

**Effect of  $3\pi$  unitarity on resonance poles  
from heavy meson decays**

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# Introduction

Why we care  $3\pi$  ( $\pi\pi K$ , etc.) scattering ?

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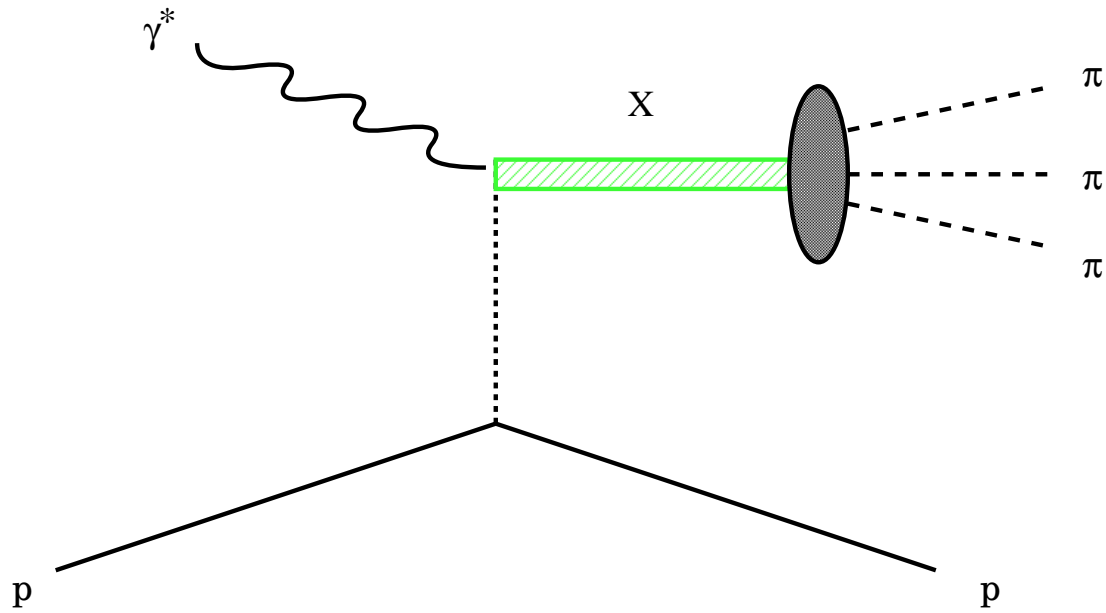
$\implies$  Information of interesting physics is hidden in  $3\pi$  from heavy meson decays

e.g.,

- \* Exotic (hybrid) mesons (e.g.,  $J^{PC} = 1^{-+}$ )      BNL(E852), JLab(GlueX)
- \* Light scalar mesons from D-decays      FNAL (E791, FOCUS), BABAR, CLEO
- \* CP-violation from B-decays      BABAR, Belle

GlueX experiment

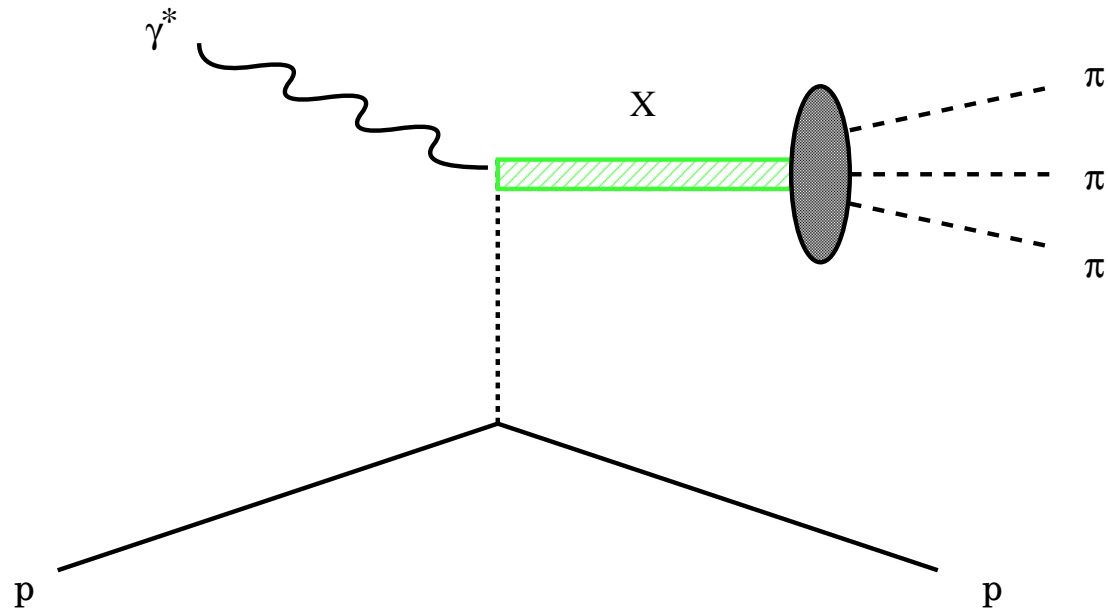
(JLab, planned after 12 GeV upgrade)



