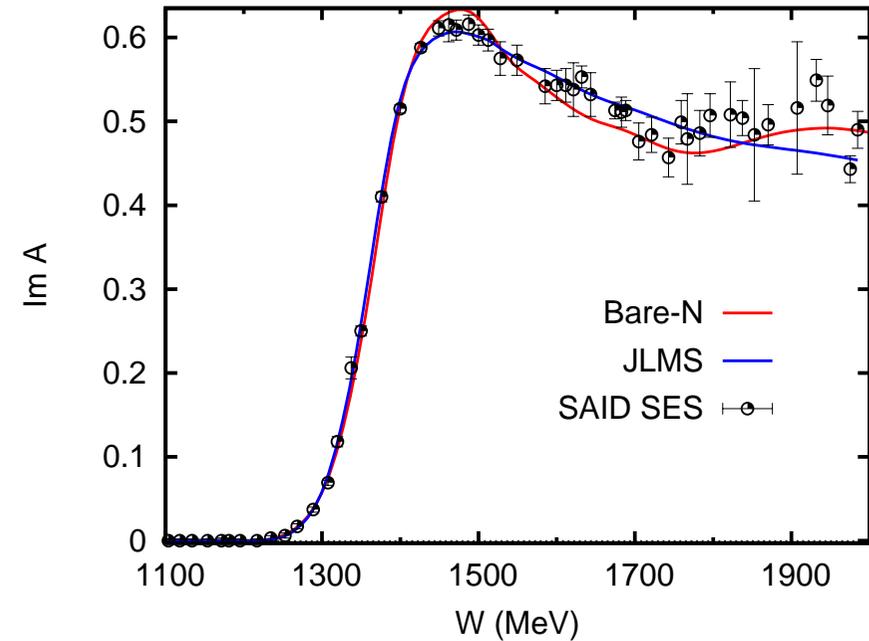
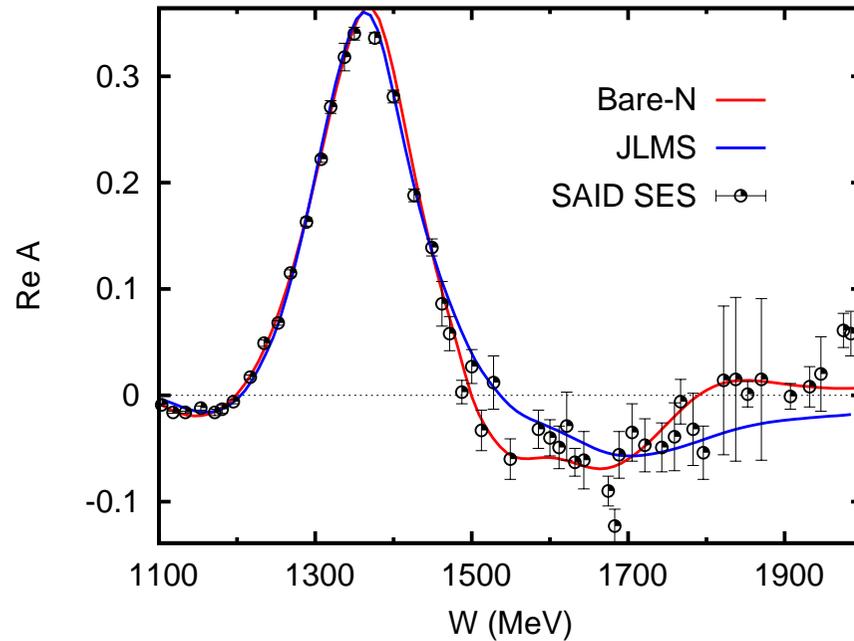
Model	$upuupp$	$upuppp$	$uuuupp$	$uuuuup$	$\chi_{pd}^2$
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	—	2.94
JLMS	(1357, -76)	(1364, -105)	—	(1820, -248)	3.55
$2N^*-3p$	(1368, -82)	(1375, -110)	—	(1810, -82)	3.28
$2N^*-4p$	(1370, -81)	(1384, -115)	(1635, -68)	(1960, -214)	3.36

\* Roper two poles are stable !

\* Additional pole in  $2N^* - 4p$  !