New EBAC project : Analysis of  $3\pi$  FSI in heavy meson decays

GlueX experiment

(JLab, planned after 12 GeV upgrade)



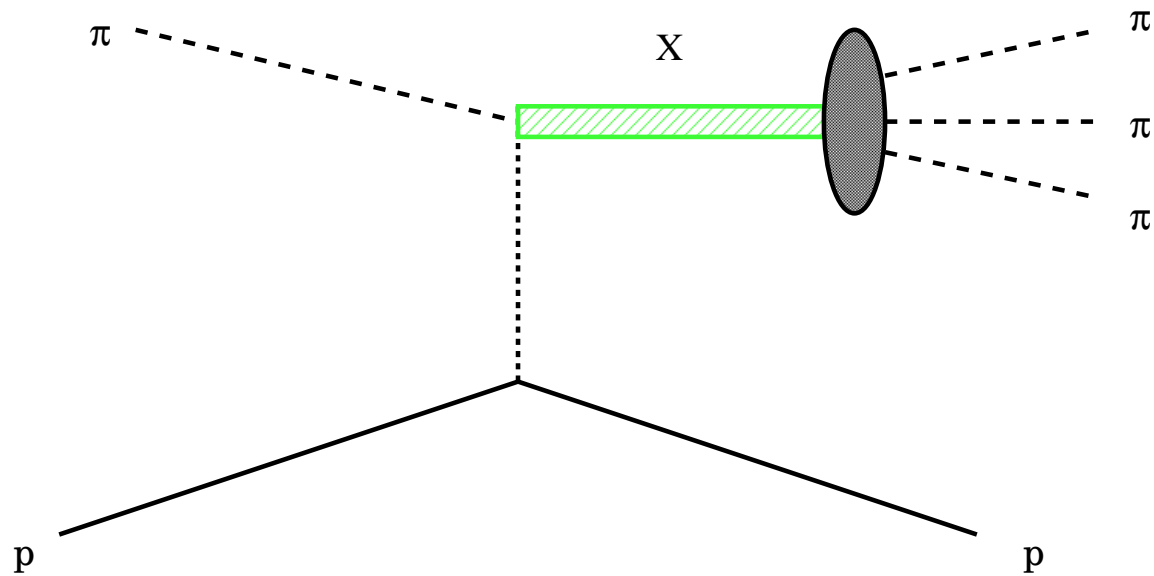
EBAC  $\longrightarrow$  EHAC ?

(Excited Hadron Analysis Center)

Previous analysis of  $3\pi$  decay of  $X$

E852 (BNL)  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$

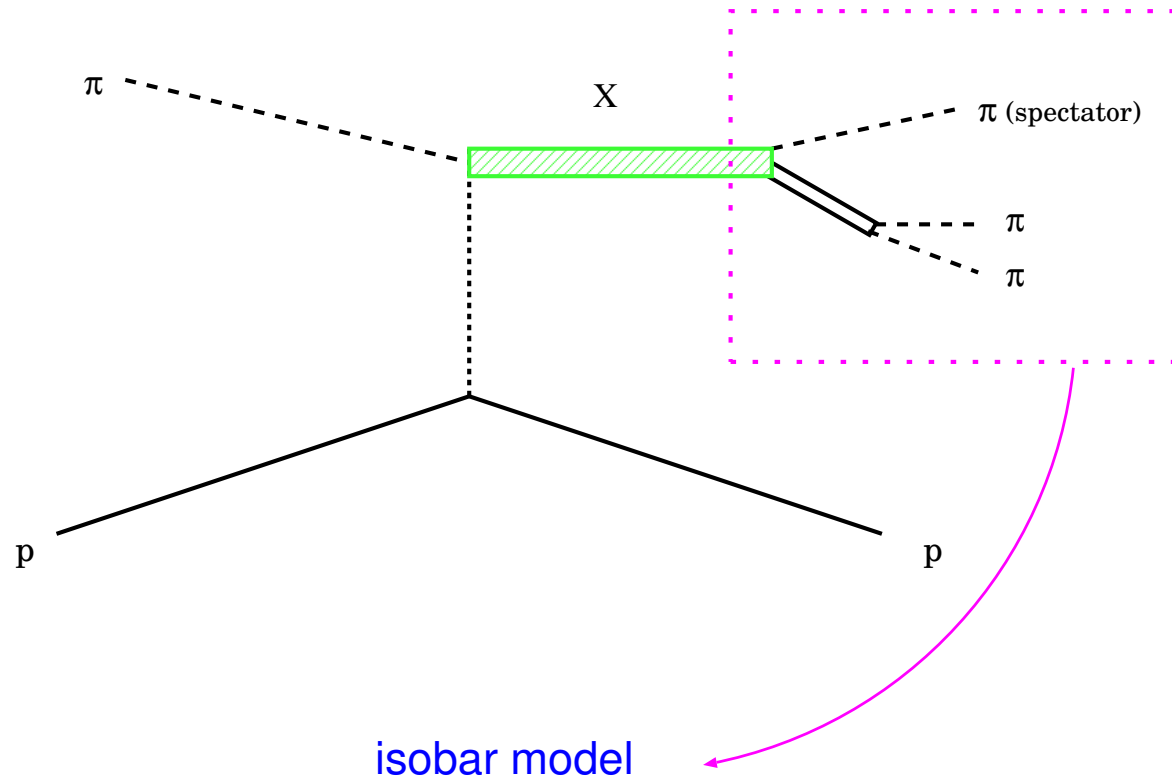
Chung et al., PRD **65**, 072001 (2001)



## Previous analysis of $3\pi$ decay of $X$

E852 (BNL)  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$

Chung et al., PRD **65**, 072001 (2001)



\*  $\pi\pi$  subsystem forms a resonance

\* 3rd  $\pi$  is a spectator



## Our objective

(This work)

Examine the effect of  $3\pi$  unitarity

$\implies$  address the basic assumption of the isobar model

(Long term)

Comprehensive development of reliable  $3\pi$  ( $\pi\pi K$  etc.) model

# Coupled-Channels Model

Matsuyama, Sato, Lee, Phys. Rept. **439**, 193 (2007)

$$\underline{X \rightarrow (\pi M_a) \rightarrow \pi\pi\pi}$$

Relevant channels  $(\pi M_a)$  :  $\pi f_0(600)$ ,  $\pi f_0(980)$ ,  $\pi \rho(760)$ ,  $\pi f_2(1270)$