\* Different high  $W$  amplitude  $\Rightarrow$  Different higher mass pole position

## More Models 2 (Bare Nucleon Model)



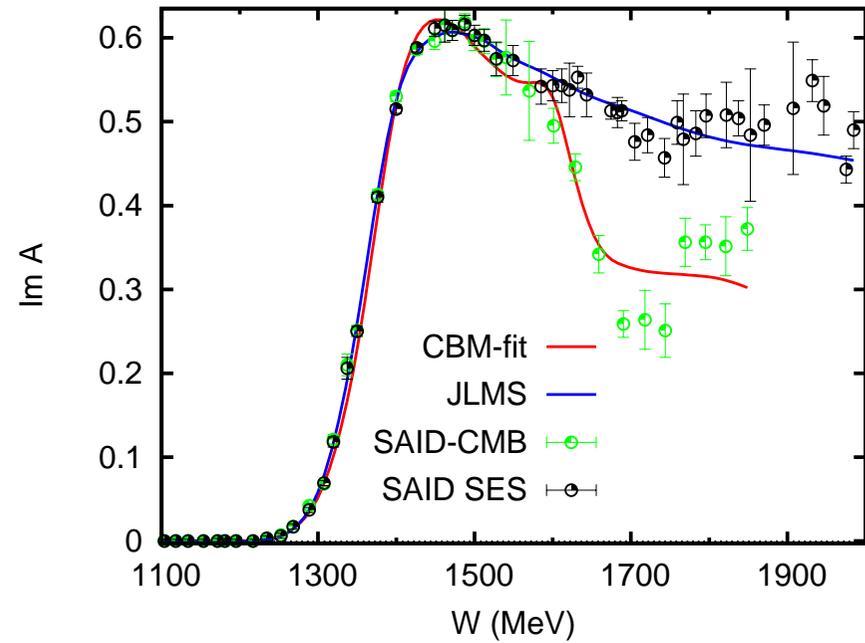
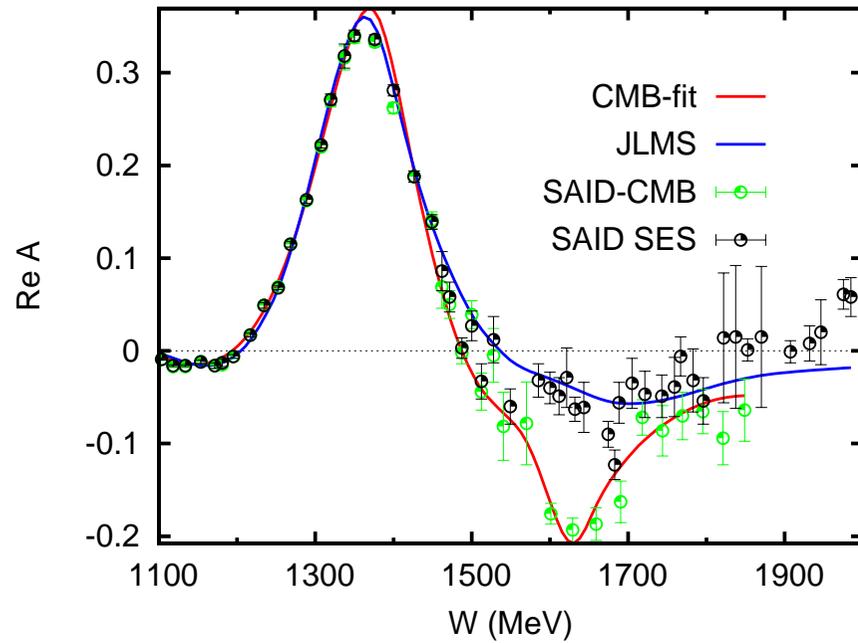
Model	$upuu\bar{p}\bar{p}$	$upu\bar{p}\bar{p}\bar{p}$	$uuuu\bar{p}\bar{p}$	$uuuuu\bar{p}$	$\chi_{pd}^2$
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	—	2.94
JLMS	(1357, -76)	(1364, -105)	—	(1820, -248)	3.55
$1N_01N^*-3p$	(1364, -81)	(1377, -129)	—	(1769, -132)	2.51