$M_a$  is a resonance found in  $\pi\pi$  scattering

$\implies$  First task is to develop  $\pi\pi$  model

## Simple $\pi\pi$ model

Coupled-channel scattering equation for  $\pi\pi$  partial wave

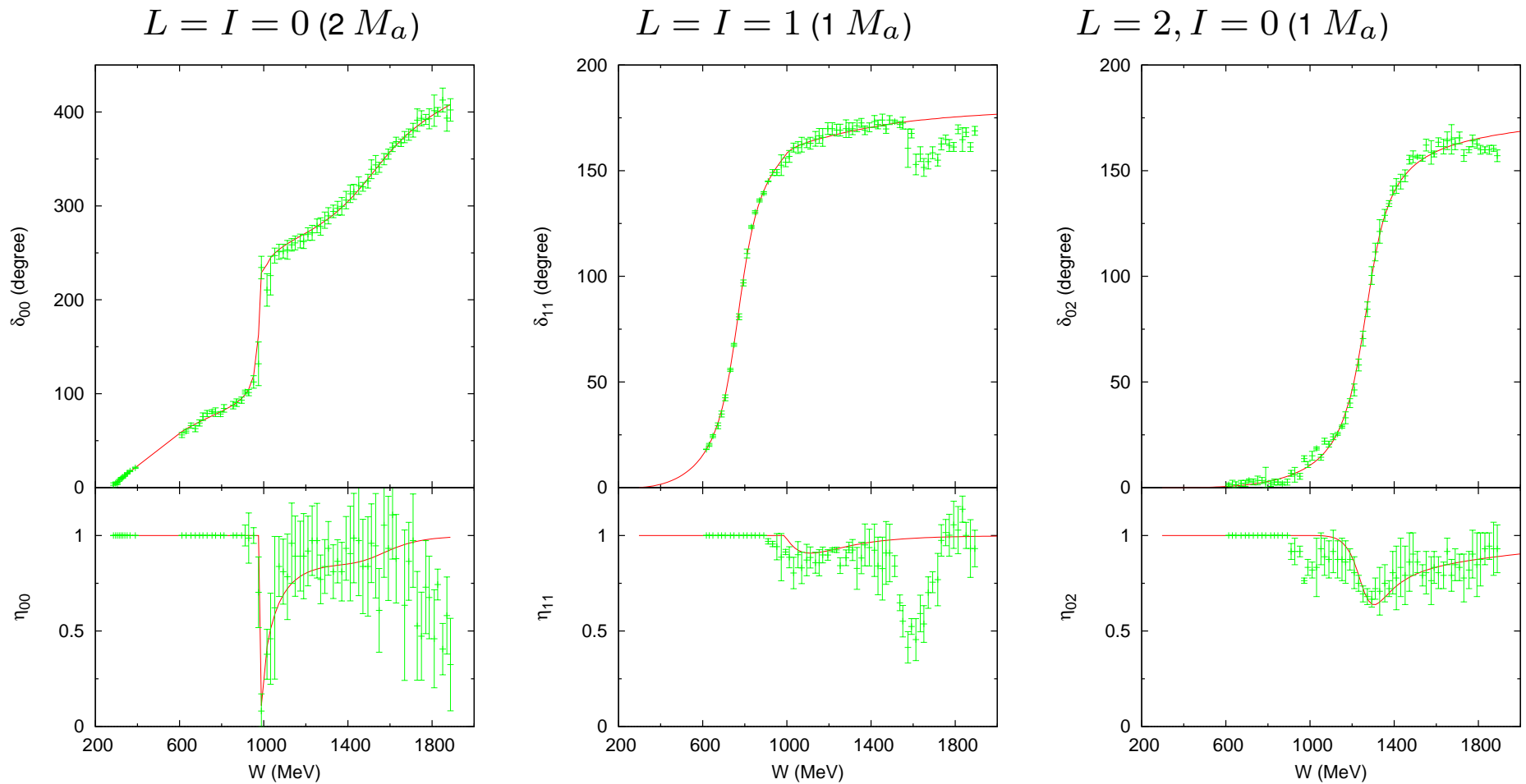
$$t_{i,j} = V_{i,j} + \sum_m \int_0^\infty q^2 dq V_{i,m}(k, q, W) \frac{1}{W - E_{1m}(q) - E_{2m}(q) + i\epsilon} t_{m,j}(q, k', W)$$

$(i, j, m = \pi\pi, K\bar{K})$

$$V_{i,j}(k, k', W) = \sum_a f_{M_a,i}(k) \frac{1}{W - m_{M_a}} f_{M_a,j}(k')$$

$$f_{M_a,i}(k) = \frac{g_{M_a,i}}{\sqrt{m_\pi}} \frac{1}{(1 + (c_{M_a,i}k)^2)} \left( \frac{k}{m_\pi} \right)^L$$

## Phase and inelasticity of $\pi\pi$ amplitude

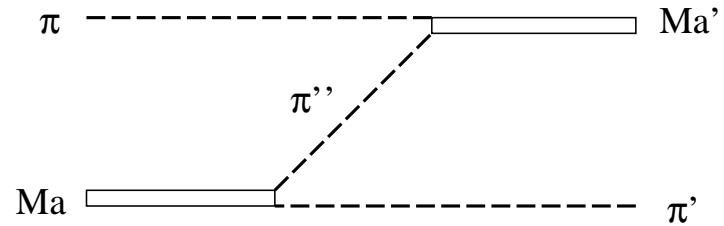


[Data: G. Gayer et al., NPB **75**, 189 (1974)]

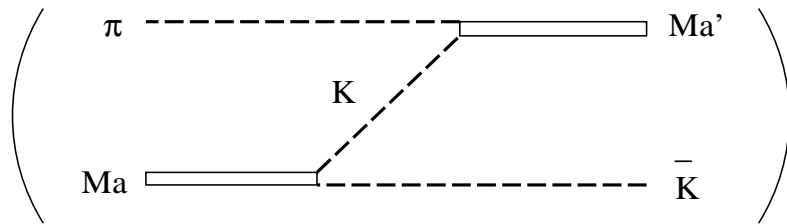
## Pole positions in $\pi\pi$ amplitude

	Re[ $M_R$ ] (MeV)		-Im[ $M_R$ ] (MeV)	
	Ours	PDG	Ours	PDG
$f_0$ (600)	437	400 – 1200	256	300 – 500
$f_0$ (980)	897	$980 \pm 10$	69	20 – 50
$f_0$ (1500)	1527	$1505 \pm 6$	237	$54.5 \pm 3$
$\rho$ (760)	769	$775.5 \pm 0.3$	81	$74.5 \pm 0.4$
$f_2$ (1270)	1249	$1275 \pm 1.2$	101	$92.5 \pm 1.3$

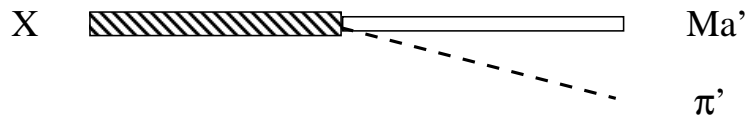
## Quasi two-particle ( $\pi M_a$ ) interaction