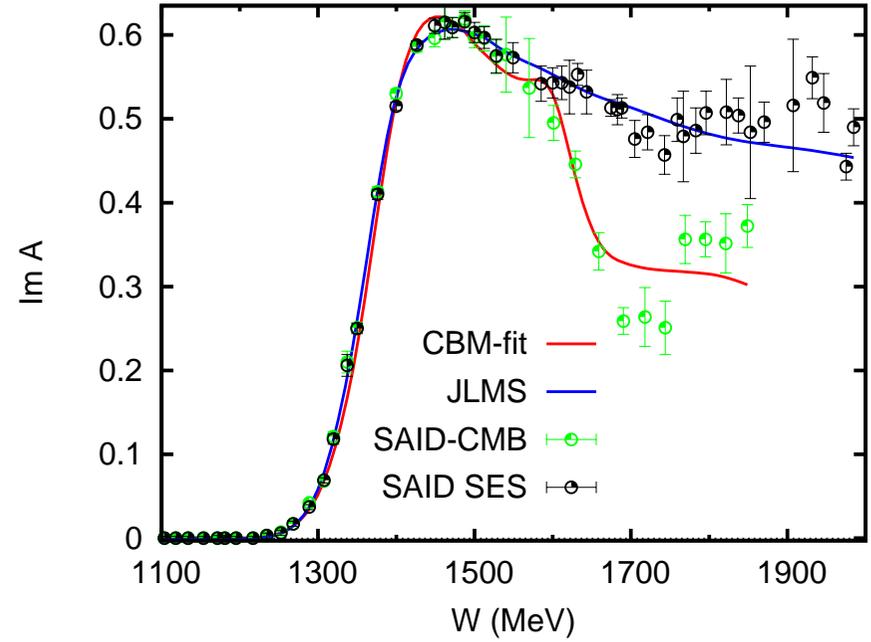
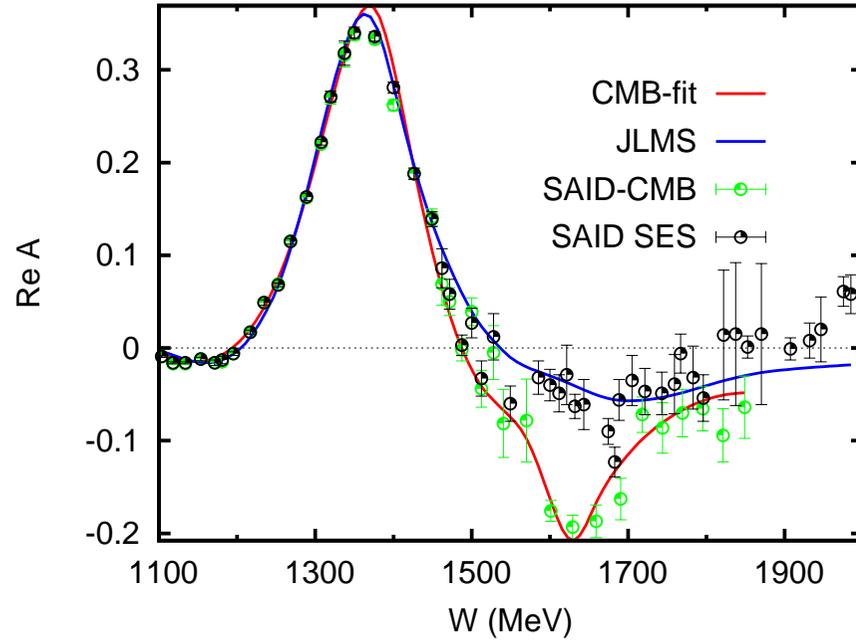
\* Roper two poles are stable !

## Different data

\* SAID-SES for  $W \leq 1.55$  GeV

\* CMB for  $W \geq 1.55$  GeV





Model	$upuupp$	$upuppp$	$uuuuup$	$uuuuup$	$\chi_{pd}^2$
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	—	2.94
JLMS	(1357, -76)	(1364, -105)	—	(1820, -248)	3.55
$2N^*-4p$ -CMB	(1379, -89)	(1386, -109)	(1613, -42)	(1913, -324)	4.91

\* Roper two poles are stable !

## Results 2 : Residues

\*  $\pi N$  scattering amplitude

$$\begin{aligned}
 F_{\pi N}(E)|_{E \rightarrow E_{\text{pole}}} &\sim (\text{kinematical factor}) \times \frac{\bar{\Gamma}(E_{\text{pole}}) \bar{\Gamma}(E_{\text{pole}})}{E - E_{\text{pole}}} \\
 &= \frac{R e^{i\phi}}{E - E_{\text{pole}}}
 \end{aligned}$$

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	[ $R, \phi(\text{deg.})$ ]		
	upuuupp	upuppp	uuuuup
JLMS	[37, -111]	[64, -99]	[20, -169]
$2N^*-3p$	[44, -95]	[75, -98]	[4, -128]
$1N_0-1N^*-3p$	[41, -108]	[88, -110]	[18, -155]

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\* Residue is more sensitive to details of amplitude !

## Summary 1

\* Roper two poles are stable against

- Large variation of parameters within EBAC-DCC ( $v \neq v_{JLMS}$ )
- Inclusion of bare nucleon state
- Fitting to different amplitude

provided amplitudes are precisely fitted to SAID SES for  $W < 1.5$  GeV

\* Average and range of Roper pole positions

$$E_{\text{pole}} = 1363_{-6}^{+6} - i79_{-3}^{+3} \quad (s_{\pi\Delta} = u) \quad , \quad 1373_{-10}^{+10} - i114_{-9}^{+15} \quad (s_{\pi\Delta} = p)$$

\* Higher mass resonance pole exists in our models

- Rather model-dependent position :  $E_{\text{pole}} = 1830_{-61}^{+130} - i190_{-108}^{+58}$

- It doesn't exist in SAID EDS

⇒ need for simultaneous fit to inelastic channel ( $\pi N \rightarrow \eta N$ )

S. Ceci et al., PRL **97**, 062002 (2006)

\* Possible pole in  $W \sim 1.6$  GeV (caused by oscillatory amplitude)

⇒ need for more precise  $\pi N$  data

\* Residue for  $P_{11}$  seems more sensitive to details of amplitude

⇒ Simultaneous fit to inelastic channels