3  $\pi$  Z-graph



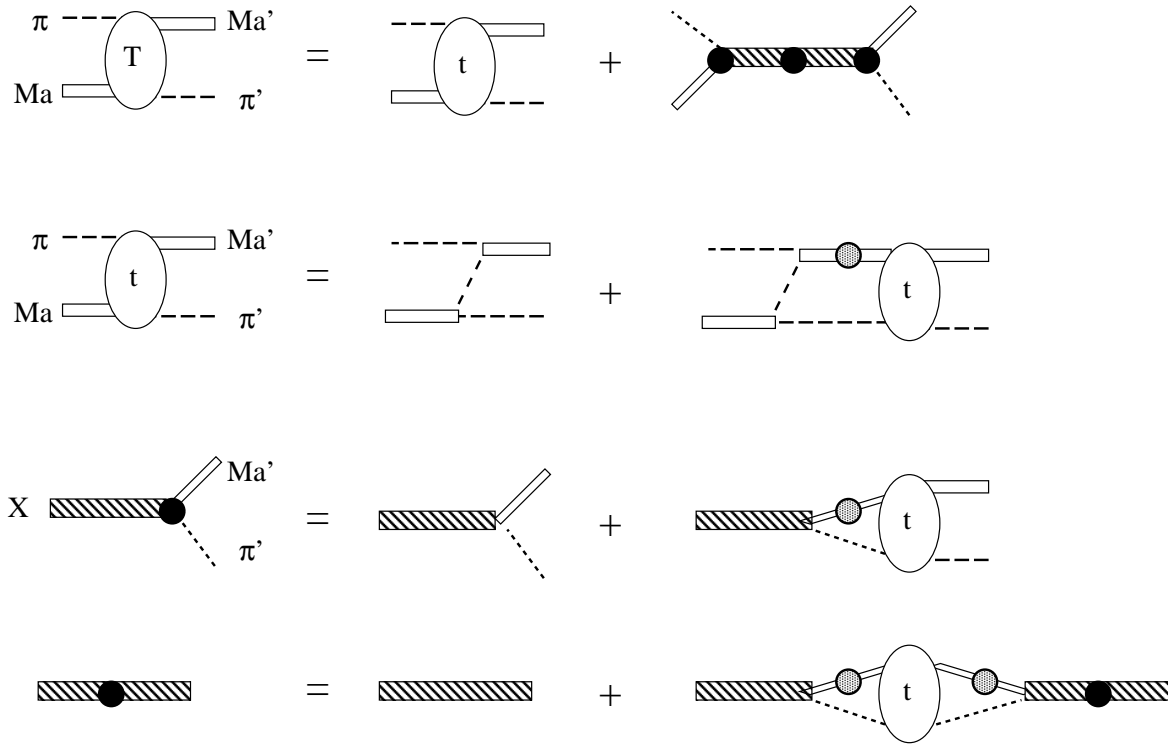
$\pi K \bar{K}$  Z-graph



$X$  graph

Z-graphs play an essential role to maintain the unitarity !

## Quasi two-particle ( $\pi M_\alpha$ ) scattering equation



Resonance pole :  $W - M_X^0 - \Sigma_X(W) = 0$

# Application

Effect of  $3\pi$  Z-graph (unitarity) on  $\pi_2(1670)$  pole

$$\pi_2(1670) : J^{PC} = 2^{-+}, \quad M_R = (1672 \pm 9.2) - (259 \pm 9)i \text{ MeV}$$

Branching ratios	$\pi_2(1670)$	$\rightarrow$	$\pi f_2(1270)$	56 %
		$\rightarrow$	$\pi \rho(760)$	31 %
		$\rightarrow$	$\pi f_0(600)$	11 %



Fit  $M_{\pi_2}^0, \Gamma_{\pi_2 \rightarrow \pi M_a}$  to the pole and branching ratios for  $\pi_2(1670)$

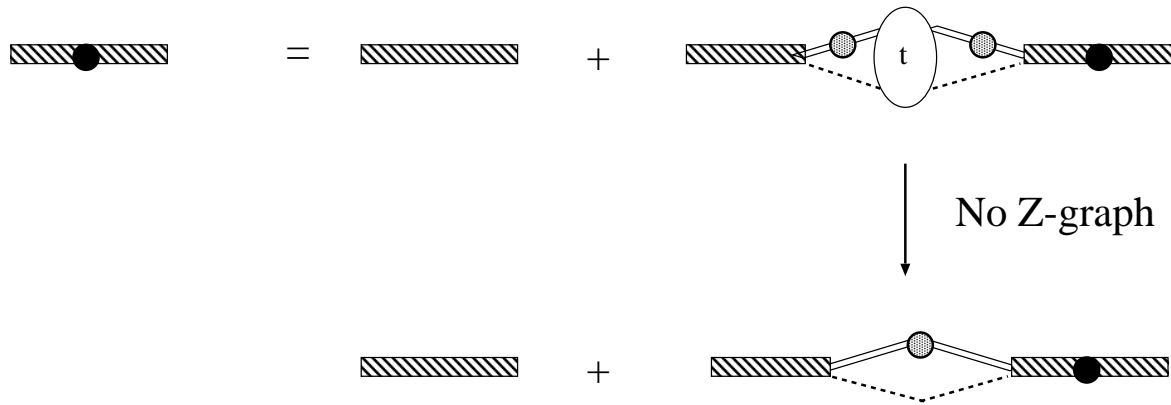
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**Pole search method** : Suzuki, Sato, Lee, PRC **79**, 025205 (2009); ibid, **82**, 045206 (2010)

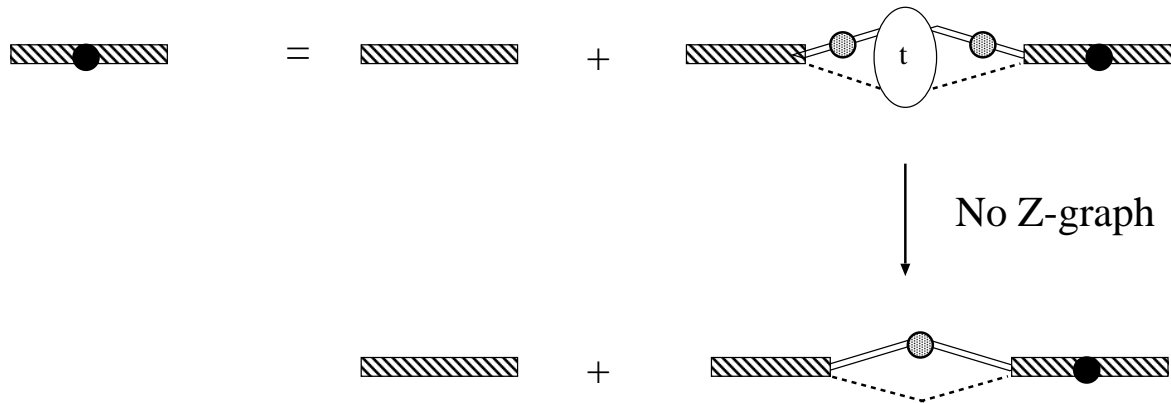
**Branching ratio** :

$$\begin{aligned} B(\pi_2(1670) \rightarrow \pi M_a) &= \frac{\Gamma_{M_a}}{\Gamma_{\text{total}}} \\ &= \frac{\text{Im} \left( \Gamma \frac{1}{W - E_\pi - E_{M_a} - \Sigma_{M_a}} \Gamma \right)}{\sum_{M'_a} \text{Im} \left( \Gamma \frac{1}{W - E_\pi - E_{M'_a} - \Sigma_{M'_a}} \Gamma \right)} \end{aligned}$$

# Effect of $3\pi$ Z-graph on the $\pi_2$ pole position



## Effect of $3\pi$ Z-graph on the $\pi_2$ pole position



w Z-graph  $M_R = 1672 - 259 i$  MeV

w/o Z-graph  $M_R = 1670 - 257.6 i$  MeV

- \* The effect of  $3\pi$  Z-graph is very small
- \*  $\pi K \bar{K}$  Z-graph could have a significant effect
- \* Effect could be seen more clearly in Dalitz plot

# Summary

$3\pi (\pi\pi K)$  from heavy meson decay contains information of interesting physics !

exotic (hybrid) meson, light scalar mesons, CP violation

EBAC ( $\rightarrow$  EHAC?) steps toward developing reliable  $3\pi$  model !

## Model

Coupled-channels quasi two-body ( $\pi M_\alpha$ ) scattering equation

- \*  $\pi\pi$  model
- \*  $3\pi$  Z-graph,  $\pi K \bar{K}$  Z-graph (essential for unitarity)
- \* bare resonance mechanism

## Effect of $3\pi$ unitarity on resonance pole

- \* Simple model
- \*  $\pi_2(1670)$ ,  $J^{PC} = 2^{-+}$ ; pole and branching ratio are fitted
- \* Shift of the pole due to  $3\pi$  Z-graph is very small

## Near future

- \* Inclusion of  $\pi K \bar{K}$  Z-graph
- \* Dalitz plot analysis of  $D \rightarrow \pi\pi\pi (\pi\pi K)$ 
  - $\Rightarrow$  Effect of  $3\pi$  unitarity on scalar resonances extracted from the analysis
- \*  $\pi\pi (\pi K, \text{etc.})$  interaction based on chiral Lagrangian